

Math Refresher for Adults

Michael Gannotti

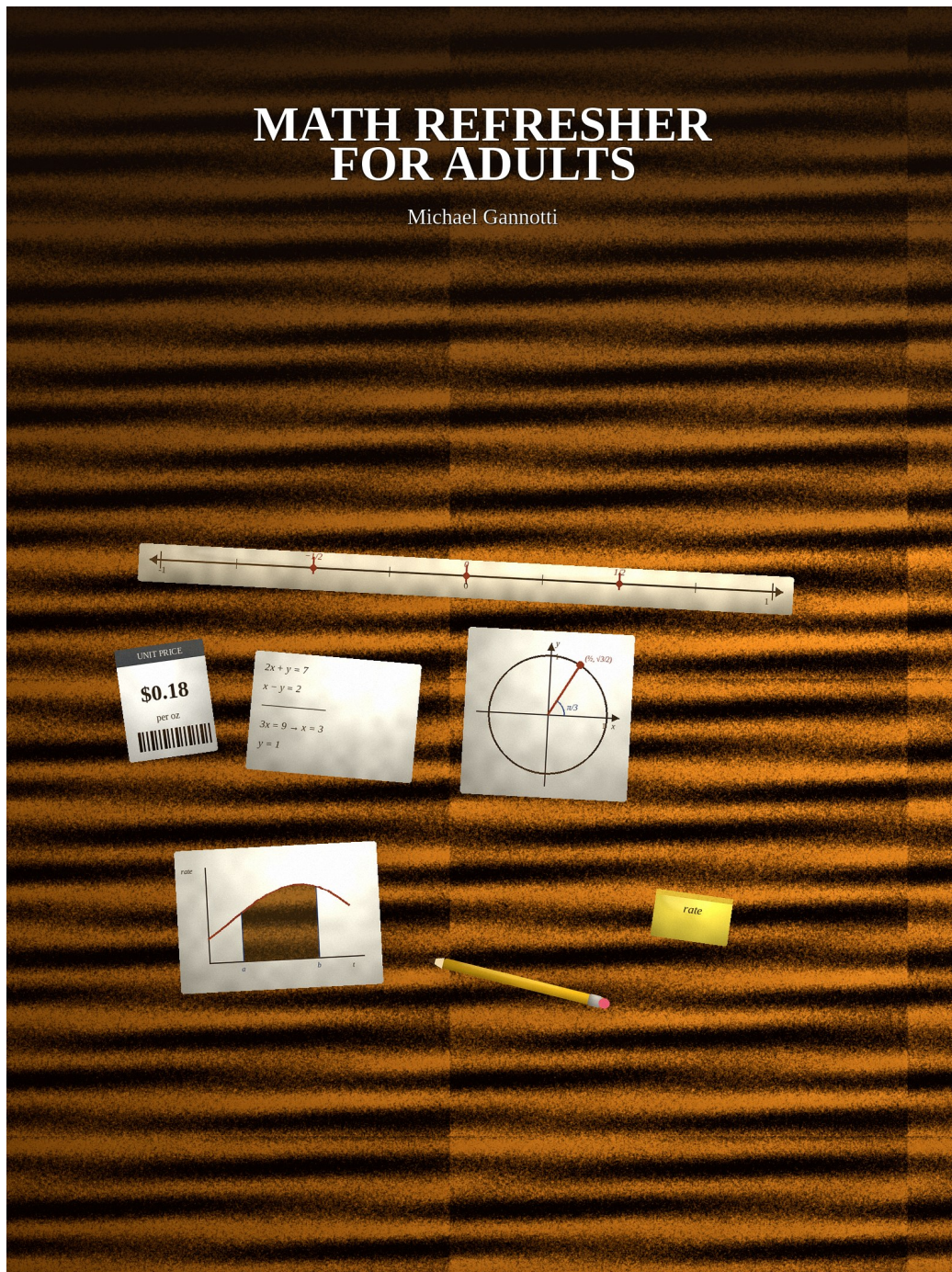


Figure 1: Cover

Math Refresher for Adults

Michael Gannotti

This illustrated edition is the teaching-manual of 5 September 2026. Endnotes run in one series. Chapter-opening images are original still-lives, not portraits of persons or children. Documents still constrain the facts.

Contents

- Welcome
- How to Use This Book This Week
- If You Only Remember Five Things
- The Refresh Hour
- Chapter 1 — Number sense, fractions, and decimals as numbers
- Chapter 2 — Percent and ratio
- Chapter 3 — Algebra refresh
- Chapter 4 — Geometry with reasons
- Chapter 5 — Functions and trigonometry
- Chapter 6 — Calculus edges
- Chapter 7 — Statistics for life and work
- Chapter 8 — Keeping track
- Chapter 9 — Resources
- A Note on Sources

Welcome

This book exists because usable mathematics is ordinary adult work — a number on a line, a base on a paycheck, a unit price on a shelf, a percent on a dashboard, a short reason for a measurement, a rate in units, a sample labelled as a sample. There is you, the object in front of you, and today’s idea. You already have a life. The hour is the work.

I wrote this for a capable, busy, willing adult. You may be on a train, at a desk, on a shift, in a trade bay, a clinic, a warehouse, a retail counter, or at a kitchen counter before work. Many adults feel rusty when they sit down to math they last saw in school. That feeling is common. It is not a verdict, and it is not proof of a math gene.¹ This book will make you fluent enough in today’s idea to hear a keyword percent that is not yet a schema, letter-moving with no structure, a formula with no reason, a sample mean offered as a population claim, a calculator output offered as sense, and a photo-to-key offered as practice — and to ask yourself one good question before you grab the answer.

You do not need to be a mathematician. You do need to hear “of means multiply” as a keyword grab, not a schema. Ordinary stakes live on the go and in deeper sits: a paycheck stub, an APR disclosure row (structure, not product advice), a unit-price tag, a cooking scale, a map scale, a DIY cut list, a sports box-score rate, a dashboard percent, a pharmacy label rate (units and rate only — not medical advice).² Money, health leaflets, cars, and work decks motivate. They do not replace the balanced equation, the ratio that means, or the rate meaning.

¹Maloney and Beilock; Beilock and Willingham on math anxiety: loads working memory; “not a numbers person” language is real. Not a clinical protocol in this book. Access date for URLs in these notes: 5 September 2026.

²CFPB: interest rate and APR are not the same; compare APR to APR. Pharmacy try-its use rate and units only — not dosing advice.

The cover says *Math Refresher for Adults*. The work is this commute, this desk, this weekend sit. A job, a degree, and years of experience are not a bicycle you now ride on every quantitative surface.³ You already compute in pockets of life and work. That is not the same as having rebuilt meaning and fluency across the skill strips this book teaches. Adulthood is not a math birthday.

A large adult skills survey places U.S. numeracy below the international average — mean 249 versus 263 — with about one in three U.S. adults at Level 1 or below.⁴ That survey is context for why a refresher exists. It is not a grade for you, and it is not a finding that this book raises a score. The stub, the shelf tag, or the number line on the table tomorrow is the work.

What a good refresh hour looks like

You sit down already knowing today's idea. An object is on the table — a stub, a shelf tag, a number-line scrap, a solved problem, a marked diagram, a dashboard tile, a recipe card. You retrieve a little. Then you talk yourself through a solved example. Then you try a parallel item unaided. Then you ask one good self-question and wait before the key. Then you leave a short sticky or dated line you could almost show a colleague. *The Refresh Hour* will teach this in full — micro-session, desk sit, weekend deep sit, talk-to-yourself shape, on-the-go and deeper try-its, and the AI rules once for adults. Later chapters will not reinvent it.

³Willingham, “Critical Thinking” (*American Educator*, 2007): thinking is not a bicycle skill. A job and a diploma are domains — not all quantitative surfaces.

⁴NCES/IES, *Highlights from PIAAC 2023* (ages 16–65): U.S. numeracy mean **249** vs international **263**; Level 1 or below **34%** (was **29%** in 2017). Survey, not a trial of this book. Not teen NAEP.



Figure 2: A refresh-hour setup

Adults can rebuild usable mathematics with meaning. Place by skill strip, not by birthday and not by diploma year.⁵ Conceptual understanding, computational fluency, and problem solving belong together.⁶ A person who can tip a bill in their head is computing. They have not finished the dual job of hearing a wrong turn and trying a parallel item before a tool finishes the attempt. We practice on surfaces you already meet.

If you used *Math for Little Thinkers*, *Math for Young Minds*, *Math for Emerging Adults*, or *Mathematics for Homeschooling* at a kitchen table, one pointer each, then this desk and this commute. Those books keep a parent in the room and a student section on the page. This book does not. You are the learner. The

⁵National Mathematics Advisory Panel, *Foundations for Success* (2008), Finding 15: place by skill strip, not "too old." Finding 4 / Critical Foundations still gate algebra.

⁶NMAP Finding 10: conceptual understanding, computational fluency, and problem solving belong together.

seven headings still work. Content names overlap in places — algebra, geometry, functions, calculus edges, stats. Audience and pedagogy do not. This is not a reprint of those lesson banks.

Critical Thinking for Adults is a different book, for a different subject. One pointer: that book teaches moves for deciding what to believe or do on ordinary work stakes, with a short written reason a colleague who missed the meeting could follow. This book teaches usable mathematics on ordinary life and work stakes, with talk-to-yourself checks and skill-strip practice. Shared house voice. Different pond. Adult numeracy survey numbers are not thinking-test brands. One pointer, then math.

What you will actually get

Each teaching chapter does seven jobs, always in the same order. You will learn why this week's idea is worth the struggle, and understand it yourself. You will get a session you can run this week: named try-its across **commute/lunch**, **desk-or-shift**, and **weekend deep sit**, and a talk-to-yourself box. Tools stay optional. "Done enough" places by skill, not age or diploma year.

The seven teaching chapters follow the skill strips, not twelve thin grade labels: number sense, fractions, and decimals as numbers; percent and ratio; algebra refresh; geometry with reasons; functions and trigonometry; calculus edges (optional, for the ready — not for everyone); statistics for life and work. Keeping track and resources come last.

Today you can put a stub, a shelf tag, a number line, or a dashboard tile on the table, name where the number lives or what the base is, try a parallel item, and leave one exit sticky.

What this book will not do

This book will not hand you 180 days of worksheets. Coverage is a map. Depth is the hour.

It will not treat "I'm not a math person" as destiny, a calculator alone as surrender, or calculus as a moral summit for every reader. It will not use teen test percentages as if they measured working adults. It will not invent Emerging Adults Math I,

Math Refresher I, or a certificate of fluency. The promise is a path through ordinary quantitative life — not a fake diploma.

It is not a reprint of the school-age math manuals named above, and it is not a sequel to *Critical Thinking for Adults*. It is not a curriculum sales pitch or a guaranteed score.

It is not medical advice, lending advice, or an investment pitch dressed as math. It does not rehearse live fights as content. We practice leaving a stub, a shelf tag, a map, a cut list, a dashboard tile, and a labelled rate. Politics, religion, identity fights, war, and medical culture-war stay off this table.

The promise

If you only remember one sentence, remember this: a path through ordinary quantitative life is the promise. A fake certificate is not.

Age is not placement. Diploma year is not a window that closed.

Three things have to hold. You understand today's idea well enough to hear a wrong turn — including from yourself. You retrieve, talk through a solved problem, and try a parallel item before any tool finishes the attempt. Any helper — including an AI tool — stays a supplement you host and constrain, not a partner for the first attempt, not a photo-to-key, not a secret friend, and not a paste of proprietary workplace data into a public chat.

You can do this. You do not have to know next week's idea today. You have to know this week's idea well enough to sit still while you struggle, then ask one good self-question. Start here. Read *How to Use This Book This Week*, then the five things, then *The Refresh Hour*. Open the chapter that matches the skill in front of you. You will know more after one chapter than you know this morning. You will have something to try on the commute, and something to try in a deeper sit. We can do this.

How to Use This Book This Week

Start at the skill in front of you, not on page one because a catalog, a diploma year, a birthday, or a well-meant colleague said so.

This book is a handbook you open to this week’s skill strip, not a novel you read cover to cover. The teaching chapters run: number sense, fractions, and decimals as numbers; percent and ratio; algebra refresh; geometry with reasons; functions and trigonometry; calculus edges; statistics for life and work. Keeping track and resources come last. Context bands — **commute/lunch**, **desk-or-shift**, and **week-end deep sit** — live *inside* each teaching chapter. They tell you the grain of the same idea. They do not tell you where *you* sit by age.

This book does not replace a community-college transcript if you need credit, or a working local method that already holds. If something is working, keep it. Use this book to rebuild meaning and fluency and to run the hour. The title is still the skill you are rebuilding — not Emerging Adults Math I, and not a fake certificate.⁷

Pick the chapter by skill, not by birthday

Open the chapter you think is right. Skip to **What “done enough” looks like** at the end of the *previous* chapter, or to **If it isn’t clicking** in the one you opened. If you can already do those checks unaided, you are too early in that chapter. If the checks from two chapters back are still failing, drop back.

If fractions still feel opaque — $2/3$ versus $3/5$, or where 0.2 lives beside $1/8$ — you

⁷Stranger-readable work products: dated practice note; worked example; skill-strip log; optional CLEP score report. Never Emerging Adults Math I, Math Refresher I, or “Certified Math Refresher.” Access date: 5 September 2026.

are still in the gate chapter, whatever your diploma.⁸ If you cannot yet name a base on a percent, you are not ready to skip to derivative meaning. If linear structure still collapses into letter-moving, stay in algebra refresh. Calculus edges are optional for ready adults. They are not a moral summit, and they are not required for every reader.

If you can already do those checks unaided, skip ahead. A publisher's chapter label is a scope, not a licence, and not a birthday.

How you use a teaching chapter

Each teaching chapter has the same seven headings, always in this order, so you always know where the explanation, the session, the named try-its, the talk-to-yourself box, and the tools box live.

Why this matters tells you what this idea unlocks. **Understand it yourself** gives one everyday picture, one precise picture, and three to five wrong answers you should be able to hear — including from yourself.

How to try it this week assumes the session shape from *The Refresh Hour*. It will not rebuild the hour. It will give you this week's named try-its — at least one on the go and one in a deeper sit — and a talk-to-yourself box with an exact opening question.

Practice that actually builds it names the try-its with time, materials, safety, and the move. Paychecks, shelves, maps, and dashboards *are* the practice. They do not replace the structure. **Tools, including AI** is optional and short. The rules live once in *The Refresh Hour*. **What “done enough” looks like** is how you leave: unaided work on the skill strip, not a perfect Tuesday and not a chatbot diploma.

You do not have to read the whole chapter tonight. You do have to read *this week's* idea before you put the object on the table.

The five-minute warm-up

Five minutes. Phone face down — or at least not on a solver. Object in reach.

⁸NMAP Finding 15 / Critical Foundations; NCES/IES PIAAC 2023: U.S. numeracy mean **249** vs international **263**; Level 1 or below **34%**. Survey context, not a grade for the reader.

1. Read today's idea until you can say it in one sentence.
2. Look at one object yourself. Ask, out loud, where the number lives, what the base is, or what stays constant. Stay off any tool until you have looked.
3. Glance at the "wrong answers you should be able to hear." Name the one you would have given last year.
4. Write one sentence you will actually ask yourself. Not a speech. Example: "What is the base?" Or: "Where does this number live?" Or: "Does the size make sense?"
5. Close the book to the try-it. You are ready.

If you are learning the idea *while* you are stuck mid-problem, you will grab the key too soon. Prepare first. Then sit still.

On the go and in deeper sits

Every teaching chapter names try-its across commute/lunch, desk-or-shift, and weekend deep sit — a stub skim, a unit-price compare, a dashboard percent, a cooking scale, a map/scale, a DIY measure, a solved-problem autopsy, a sample-versus-population sticky. The talk-to-yourself box travels. The live fight does not. If a real workplace number is too hot today, pick a composite stub, a shelf tag, a recipe, or a labelled household rate.

Math for Little Thinkers, *Math for Young Minds*, *Math for Emerging Adults*, and *Mathematics for Homeschooling* stay at their tables. This book does not reprint those lesson banks. Pictures show objects — not a classmate to copy. Notes live at the back.

Safety is ordinary. No public quiz. No undercover workplace audit. No medical or lending advice. No proprietary paste into public tools. A pharmacy try-it uses a labelled composite or household OTC label — rate and units only.

When to skip ahead

Skip ahead when this chapter's "done enough" checklist is already true *unaided*. Slow down when the same keyword grab repeats after a clear look at the base, or when a finished-looking chatbot step list still stands in for your attempt. **If it isn't clicking** will give you three likely causes and a next move.

A stranger-readable record is a dated practice note and a worked check — unit price, APR structure, dashboard, dose-rate units, scale — inside a skill-strip log or a college course name a stranger already understands (College Algebra, Intermediate Algebra, Quantitative Reasoning). Never Emerging Adults Math I. Never Math Refresher I. Never a chatbot certificate of fluency.⁹ Optional CLEP is a bridge when credit is the goal — not the spine of this book, and not mandatory.

You hold the account. A chatbot is never the only partner during the first attempt. Try unaided first. The full allow-and-ban list lives in *The Refresh Hour*.

For this week: pick the chapter by skill. Do the five-minute warm-up. Run the hour as *The Refresh Hour* describes it. Space across days — a few micros beat one massed Sunday dump.¹⁰ Stop peeking sooner than you want to.

⁹CLEP is optional and institution-set; sitting an exam is not finishing this book.

¹⁰Murray et al. (2025) spacing meta-analysis: $g \approx 0.28$. Useful study, not a promise that every adult will see the same result. Prefer retrieve-and-space over reread. Session minutes are pedagogical design, not a national minutes table.

If You Only Remember Five Things

Keep this page. The chapters will add wording, objects, and this week's questions. They will not replace these.

1. Skill, not age. Usable mathematics is meaning plus fluency on numbers, ratios, structures, reasons, rates, and claims you actually meet — not a talent you either have or lack, and not a birthday that closed.¹¹ Rust is ordinary. Anxiety is real. “I’m not a math person” is a stop-rule you do not have to obey. A forty-five-year-old with fraction fluency can learn algebraic structure. A twenty-eight-year-old without foundations cannot skip to calculus because a podcast said so. Place by skill strip and life demand. Start now. Give yourself one strip you somewhat know.

2. Concept, fluency, and problem solving together. Understanding what a quantity means, getting accurate flexible execution, and solving a problem that could come out otherwise belong in the same week — not as three rival camps.¹² Representation before procedure when the idea is new. Mixed practice when you already know when to use it.

3. Ask, wait, try. You talk to yourself. You ask one good self-question and wait before the key. Then you try a parallel item.¹³ Struggle before rescue: try unaided first. Ask, wait, hint, then look. You do not grab the answer at one second. You do not photo-to-key mid-attempt. A helper, including an AI tool, is a supplement you host. It does not sit in the chair during the first attempt. You hold the account. A chatbot is never the only partner.

¹¹NMAP Finding 15: place by skill, not “too old.” Maloney/Beilock: anxiety loads working memory — not a clinical protocol here. Access date: 5 September 2026.

¹²NMAP Finding 10: concept, fluency, and problem solving reinforce each other.

¹³Stahl think-time (1994): ~three seconds, classroom-origin — not an adult math RCT. Star et al. algebra guide (2015, grades 6–12): solved problems / structure / strategies — portable autopsy grain.

4. Reasons and meaning, not keywords. A fluent calculator output is not a check. A fluent chatbot step list is not your work during the attempt. Estimate, compute, check.¹⁴ Hear “of means multiply,” letter-moving with no structure, a formula with no reason, stacked percents added as rates, and a sample offered as a population — then ask a better question. Correct a wrong reason without crushing the attempt — applied to yourself. Silence in the face of your own wrong reason is not kindness.

5. A path, not a fake certificate. Practice on your stub, shelf tag, dashboard, cut list, recipe — commute/lunch, desk-or-shift, weekend deep sit. Ordinary stakes only. No live controversy. No medical or lending advice dressed as math. COPPA’s under-13 line does not apply here. That is not a math birthday and not a licence for an unsupervised chatbot as a secret friend.¹⁵ A path through ordinary quantitative life is the promise. A fake diploma is not. Optional CLEP is a bridge when credit is the goal — not the spine. There is no Emerging Adults Math I. There is no window that closed when you turned twenty-five.

You ask, wait, and try before the tool. You hold the account. Five things. Then sit down and try.

¹⁴Reyna and colleagues: percent meaning matters. Estimate, compute, check. Full AI rules live in *The Refresh Hour*.

¹⁵COPPA under-13 does not apply — not a companion licence. Never Emerging Adults Math I, Math Refresher I, or a chatbot fluency certificate.

The Refresh Hour

The hour has a shape. Learn it once. Later chapters will give you today’s idea. They will not rebuild this hour. When a chapter says “run the session,” it means this.

You do not need a school bell. You need the idea on the table, a try that could come out otherwise, you doing the retrieving and the trying, a record you could almost show a colleague, talk-to-yourself about meaning, just-in-time naming when the representation needs a word, and stop. Three shapes carry the week: a **micro-session** on the commute or at lunch (~15–20 minutes), a **desk sit** (~25–40 minutes), and a **weekend deep sit** when needed (~45–75 minutes). Space them across days when you can. Cramming one Sunday feels productive. Spacing feels harder and usually wins more for retention.¹⁶ Three to five micros plus one desk sit in a week, with an occasional deep sit, is the honest ambition — not under-running of a proven adult commute RCT, because there isn’t one at this grain.

Sit down having already done the five-minute warm-up from *How to Use This Book This Week*. The object changes. The shape does not.

¹⁶Lindsay A. Murray and colleagues, mathematics spacing meta-analysis (2025): spacing versus massing overall $g \approx 0.28$ (27 studies). Isolated and embedded estimates differ; testing versus restudy was smaller and the confidence interval crossed zero — hedge on retrieval as a sole engine. Useful study, not a promise that every adult will see the same result. Access date for URLs in these notes: 5 September 2026. Session minutes in this chapter are pedagogical design, not a national adult-math-minutes table. What Works Clearinghouse DCMP findings (+21 / +8) are community-college developmental-education progression outcomes — not a solo commute proof.

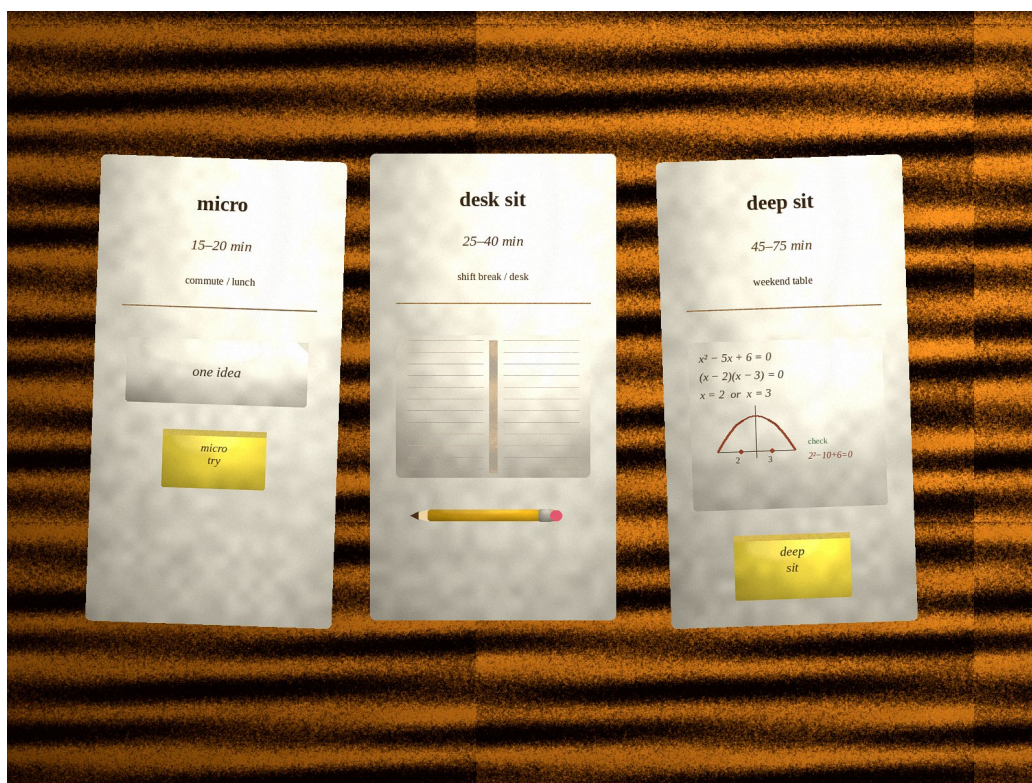


Figure 3: Three session shapes

1. Micro-session (~15–20 min, commute/lunch)

One idea. Brief feedback. No high-stakes countdown timer as default rigor. Interruptible without shame. If the session becomes scrolling, it is not a session — set a fifteen-minute clock, one object, stop.

Retrieve (2–3 min). Close the book. Write or say what yesterday’s idea was. One magnitude flash, one base, one name for a number. Prefer retrieve over reread.¹⁷

Solved-problem autopsy (5–7 min). Look at a worked twin — Star and colleagues’ algebra practice guide licences this grain for self-study; the guide is grades 6–12 origin, portable here, not an adult RCT.¹⁸ Cover the next line. Ask:

¹⁷John Dunlosky and colleagues’ technique ratings: practice testing and distributed practice rate high utility relative to rereading. Prefer retrieve-and-space over highlight-and-hope.

¹⁸Jon R. Star and colleagues, *Teaching Strategies for Improving Algebra Knowledge in Middle*

What was given? What stayed constant? Where could someone go wrong? Name the structure before you praise the answer.

Parallel try unaided (5–7 min). Same structure, new numbers. Hands off the key. Hands off the camera solver. If you stall: ask yourself one question, wait, hint, then look — not the reverse.

Exit sticky (1 min). One line dated: the idea, one check, one wrong twin you now hear. Put the phone away.

2. Desk sit (~25–40 min)

Warm-up retrieval mixed with last week’s strip. Then representation first — table, graph, diagram, units — for eight to twelve minutes before procedure. Short model or incomplete worked example (about five minutes); fade a step so you finish it. Two tries plus one mixed interloper (ten to twelve minutes) so you practice *when* to use the move, not only *how*. One good self-question, wait, written reason (about three minutes). Exit ticket: unaided, book closed.

Start from an artifact when you can: a stub already in your bag, a shelf photo you took, a dashboard tile with proprietary names blurred, a cut list, a recipe card. Name the quantity. Name the decision the number supports. Sense-check before you trust the calculator.

3. Weekend deep sit (~45–75 min, when needed)

Open with a diagnostic wrong — the keyword percent, the letter-moving twin, the formula without reason, the sample offered as population. Longer solved problem with structure talk. When two methods are legal, compare alternative strategies (same Star guide, recommendation on alternatives — moderate evidence tier, school-labelled).¹⁹ Mixed set of four to six with interlacing. Run the chapter’s

and High School Students (What Works Clearinghouse practice guide, 2015): Recommendation 1 solved problems (minimal evidence); Recommendation 2 structure (minimal); Recommendation 3 alternative strategies (moderate). Grades 6–12 origin. Portable autopsy grain for self-study; not an adult refresher RCT.

¹⁹Jon R. Star and colleagues, *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (What Works Clearinghouse practice guide, 2015): Recommendation 1 solved problems (minimal evidence); Recommendation 2 structure (minimal); Recommendation

“done enough” checklist. Schedule next week’s three micro cues on a calendar — not vibes.

A deep sit that becomes “finish chapter 7 of a PDF” is a textbook dump wearing a weekend hat. One structure deeply beats seven skimmed.

The talk-to-yourself shape, once

Later chapters fill the exact opening. The shape lives here.

Opening question. Authentic. About today’s strip. No recitation answer already written on your page. Use one: Where does this number live — and what is one name that matches it? What is the base — and am I finding part, whole, or rate? If this is linear, what stays constant — the rate or the second difference? What are we given? What must we show? What bridge gets us there? Is this table linear, quadratic, or exponential — how do I know? As the interval shrinks, what is this average rate becoming — in what units? Is this number about the sample, or a claim about the population?

Three to five follow-ups. Pick; you do not need all five every Tuesday. Meaning: What do you mean, *base*? Representation: What does the table, line, or diagram show before the formula? Alternative: Is there another legal strategy? Size-check: Does the size make sense — estimate first? Revise or diagnose: You may change your mind — where could someone go wrong?

How to wait alone. After you ask yourself, cover the answer. A slow three is a convention, not a sacrament — Stahl’s think-time is classroom-origin; Rowe’s wait-time is elementary science class; neither is an adult math RCT.²⁰ After you stop mid-reason, wait again before peeking. If the silence is hard, look at the stub, the shelf tag, the graph, or the clock — not at the phone’s answer key. Rescue at one second is still rescue. Abandonment for forty minutes of scrolling is not a micro-session.

What a stuck silence usually means. The question was vague; wait-time after

3 alternative strategies (moderate). Grades 6–12 origin. Portable autopsy grain for self-study; not an adult refresher RCT.

²⁰Robert J. Stahl, “Using ‘Think-Time’ and ‘Wait-Time’ Skillfully in the Classroom,” ERIC Digest ED370885 (1994): about three seconds as a convention. Mary Budd Rowe, “Wait-Time and Rewards as Instructional Variables,” NARST paper, ERIC ED061103 (1972): elementary science class. Neither is an adult math randomized trial. Translation here: pause before lookup.

the question was zero; you are searching for a vocabulary word; you are guessing what a textbook would want; you offered a keyword where a schema was asked for; you think “math person” means leave; you think a finished-looking chatbot paragraph is already a check; anxiety is eating working memory — shorten the session, win one magnitude item, come back tomorrow.²¹ Your job is to pose, wait, and point back at the object — not to fill the silence with the right answer from a photo-solver.

Do not stack five questions. Do not perform Socrates. Do not switch to a live controversy. Do not photo-to-key mid-attempt.

The on-the-go try-it shape, once

Time. Fifteen to twenty minutes, then stop.

Materials. A paycheck stub, an APR disclosure row, a shelf tag, a dashboard screenshot (blur proprietary names), a box score, a recipe, a labelled composite pharmacy label or a household OTC label you already use.

Safety. Curious, not courtroom. Not medical advice. Not lending or investment advice. No proprietary paste into public tools. No public humiliation of yourself or a colleague. Noticing is not a sting.

The move. Name where the number lives, what the base is, what the rate is, or whether the size makes sense. Estimate first. Then compute. Then check.

Types you will meet, not a lesson bank: paycheck or APR skim (name the base; one structure row); unit-price compare; dashboard percent with old value explicit; sports rate from a box score; cooking scale; pharmacy label rate — units and rate only.

The deeper try-it shape, once

Desk or weekend. Same safety. More room for a diagram, a wrong twin, and a written reason.

²¹Erin A. Maloney and Sian L. Beilock; Sian L. Beilock and Daniel T. Willingham on math anxiety: loads working memory; socially acceptable “not a numbers person”; college estimates vary by setting. Short sessions and early wins — not a clinical protocol in this book.

Types you will meet: map or scale; DIY measure and cut list; APR amortization row (interest versus principal portion — literacy, not product advice); unit-circle card; rate-to-accumulation story; sample-versus-population sticky; wrong solved autopsy; function-family sort.

These are not the hour: a gotcha audit of a workplace; a live-controversy field trip; a 180-day worksheet dump; a photo-to-key loop called “practice.”

The AI rules, once

Later chapters will point here. You talk to the page and to the check. A chatbot is never the only partner during the first attempt. An agent is not the math partner for the first attempt. The unaided try is still yours. You hold the account. COPPA’s under-13 line does not apply to an adult reader. That is not a math birthday and not a licence for unsupervised chat or a secret friend.²²

After you have tried, you **may**:

- Explain this idea *to yourself* from a **named** lesson (title, page, chapter object, today’s micro-goal).
- Ask for **extra isomorphic practice**, with answers on **YOUR** page (practice sheet plus key).
- Use a labelled **SCRIPT** only **after you tried** — a short spoken or written sequence for *you* to say, not a paragraph to copy as your work.
- Ask for a **hint** after an attempt. A hint is not the finished answer.
- Ask a tool to **diagnose work you already did** (crop to paper or note — no face).
- Use a labelled **calculator**, Desmos, GeoGebra *dynamic* tool, Wolfram, or CAS **after** the try to check or explore.

You **may not**:

- Paste tonight’s worksheet or an employer sheet and ask the tool to complete it.
- Ask “what’s the answer” or “just give me the steps” during the attempt.

²²Children’s Online Privacy Protection Act, 15 U.S.C. §§ 6501 *et seq.*; 16 C.F.R. Part 312. Bright line: children under 13. An adult reader is not under 13. That is an account fact, not a companion licence.

- **Photo-to-key** — Photomath-class loops, Mathway, Symbolab, camera solvers, GeoGebra Math Solver photo flow. The loop is the ban. Vendor “designed for learning” language does not make a scan-to-solution your practice.
- Park an **unsupervised chatbot** as your only partner during the attempt.
- Treat “I am an adult / I hold the account / COPPA is off” as a permission slip or companion licence.
- Ask the tool to **write your worked solution** as yours.
- **Invent facts** (the receipt total, the axis start, the contract rate).
- Paste **proprietary employer data** into a public consumer tool. Strip proprietary rows to invented isomorphic numbers on YOUR practice sheet, or use an enterprise tool your employer allows. Open *your* acceptable-use page. This book will not invent a federal workplace chatbot statute for you.
- Hand yourself an **agent** (Hermes Agent, Grok Bot, or any cloud-computer worker) as a math partner for the first attempt.
- Print a **certificate of fluency** from a chatbot.
- **Detector-grade** your own unaided write-up. Detectors have mislabelled a large share of human writing as machine-written in published tests; a detector score is not a grade for your practice note.²³
- Trust a model bibliography without opening the sources. Language models invent citations.²⁴

One mechanism sentence, labelled ON-AGE-adjacent, then teach. In a high-school math field experiment (grades 9–11, nearly 1,000 students, Turkey), an unguarded chatbot made practice look better (+48%) and left students worse on the closed exam (−17%); a guarded tutor that withheld the answer made practice look much better and left unaided performance about the same as no AI.²⁵ **Not an**

²³Weixin Liang and colleagues, “GPT Detectors Are Biased against Non-Native English Writers,” *Patterns* 4, no. 7 (2023): average false-positive rate on human TOEFL essays **61.22%** in that study.

²⁴William H. Walters and Esther Isabelle Wilder, “Fabrication and Errors in the Bibliographic Citations Generated by ChatGPT,” *Scientific Reports* 13 (2023): **55%** of GPT-3.5 citations and **18%** of GPT-4 citations were fabricated in that study. If you asked a tool for a paper, open the paper.

²⁵Hamsa Bastani, Osbert Bastani, Alp Sungu, Haosen Ge, Özge Kabakcı, and Rei Mariman, “Generative AI without Guardrails Can Harm Learning: Evidence from High School Mathematics,” *Proceedings of the National Academy of Sciences* 122, no. 26 (2025): e2422633122. Field experiment, nearly 1,000 Turkish high-school math students (grades 9–11): unguarded chatbot raised assisted practice about **48%** relative to control, then cut unaided exam performance about **17%**; guarded tutor raised practice sharply and left unaided performance about the same as control.

adult RCT. The lunch-break rule it supports: the model may prepare you and the next problem; it may not do your problem during the attempt. Close the window for the unaided try.

Names, in ordinary language. Hermes 4 is a chat model. Hermes Agent is a different runtime — a computer that can keep going. Grok is a chat assistant. Grok Bot is a cloud-computer agent. A public-school partnership is a third Grok. Game artist makes pictures. None of these is a math refresher class. A chatbot answers in a window. An agent has tools and can keep going. Neither is a partner for the first attempt. There is no Grok Bot education SKU on pages this book opened.

Not a secret friend. A tool that talks in the first person is still a tool. It does not get a private channel, a promise to keep secrets from a manager or a colleague, or a seat in the chair during the first attempt.

You can practice this entire book with no AI. Many readers will.

A life of the habit

Growing fluency is more miles on the same road, not a new faculty and not a math birthday. Revisit the same strip next Tuesday. Mix last week's item into this week's warm-up. Space across days. Stop when the exit sticky is honest. Placement is by skill strip, not diploma year. A fake certificate is not. Optional CLEP is a bridge when credit is the goal — sitting an exam is not finishing this book.²⁶ There is no window that closed when you were hired or when you turned thirty. The honest pitch is *often enough that the question is familiar*.

Anxiety is common on adult education pathways and can disrupt working memory. It is not proof of low ability. This book uses short sessions, early magnitude and percent wins, spaced practice, and refuses timed humiliation as default rigor. That is posture, not a therapy manual. If a session is eating you, shorten it, win one item, return tomorrow.

ON-AGE-adjacent — not an adult RCT. Mechanism: crutch. Close the window for the attempt.

²⁶College Board CLEP: College Algebra, Precalculus, and Calculus listed at **\$97.00** each plus administration or remote-proctor fees (opened 5 September 2026). Credit-granting scores are institution-set. Optional. Never Emerging Adults Math I. Never Math Refresher I. Never a chatbot certificate of fluency. Hermes 4 is not Hermes Agent. Grok is not Grok Bot and is not a public-school Grok-as-tutor partnership. There is no Grok Bot education SKU. Agents are not chatbots. Game artist makes pictures. Open *your* employer's acceptable-use page when workplace examples appear.

A Tuesday, said plainly

Here is an illustration, not a reported workplace. Someone opens a stub on the train, names the base on one percent row, waits before peeking at a worked twin, tries a parallel tip estimate unaided, and leaves a dated sticky: *base = gross; stacked 20% then 10% $\neq$ 30% off*. Later chapters will say: run the session as in *The Refresh Hour*.

Chapter 1 — Number sense, fractions, and decimals as numbers

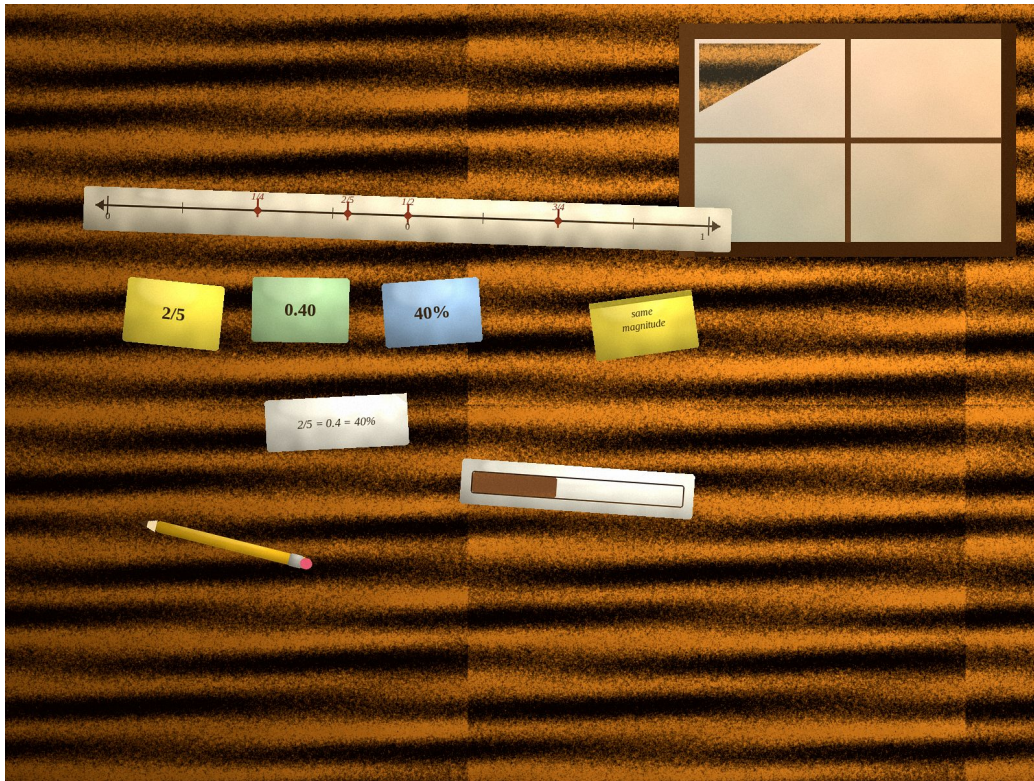


Figure 4: A still-life: number-line scrap with stacked fraction–decimal–percent stickies; recipe card and cut-list scrap. No people. No logos. No readable UI.

Why this matters

You already use numbers every day. A receipt total. A half-hour left on a break. A cut that needs to be three-quarters of a board. A dashboard tile that says 0.18. A temperature that dips below zero. The trouble is not that adults “cannot do math.” The trouble is that magnitude goes soft when life is loud — and soft magnitude makes every later strip harder.

This chapter is the gate. Algebra, percent schemas, and multi-step workplace numeracy all lean on the same quiet skill: knowing where a number lives, and being able to rename it without losing its size. If two-thirds and three-fifths still feel like rival slogans rather than points on a line, skipping ahead does not save time. It only moves the fog.

Many adults want the “real” topics — linear models, dashboard percents, even a calculus edge — while $2/3$ versus $3/5$ still feels opaque under a little pressure. That is not a character flaw. Fractions, decimals, and percents are named in research as critical foundations for later algebra for a reason: they are the shared language of part, whole, and size.²⁷ Multi-step proportion work sits above the most basic adult numeracy ceiling in large surveys; magnitude and part-whole are not optional garnish.²⁸ You do not need those survey numbers as a grade. You need them as a sober reminder that this strip is load-bearing.

You do not need a pizza chapter from childhood. You need adult objects: cuts, time fractions, debt as direction, recipe scales, shelf decimals that are really percents in disguise. The number line is still in play. Benchmarks still matter — 0, $1/2$, 1, and the familiar neighbors $1/3$, $1/4$, $1/5$, $1/8$, $1/10$. Fractions, decimals, and percents are names for the same magnitudes — not three unrelated school subjects you “finished” in different years.

Place yourself by skill, not by diploma year. Having sat Algebra II once is not a magnitude test. Feeling rusty is ordinary. Anxiety around “simple” comparisons

²⁷National Mathematics Advisory Panel, *Foundations for Success* (2008): Critical Foundations include fractions (with decimals, percents, and negatives) as gates to algebra; Finding 10 — conceptual understanding, computational fluency, and problem solving mutually reinforce; Finding 15 — developmental appropriateness is largely contingent on prior opportunities to learn; place by skill, not “too old.” Access date for URLs in these notes: 5 September 2026.

²⁸NCES/IES, *Highlights from PIAAC 2023* (ages 16–65): U.S. numeracy mean **249** vs international **263**; Level 1 or below **34%**. Multi-step proportion work sits above the most basic adult numeracy ceiling in that survey pond. Survey context, not a grade for the reader and not a trial of this book. Not teen NAEP.

is real and common; it loads working memory; it is not proof of a math gene; and this book is not a clinical protocol.²⁹ Short sessions and early wins belong here on purpose. Untimed magnitude beats a countdown timer dressed up as rigor.

What this chapter unlocks is usable size. When you can place a number, rename it, and catch a digit-size error in your own head, percent stops being a keyword hunt and algebra stops being letter theater. That is worth the struggle for ordinary adult life — paycheck structure later, unit price later, a cut list this weekend, a dashboard tile on Monday.

Rust is not failure. Refresh means rebuild usable meaning and fluency. Start where the line is soft. The rest of the book will still be there when size is solid.

One more reason this strip earns a full chapter: returners often arrive procedure-only (“I memorized convert steps, then forgot”) or discovery-frustrated (“we talked about shares, I never got fluent”). Conceptual understanding, computational fluency, and problem solving belong together.³⁰ In a paperback, conceptual means: What quantity? Where does it live? What rename keeps the size? Procedural means accurate, flexible execution of the renames and comparisons you need. Worked examples glue both. This chapter refuses choosing sides.

Understand it yourself

Everyday picture. Think of a kitchen ruler or a scrap of paper with a line from 0 to 1. Every fraction you care about this week — half a cup, three-quarters of an hour, two-fifths of a tank — lives somewhere on that line. A decimal is another name for a point on the same line. A percent is yet another name for a point on a related scale that runs to 100. When someone says “0.2 looks smaller than $\frac{1}{8}$ because 2 is small,” they are reading digit shapes instead of size. The line refuses that trick.

Hold a mental picture of three stickies stacked on one point: **$\frac{2}{5}$, 0.4, and 40%**. Same location. Three jackets. If those three ever feel like different animals, slow down and put them back on the same point before you chase any procedure.

²⁹Erin A. Maloney and Sian L. Beilock, “Math Anxiety,” *Trends in Cognitive Sciences* 16, no. 8 (2012); Sian L. Beilock and Daniel T. Willingham, *American Educator* (Summer 2014). Anxiety loads working memory; short sessions and early wins. Not a clinical protocol in this book.

³⁰NMAP Finding 10 (see note 26): concept, fluency, and problem solving belong together — refuse “only tricks” and refuse “only discover with no practice.”

Precise picture. Magnitude is the size of a number relative to a unit. A fraction a/b (with b not zero) is a equal parts of a whole divided into b parts — or, equivalently, a point you can place by partitioning the segment from 0 to 1. You can also think of a/b as the result of dividing a by b ; both readings should land on the same place.

A terminating or repeating decimal is a base-ten rename of that same magnitude. Tenths, hundredths, thousandths are just finer partitions of the same segment. A percent is “per hundred”: 40% is $40/100$, which simplifies to $2/5$, which is 0.4. Renaming does not change location. That is the whole point of the game.

Negatives extend the line through zero. -3° is three units on the cold side of zero. A \$40 overdraft is forty dollars in the debt direction. Direction and debt are meaning, not “advanced algebra” you must postpone until a letter appears. You do not need a full signed-number course to place -8 relative to 0 and 5. You need one honest sentence and a short line.

Benchmarks are memory anchors, not decorations. If you know where $1/2$, $1/3$, $1/4$, $1/5$, and $1/10$ live, most adult comparisons become “near which neighbor?” before they become calculator work. $3/8$ is a little under $1/2$. 0.05 is half of a tenth. $2/3$ is past $1/2$ and short of 1. Say the neighbor first; refine second.

Wrong answers you should be able to hear (including from yourself).

1. **“0.2 looks smaller than $1/8$ because 2 is small.”** Digit-size heuristic. Two is a small digit; eight is a larger digit; the eye votes for $1/8$ as bigger. On a line, $0.2 = 1/5 = 0.200\dots$, and $1/8 = 0.125$. Zero-point-two lives farther from zero. The move: estimate, place both, then peek at a convert if you need confirmation. Name the trap when you feel it.
2. **“Fractions are pizza only.”** Food pictures can help once. Adult life uses cuts, time, doses-as-parts on a labelled household bottle you already use (rate and units only — not medical advice), debt fractions, and scale factors. Pizza is a picture, not the identity of fractions. If your only fraction story is food, widen the objects this week.
3. **“Decimals and percents are unrelated topics.”** They are names for the same magnitudes. Converting is renaming. If $2/5$, 0.4, and 40% feel like three different chapters from three different years, the line has left the room. Bring it back.
4. **“Negatives can wait until real algebra.”** Temperature, direction, and debt need them early as meaning. Waiting until “letters” appear teaches you to

fear a sign that already shows up on a weather app.

5. **“I had Algebra II, so skip this.”** Diploma year is not a magnitude test. If $3/5$ vs $2/3$ still feels opaque under a little pressure, you are still in the gate — skill strip first. Place by rust and need.

Five-minute warm-up before you try. Draw a scratch line. Mark 0, $1/2$, and 1. Place one rusty fraction you actually mis-order (maybe $2/5$ or $3/8$). Say one other name for each mark — fraction, decimal, or percent. Then pick two decimals — say 0.3 and 0.28 — and estimate which is larger *before* you calculate. Cover any answer key. That is the warm-up: place, rename, estimate. If anxiety spikes, stop after the warm-up and count it as a win. Come back for the try later the same day or tomorrow.

What “fluent” is not. A fast calculator output is not a check. Estimate, compute, check. A fluent chatbot step list is not your work during the attempt. Speed without placement is theater. Placement without any fluent rename will stall you on a noisy commute. Build both, in short doses, on numbers you actually meet.

Common adult scars, named gently. Some of you were timed until panic. Some of you were told to “just move the decimal” until meaning left. Some of you were praised for speed and never asked where the number lived. None of that is a life sentence. This week’s repair is small: line first, rename second, tool third. If a scar flares, shorten the session and keep the early win. You are allowed to be an adult who rebuilds a foundation without apology.

More rename fluency without losing meaning. Practice saying the chain in both directions. Start from a percent you see on a tip jar sign in your head — 15% — and walk to 0.15 to $15/100$ to $3/20$. Start from a fraction on a cut list — $5/8$ — and walk to 0.625 to 62.5%. If $5/8$ feels hard, place it between $1/2$ and $3/4$ first: half is $4/8$, three-quarters is $6/8$, so $5/8$ sits in between. Location first; digits second.

Equivalence without theater. $2/4$ and $1/2$ are the same magnitude. Multiplying or dividing numerator and denominator by the same non-zero number renames without moving the point. That is why simplifying is safe when you understand it as renaming. If simplifying feels like a magic trick that “makes a different number,” return to the stacked sticky on one point until the fear quiets.

Estimation as adult manners. Before a tool, ask: is this nearer 0, $1/2$, or 1? Is this a little more or a little less than a benchmark I trust? Estimation is not anti-precision. It is the sense check that catches a moved decimal before it becomes a

wrong order, a wrong cut, or a wrong story in a meeting.

How to try it this week

Use the refresh-hour shapes you already have: a micro-session on the commute or at lunch, a desk-or-shift sit when you have a quiet surface, and a weekend deep sit when you need a longer rebuild. This week's strip is magnitude and rename. Do not turn Sunday into a textbook dump wearing a weekend hat. Three to five micros plus one desk sit, with an optional deep sit, is the honest ambition — not under-implementation of a proven adult-commute RCT, because there is none at that grain.

Session shape (all bands). Warm-up retrieval: cover notes; place two numbers; rename one. Short model: one solved twin that places $\frac{2}{5}$ and 0.4 on the same line and labels both. Your attempt: a parallel pair. One good self-question (the locked opening below). Mixed practice only after a few blocked wins — same move, different jackets. Exit sticky: *I can place ____ / I almost / I cannot yet — next cue is ____.*

Fade help on purpose. First item: you may use a convert after your estimate. Second item: estimate and place before any tool. Third item: unaided, then check. When to stop looking up: when your estimate and the rename agree twice in a row on numbers you used to mis-order.

Commute / lunch (about 15–20 minutes). Materials: phone note or scrap paper; two decimals and one fraction you already bump into (a dashboard 0.18, a receipt tax line written as a decimal, a half-hour). Safety: no public quiz that humiliates you or a colleague; no photographing someone else's work as a sting; noticing is not an audit. The move: two comparisons and one rename. Example seed: place 0.2 and $\frac{1}{8}$ mentally; name which is larger; rename 0.2 as a fraction or a percent. Say the locked opening once. Wait. Exit sticky before you scroll. If the session becomes scrolling, it stopped being a session — set a 15-minute boundary and one object.

Desk or shift (about 25–40 minutes). Materials: a recipe you will actually scale, or a cut list with a half and a remainder; scratch line; pen. Safety: ordinary DIY caution if you are cutting material — measure twice; this chapter teaches size, not shop certification. Do not paste proprietary employer data into a public consumer tool; invent isomorphic numbers if a work tile is sensitive. The move: represen-

tation first. Scale 4 servings $\rightarrow$ 6 (factor $3/2$). Write the factor as a fraction, a decimal, and — if it helps — a percent of the original. Check one quantity on a drawn line (does “half again as much” land where you think?). Include one interloper only after two clean scales: a magnitude comparison slipped into the same sit so you choose the move.

Weekend deep sit (about 45–75 minutes when needed). Materials: five numbers in mixed forms; one intentional wrong twin (a moved decimal, labelled wrong); a drawn line; calendar for next week’s three micro cues. The move: convert among fraction–decimal–percent for five numbers; check one on the line; diagnose the wrong twin out loud or on paper (“the decimal point moved — magnitude changed”). Compare one alternative rename path when both are legal (simplify then convert, or convert then simplify) and say why both land on the same point. Schedule Mon/Tue/Fri micro cues — spacing across days beats one heroic dump. A useful mathematics meta-analysis of spacing supports distributing practice; it is not a promise that every adult will see the same result.³¹

Exact wording you can say to yourself. “Where does this number live — and what is one name that matches it?” Then wait. Cover the answer. A slow three is a convention, not a sacrament (classroom think-time research is the neighbor; this is an adult pause-before-lookup, not a math RCT).³² Look at the line or the object, not at a photo-solver.

First try-it this week (on the go). Place two decimals and one fraction on a mental line; name which is larger; give one rename. That is enough for a micro-session. On a deeper sit later: five F–D–P renames with one line check and one labelled wrong twin.

A sample micro, narrated once (composite). You open a note on the train. Retrieval: “Where does $3/8$ live?” Near $1/2$, a little under. Solved twin glance: yesterday you wrote $2/5 = 0.4 = 40\%$ on one point — first useful move was the shared line, not a convert chant. Parallel try: is 0.3 larger than $1/4$? Estimate yes; $1/4 = 0.25$; confirm. Exit sticky: *I can place 0.3 vs $1/4$; next cue is 0.08 vs $1/10$.* Close the note. That is a refresh hour micro — not a chapter binge.

³¹Lindsay A. Murray and colleagues, mathematics spacing meta-analysis (2025): spacing versus massing overall $g \approx 0.28$. Useful study, not a promise that every adult will see the same result.

³²Robert J. Stahl, “Using Think-Time and Wait-Time Skillfully in the Classroom,” ERIC Digest ED370885 (1994) — about three seconds as a convention; Mary Budd Rowe wait-time (1972) was science class. Classroom-origin neighbor for an adult pause-before-lookup — not an adult math RCT.

Worked pictures you can steal (then put away)

These are composite illustrations, not reported workplaces. Use them as models. Then close them and try a parallel item unaided.

Picture 1 — Stacked names on one point. Draw 0————1. Mark a point a little under halfway. Label it three ways: $\frac{2}{5}$, **0.4**, **40%**. Say aloud: “Same place. Three names.” If you feel the urge to treat 40% as “bigger because forty,” notice that urge — percent jackets can fool size sense until the line is back.

Picture 2 — Digit-size autopsy. Write the wrong claim: “ $0.2 < \frac{1}{8}$ because $2 < 8$.” Cross it out. Underneath: $\frac{1}{8} = 0.125$; $0.2 = 0.200$; so $0.2 > \frac{1}{8}$. On the line, 0.125 sits near $\frac{1}{10}$; 0.2 sits at $\frac{1}{5}$. The first useful move was not a chant. It was a place.

Picture 3 — Scale factor 4 → 6. Original dry ingredient: 2 cups. Factor $\frac{6}{4} = \frac{3}{2}$. New amount: $2 \times \frac{3}{2} = 3$ cups. Benchmark check: you added half of the original again (half of 2 is 1; $2 + 1 = 3$). If your scaled amount landed near double when you only meant “half again,” the factor went wrong — return to $\frac{3}{2}$ on the line between 1 and 2.

Picture 4 — Decimal that is a percent in disguise. A dashboard tile shows **0.18**. Rename: $0.18 = \frac{18}{100} = 18\% = \frac{9}{50}$. Story check: is “about one-fifth” fair? One-fifth is 0.20, so 0.18 is a little under one-fifth. That sentence is number sense doing work before any meeting talk.

Picture 5 — Negative as direction. Line: -10 — -5 — 0 — 5 — 10 . Mark -3 . Sentence: “Three units on the below-zero side.” Mark a composite balance of -40 dollars. Sentence: “Forty dollars in the debt direction.” You are not choosing a bank product. You are placing a signed magnitude.

Before each picture becomes a crutch, close the page and invent a parallel: a different fraction on the same line; a different scale; a different dashboard decimal. Struggle before rescue. Try unaided first.

Practice that actually builds it

Blocked practice first: several magnitude comparisons in a row, then several re-names. Mixed practice later: a comparison, a rename, a scale factor, a negative-as-direction sentence — so you have to choose the move. Paychecks, shelves, and

recipes *are* the practice surfaces. They do not replace the line.

Knowledge first, then fluency on the same objects. Hearing a wrong reason — including your own — then asking a better question, then naming a better reason, is the work.

Talk-to-yourself box

Opening (locked): “Where does this number live — and what is one name that matches it?”

Follow-ups: 1. What are two other names for this number? 2. Which benchmark is it near — 0, $\frac{1}{2}$, or 1? 3. If you place it on a line, what sits on either side? 4. Where could someone go wrong converting? 5. Does the size make sense for the story (recipe, cut, money)?

How to wait alone. After you ask, cover the answer. Count a slow three. If you stop mid-reason, wait again before peeking. Look at the stub, the shelf tag, the scratch line, or the clock — not at a camera solver. Rescue at one second is still rescue. Abandonment for forty minutes of scrolling is not a micro-session.

What a stuck silence usually means. The question was vague; wait-time one was zero; you are hunting for a vocabulary word instead of a location; you are guessing what a textbook would want; you offered a digit-size vote where magnitude was asked; you think “math person” means leave; you think a finished-looking chatbot paragraph is already a check; anxiety is eating working memory — shorten the session, win one magnitude item, come back tomorrow. Your job is to pose, wait, and point back at the object — not to fill the silence with a photo-solver’s answer.

Named try-its

Try-it A — Two-number magnitude flash (commute / lunch)

Time: 8–12 minutes.

Materials: phone note or mental list; two pairs you actually confuse (for example 0.3 vs $\frac{1}{4}$; 0.08 vs $\frac{1}{10}$).

Safety: private; no public quiz.

The move: For each pair, estimate which is larger *before* converting. Place both on a mental line between 0 and 1. Then rename the winner once (fraction $\leftrightarrow$ decimal or percent). Write one sentence: “___ is larger because it lives nearer ___.”

Try-it B — Estimate-then-peek: is 0.2 larger than 1/8? (commute / lunch)

Time: 5–8 minutes.

Materials: scrap or note.

Safety: ordinary.

The move: Answer without a calculator first. Then convert $1/8$ if needed. Label the digit-size trap if you felt it (“2 looks small”). Exit sticky: *I caught / I almost caught / I still buy digit size.*

Try-it C — Convert one fraction through the chain (commute / lunch)

Time: 5–10 minutes.

Materials: one fraction from life (half an hour; three-quarters of a tank; two-fifths left on a progress bar described without product chrome).

Safety: ordinary; blur proprietary names.

The move: Fraction $\rightarrow$ decimal $\rightarrow$ percent $\rightarrow$ back. Check that all names share a location. If you only chant a convert rhyme without placing, stop and draw the line.

Try-it D — One F–D–P chain from a dashboard decimal (desk or shift)

Time: 10–15 minutes.

Materials: one number from life (a tip percent you already use as a decimal; a half; a dashboard 0.25).

Safety: do not paste proprietary employer screens into a public tool; invent isomorphic numbers if the real tile is sensitive.

The move: Write fraction $\rightarrow$ decimal $\rightarrow$ percent $\rightarrow$ back to fraction. Check that all four names point to the same place on a drawn line. Ask: which form makes the story easiest to say aloud?

Try-it E — Recipe or cut-list scale with unit sense (desk or shift)

Time: 20–30 minutes.

Materials: a real recipe or a cut list; measuring tools you already own.

Safety: food safety and tool safety are yours; this is scale math, not a cooking class or a shop cert.

The move: Scale $4 \rightarrow 6$ ($\times 3/2$) or half a length with a remainder. Write the factor. Check one quantity against a benchmark (“is this closer to $1/2$ or 1 cup?”). If a unit conversion appears (cups $\leftrightarrow$ tablespoons you already know), keep units matched in one sentence. Estimate the scaled amount before measuring when you can.

Try-it F — Benchmark nearness walk (desk or lunch)

Time: 10 minutes.

Materials: five numbers; sticky labels “near 0 / near $1/2$ / near 1.”

Safety: ordinary.

The move: Sort without converting first. Then spot-check one convert. Wrong twin to plant: sorting 0.05 as “near $1/2$ ” because 5 feels mid — diagnose why near 0 is right.

Try-it G — Negative as direction or debt (desk or weekend)

Time: 10 minutes.

Materials: a weather number below zero, or a composite overdraft labelled composite.

Safety: ordinary stakes; no live controversy; no financial product pitch; no lending advice.

The move: One sentence: “ -8 means eight units on the cold side of zero” or “ $-\$40$ means forty dollars in the debt direction.” Place -8 , 0, and 5 on a short line. That is enough. You are building meaning, not a full integer curriculum.

Try-it H — Five renames + wrong twin (weekend deep sit)

Time: 45–60 minutes.

Materials: five mixed numbers; one wrong twin that moves a decimal (for example treating 0.4 as 0.04); drawn line; calendar.

Safety: label the wrong twin clearly so you do not encode it as correct when tired.

The move: Convert each among F–D–P. Place at least three on the line. Diagnose the wrong twin in writing: what changed, and why the magnitude is wrong. Schedule three micro cues for next week (spacing). Optional alternative path: for one number, convert two different legal ways and confirm the same landing.

Incorrect example to diagnose (blocked, then name the error).

Someone says: “0.2 is less than $1/8$ because 2 is less than 8.”

What went wrong: digit-size comparison instead of magnitude.

Repair: convert or place. $1/8 = 0.125$; $0.2 = 0.200$; $0.2 > 0.125$. Or place both between 0 and $1/4$.

Say the repair out loud. Silence in the face of your own wrong reason is not kindness.

Second incorrect twin (for the deep sit).

Someone renames $3/5$ as 3.5% because they “saw a 3 and a 5.”

What went wrong: digits reused as a percent without per-hundred meaning.

Repair: $3/5 = 0.6 = 60\%$. Percents are hundredths. Place 0.6 near $1/2$ -plus.

Mixed pair (after a few blocked wins). One magnitude comparison + one rename + one scale factor in the same sit. The point is choosing the move, not racing. Near

transfer is the honest hope: the same size sense on a recipe *and* a dashboard *and* a cut list in the same week. Far transfer to an unseen domain without knowledge is not promised — vary jackets, keep structure.

How to keep a tiny record. After a micro, one dated line is enough: date; pair compared; one rename; exit sticky words. After a desk sit, keep the scaled recipe factor and the line sketch. After a deep sit, keep the wrong twin with the repair sentence. A colleague who missed your session should almost be able to follow the note. That is the work product — not a fake certificate, not a percentile, not “Emerging Adults Math I.”

Spacing the week (seed, not a 180-day plan). Mon micro: two comparisons. Tue micro: one F–D–P chain. Wed desk: recipe or cut scale with line check. Thu off or light retrieval of yesterday’s sticky. Fri micro: digit-size wrong twin. Optional Sun deep sit: five renames + labelled wrong twin + calendar cues for next week. If you miss Tuesday, do not punish yourself with a double dump; resume from the last sticky. Consistency of cueing beats perfection of minutes.

Maintenance after done-enough. When this strip clears, do not ghost it. Two micros a week of retrieval plus one interloper keeps the line warm while you open percent. Rebuild weeks return when a diagnostic wrong reappears — digit-size relapse, rename without location, diploma-year skip urge.

If it isn’t clicking

Diagnostic 1 — You keep peeking in under a second.

Likely meaning: wait-time one was zero; anxiety is loading working memory. Next move: cover the answer; shrink to one pair; untimed; win one magnitude item; stop. Point back to the refresh hour: short sessions beat marathon “prove you’re tough” sits. Come back tomorrow. Optional light calm-down before the next try is fine; this book does not promise a cure for anxiety and does not turn into therapy.

Diagnostic 2 — Renames feel like three subjects.

Likely meaning: you are memorizing convert recipes without a shared location. Next move: draw the line every time for three days. Stack labels: $\frac{2}{5} = 0.4 = 40\%$ on one point. Say “same place, three names” before any algorithm. Procedure comes after the picture. If a convert tool finishes the attempt for you, you practiced tool use, not rename.

Diagnostic 3 — You want to skip because you “already had” algebra.

Likely meaning: diploma year is standing in for a skill check.

Next move: give yourself a three-item diagnostic — two comparisons, one rename under light time pressure you choose (not a humiliation timer). If any item fails, stay. Foundations still gate algebra; that is a placement fact, not a scolding. A forty-five-year-old with fraction fluency can learn algebraic structure; a twenty-eight-year-old without foundations cannot skip to calculus because a podcast said so. Place by skill strip.³³

Extra signals and next moves. If you only succeed when the problem looks like a worksheet, change the jacket — use a receipt decimal or a cut. If you freeze at negatives, stay with one temperature sentence until the sign feels like direction. If a marathon sit leaves you worse, you overdosed — return to the micro shape.

When to slow down. If digit-size errors return after a good day, return to blocked line placements. If anxiety spikes, cut the session in half and keep the early win.

When to go ahead. When you place common fractions and decimals without a tool, rename among forms, catch one digit-size error in a wrong twin, and can say one negative as direction or debt, you are ready to lean into percent and ratio with a live base.

When to ask a human. A colleague who is good at explaining, a tutor, or a class can help when self-talk loops. Bring a concrete wrong twin, not “I’m bad at math.” Correct the reason without crushing the attempt — applied to yourself. “Never correct yourself” is not this book’s rule.

Tools, including AI

Keep tools short and late. Point back to the refresh-hour AI box for the full allows and bans. Do not turn this chapter into an AI policy paper.

A labelled calculator may check a convert *after* your estimate. A number-line sketch on paper beats a flashy app during the attempt. Dynamic graph tools are

³³National Mathematics Advisory Panel, *Foundations for Success* (2008): Critical Foundations include fractions (with decimals, percents, and negatives) as gates to algebra; Finding 10 — conceptual understanding, computational fluency, and problem solving mutually reinforce; Finding 15 — developmental appropriateness is largely contingent on prior opportunities to learn; place by skill, not “too old.” Access date for URLs in these notes: 5 September 2026.

optional later; they are not required for this gate. A convert site is a check, not a partner during the first try.

After you have tried: you may ask a chatbot to explain *this idea* to yourself from a named lesson (title, page, today’s micro-goal). You may ask for isomorphic practice with answers kept on *your* page. You may ask for a labelled SCRIPT — a short sequence for *you* to say, not a paragraph to copy as your work. You may ask for a hint after an attempt — a hint is not the finished answer. You may ask it to diagnose work you already did (crop to the paper — no face).

You may not photo-to-key during the attempt (camera solvers and photo-to-steps loops). You may not paste a worksheet and ask for completion. You may not ask “what’s the answer” during the attempt. You may not treat a finished-looking step list as your work. You may not invent the receipt total or the axis start. You hold the account; unaided first. COPPA’s under-13 line does not apply to you — that is not a companion licence and not a secret-friend permission. Do not paste proprietary employer data into a public consumer tool; use invented isomorphic numbers. Do not hand yourself an agent as a first-attempt math partner. Do not print a fluency certificate from a chat.

One mechanism sentence, labelled ON-AGE-adjacent, then teach: in a high-school math field experiment (grades 9–11), an unguarded chatbot made practice look better and left students worse on the test with the window closed. Not an adult RCT.³⁴ The lunch-break rule it supports: the model may prepare you and the next problem; it may not do your problem during the attempt.

What “done enough” looks like

Use this skill-strip checklist before you move on. Placement is by skill, not birthday and not diploma year. A chatbot certificate of fluency is receipt theater — this book never invents one, and never invents “Math Refresher I” as a credit title.

- ☐ I can place common fractions and decimals (at least $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{10}$, and two decimals I used to mis-order) on a line between 0 and 1.

³⁴Hamsa Bastani and colleagues, *Proceedings of the National Academy of Sciences* 122, no. 26 (2025): e2422633122. High-school math field experiment (grades 9–11, Turkey); unguarded chatbot raised assisted practice and cut unaided exam performance relative to control. **ON-AGE-adjacent — not an adult RCT.** Mechanism: crutch. The model may prepare you; it may not do your attempt.

- ☐ I can rename among fraction, decimal, and percent for five numbers and land on the same magnitude.
- ☐ I can catch a digit-size error in a wrong twin and say what went wrong.
- ☐ I can use one negative as direction or debt in an ordinary sentence and place it relative to zero.
- ☐ I can scale a simple recipe or cut with a factor I can name (including $3/2$ or $1/2$) and check size with a benchmark.
- ☐ I can say which benchmark a new number is near before I convert.
- ☐ My exit sticky this week names one cue for next Tuesday — not a fake diploma.

If fractions still feel opaque under a little pressure, stay here. The next chapter needs a live sense of size before “base,” “part,” and “rate” will stick. Revisit magnitude next week in three micros even after you open Chapter 2 — maintenance is part of the habit. Life-of-the-habit tips live in the refresh hour, in this checklist, and in keeping track — not in a tenth teaching chapter.

Optional later paths (community-college courses, placement tools, credit exams) are bridges when you need them. They are not required to open or finish this gate. Sitting a credit exam is not the same as finishing this book’s skill strip.

You do not need to be a mathematician. You do need to hear “2 is small so 0.2 is small” as a wrong twin, place the number, and rename it. On the commute, place two decimals and one fraction. At the desk, scale one recipe with a line check. This weekend, if you need the rebuild, run five renames and label one wrong twin. That is done enough for the gate.

A closing walk-through for this week

Here is one coherent week you can adapt. It is not a curriculum contract. It is a shape.

Monday, commute. Warm-up: place $1/2$ and 0.4. Try-it A on 0.3 vs $1/4$. Exit sticky names Tuesday’s cue.

Tuesday, lunch. Locked opening on $3/8$. Rename to decimal and percent. Peek only after estimate. Sticky: *near $1/2$, a little under.*

Wednesday, desk. Recipe scale $4 \rightarrow 6$. Write $3/2$. Check one amount on a line. Interloper: is 0.08 larger than $1/10$? (No — $0.08 < 0.10$.)

Friday, commute. Wrong twin: “ $0.2 < 1/8$.” Repair in one sentence. Retrieve Monday’s sticky without looking.

Sunday, optional deep sit. Five F–D–P renames. One moved-decimal wrong twin labelled. Calendar three micros for next week. Done-enough checklist if you are ready; if not, stay another rebuild week without shame.

If the week slips, keep the object small. One comparison done with a wait is better than a PDF chapter highlighted in a fog. You are rebuilding a gate, not performing toughness.

When size feels usable — not perfect, usable — open Chapter 2 with the same respect for foundations. Percent will ask for a base. The base is a magnitude with a role. You will be ready to name it.

Keep the tone kind when you correct yourself. You are not on trial. You are tuning an instrument you still need.

If you teach teenagers at home as well, remember this chapter still addresses *you* as the learner refreshing your own math. A different book serves the parent-and-teen kitchen table. One pointer is enough; then return to your line.

Chapter 2 — Percent and ratio

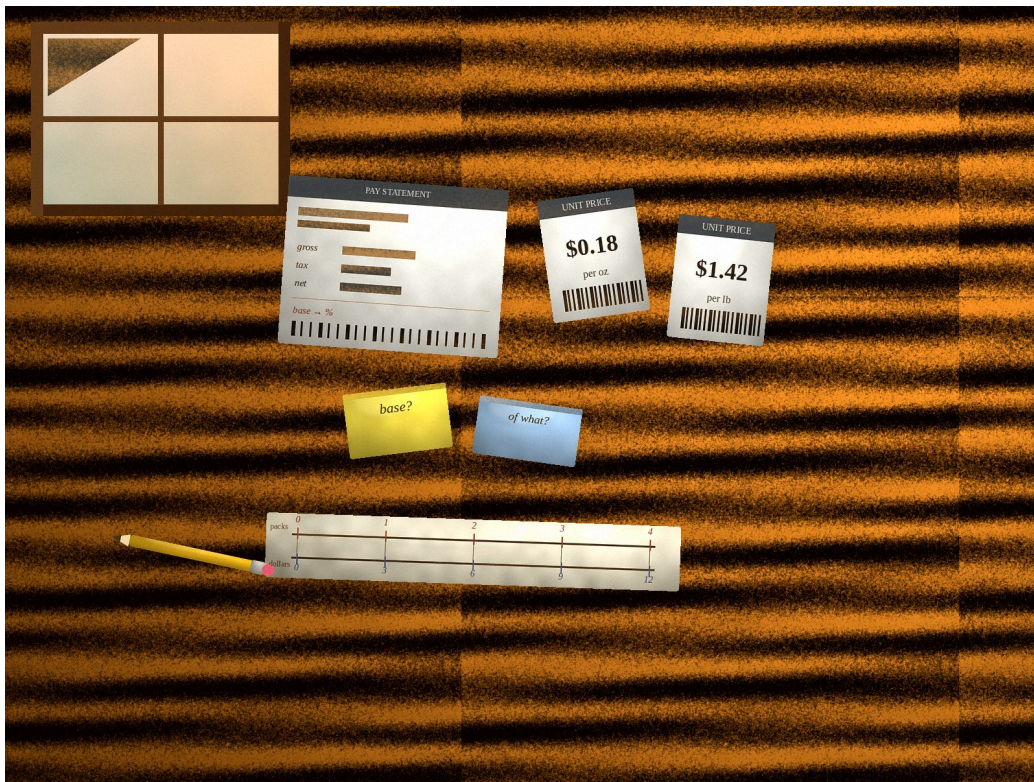


Figure 5: A still-life: paycheck stub with greeking, unit-price shelf tags, and a base sticky; toy APR vs interest row. No people. No logos. No readable UI.

Why this matters

Percent and ratio are the adult spine. Tip, tax, discount, withholding, unit price, dashboard change, a scale factor in a recipe, a rate on a label — these are not side topics. They are how size shows up when something is “out of a hundred,” “per one,” or “compared with what it was.”

If Chapter 1 rebuilt where a number lives, this chapter asks what role the number plays. Is it the base? The part? The rate? Are you finding a percent *of* something, or a percent *change from* an old value? Keyword theater fails here, loudly, in exactly the places adults actually compute. “Of means multiply” is a slogan, not a schema. It breaks on paycheck withholding, on “20% more than,” on stacked discounts, and on unit prices with mismatched units.

This module is load-bearing in adult numeracy for a reason. Multi-step proportion work sits above the most basic survey ceiling; you do not need a percentile as a grade.³⁵ You need a sober sense that naming a base is not optional garnish. Meaning of percent also matters for life decisions — including how risk and benefit language is framed later in the statistics chapter. Here, stay with ordinary stakes and structure.

Paycheck and APR appear as *structure*: what is the base; which row is interest; which row is APR. That is literacy, not tax politics and not product advice. The Consumer Financial Protection Bureau’s plain distinction is worth one careful pass: an interest rate is the cost of borrowing on the principal; an APR folds in certain additional fees charged when the loan is made; compare APR to APR, not APR to interest.³⁶ Then stop. No lender chosen. No “best card.” No refinance sermon.

Unit price and dashboard percent are practice surfaces. Cooking scale is a ratio table you can taste. A pharmacy label try-it, if you use one, is rate and units on a labelled composite or a household OTC label you already use — safety hedge, not medical advice. If the real question is clinical, ask a pharmacist or clinician.

What this chapter unlocks is a stable habit: name the base, name the question (part, whole, or rate), then compute. The habit travels. A tip on Thursday and a

³⁵NCES/IES PIAAC 2023 adult numeracy context (mean **249** vs international **263**; Level 1 or below **34%**): multi-step proportion work sits above the most basic survey ceiling. Survey pond, not a grade for the reader. Access date: 5 September 2026.

³⁶Consumer Financial Protection Bureau, Ask CFPB: interest rate versus APR — interest is the cost of borrowing on the principal; APR folds in certain additional fees when the loan is made; compare APR to APR. Literacy, not lending advice.

dashboard on Friday should feel like the same spine in different jackets, not like unrelated school units you once survived. Successive percents multiply factors — they do not add rates as if the world were a keyword list. When you can catch that wrong twin in yourself, you are fluent enough to hear percent talk without being pushed around by it.

Understand it yourself

Everyday picture. A store tag says 20% off. Off of what? The original price is the base. Twenty percent is the rate. The dollars saved are the part. If a second tag says “take an extra 10% off,” that second ten percent is off the *already reduced* price — a new base — not off the original. Adding 20 and 10 to claim “30% off” is the classic wrong twin. The everyday picture is a stack of bases, not a pile of percent signs.

Precise picture. A percent is a rate per hundred. That sounds small. It is not. Once the per-hundred meaning is steady, every adult surface that flashes a % sign becomes a question about base and role instead of a panic about school memory. In the part–whole–rate family:

- **part = rate $\times$ whole** (when the rate is written as a decimal or fraction)
- **rate = part $\div$ whole**
- **whole = part $\div$ rate** (when the rate is not zero)

Percent *of* uses that family with a clear whole (the base). Percent *change* needs an old value: $\text{change} \div \text{old value}$, often written as a percent. Percentage *points* are a different jacket: moving from 10% to 12% is a two percentage-point rise, which is a 20% relative rise of the old rate — those are not the same sentence.

Unit rate means “per one”: cents per ounce, miles per hour, milligrams per tablet on a label you already understand as units. Matched units first. Ratio tables and double number lines keep the correspondence visible before cross-multiplying as a reflex.

Successive discounts (or successive increases) multiply: a 20% discount leaves 80% of the price, written 0.80; then a 10% discount leaves 90% of *that*, written 0.90; together $0.80 \times 0.90 = 0.72$, so 28% off the original — not 30%. The same structure with increases uses factors like 1.20 and 1.10.

Schemas to name (not keywords): part-of-whole; change-from-old; unit rate (per

one); scale factor; successive multipliers of $(1 - p)$ or $(1 + p)$. Keyword theater (“of means multiply,” “is means equals,” “per means divide”) fails on paycheck withholding, unit price, and “20% more than” versus “20% of.”

Wrong answers you should be able to hear (including from yourself).

1. **“Of always means multiply.”** Keyword theater. Sometimes you multiply. Sometimes you are finding a rate or a whole. Name part, whole, rate first.
2. **“20% then 10% off equals 30% off.”** Adding successive discount rates. Multiply remaining factors: $0.80 \times 0.90 = 0.72$.
3. **“Tip on whatever number is largest on the receipt.”** Wrong base. Tip is usually on the pre-tax food-and-drink total your context uses — know which base you mean, then compute.
4. **“Interest rate and APR are interchangeable.”** They are not. Interest is cost on principal; APR includes certain fees; compare like with like (CFPB literacy).³⁷
5. **“Up 12%” with no old value.** Percent change without a base is an incomplete story. Ask for the old value before you interpret the leap.

Five-minute warm-up before you try. Pick one percent you already see this week — a tip, a discount, a dashboard tile. Write: $\text{base} = ___ \times$. Say whether you are finding part, whole, or rate. Estimate the size before you calculate. Cover the answer. That is the warm-up: base, role, estimate.

How to try it this week

Use the refresh-hour shapes: micro on commute/lunch, desk-or-shift sit, weekend deep sit when needed. This week’s strip is base / part / whole / rate, with one successive-percent autopsy and one unit-price compare. Space the work across days. A useful mathematics meta-analysis of spacing supports distributing practice rather than one massed dump; it is not a promise that every adult will see the same result.³⁸ Retrieval before reread still earns a seat at the table even when the math-

³⁷Consumer Financial Protection Bureau, Ask CFPB: interest rate versus APR — interest is the cost of borrowing on the principal; APR folds in certain additional fees when the loan is made; compare APR to APR. Literacy, not lending advice.

³⁸Murray and colleagues (2025) mathematics spacing meta-analysis: spacing versus massing **g** $\approx$ **0.28**. Useful study, not a promise that every adult will see the same result. A college financial

specific testing effect is hedged — cover the note, name the base, then look. A college financial spaced-retrieval study is a labelled neighbor for “short phone reviews beat one dump” — not a promise for every paycheck try-it, and not an adult ABE RCT.

Session shape (all bands). Retrieve: name a base from yesterday. Solved autopsy: one worked percent-of or percent-change with the base underlined. Parallel try: isomorphic item unaided. Self-question with the locked opening. Mixed practice after blocked wins — slip in a unit-price or a stacked-discount twin. Exit sticky: *I can name base on ____ / I almost / I cannot yet — next cue is ____.*

Commute / lunch (15–20 minutes). Materials: a pay stub with greeking or a receipt; phone note. Safety: structure only — not tax advice, not lending advice; no public humiliation; no proprietary paste into public tools. The move: write *base = ____* for one percent; compute one part or check one rate. Optional skim: on a toy disclosure, notice which number is labelled interest and which is APR — compare APR to APR if two APRs appear. Stop. No product chosen.

Desk or shift (25–40 minutes). Materials: two shelf tags with unit prices (or a photo you took of tags you may buy); scratch paper; optionally a dashboard screenshot with names blurred. Safety: noticing prices is not a sting; blur employer chrome. The move: representation first — a double number line or ratio table for one proportion; unit-price compare with matched units sentence; dashboard percent change with old value explicit and a glance at whether the axis starts at zero (chart critique stays light here; deeper stats live later).

Weekend deep sit (45–75 minutes when needed). Materials: pencil; calculator only after estimate; one successive-discount wrong twin; optional toy amortization row (interest vs principal portion as literacy); optional pharmacy composite label. Safety hedges: APR row is literacy, not advice; pharmacy is rate/units only — if clinical, ask a pharmacist or clinician; invent isomorphic numbers for anything proprietary. The move: prove to yourself why 20% then 10% off is not 30% off; label the wrong twin; optional second strategy on a proportion (table vs equation) and say why both are legal.

Exact wording. “What is the base — and am I finding part, whole, or rate?” Wait. Cover. Look at the stub or tag.

spaced-retrieval study is a labelled neighbor for “short reviews beat one dump” — not an adult ABE RCT of this chapter’s objects.

Fade help. First item: underline the base with a hint allowed after thirty seconds of honest try. Second: no hint until after your written estimate. Third: unaided, then check. Stop looking up when you can name base and role twice on new surfaces without a slogan.

First try-it this week (on the go). On one stub or receipt, write $\text{*base} = \underline{\hspace{1cm}}^*$ and compute one percent. Deeper sit: show why stacked 20% then 10% is not 30% off, with a wrong twin labelled.

Sample micro (composite). Train seat. Retrieval: yesterday's base was pre-tax total \$40. Solved glance: 15% of 40 is $0.15 \times 40 = 6$ — first useful move was naming the base, not the word “of.” Parallel: 18% of 40 estimate ~ 7 , compute 7.20. Exit sticky: *base before of*. Close.

Worked pictures you can steal (then put away)

Picture 1 — Part of whole. Base (whole) = \$80. Rate = 15% = 0.15. Part = $0.15 \times 80 = \$12$. Sentence: “Fifteen percent of eighty is twelve.”

Picture 2 — Percent change. Old = 50. New = 60. Change = 10. Rate of change = $10/50 = 0.20 = 20\%$ increase. Wrong twin: calling it a 10% increase because the change was 10 — that confuses amount with rate.

Picture 3 — Successive discounts. Price \$100. 20% off $\rightarrow$ \$80. Then 10% off of \$80 $\rightarrow$ \$72. Total off = \$28 = 28% of original. Wrong twin: $20 + 10 = 30\% \rightarrow$ \$70. Label the wrong twin so you do not encode it on a tired commute.

Picture 4 — Unit price. Brand A: \$3.00 for 12 oz $\rightarrow$ 25c/oz. Brand B: \$4.00 for 20 oz $\rightarrow$ 20c/oz. Matched units sentence: “Per ounce, B costs less.” If sizes were mixed pounds and ounces without converting, stop and match units first.

Picture 5 — Interest $\neq$ APR (CFPB shape, toy disclosure). Toy row labelled composite: Interest rate 6.0%. APR 6.4%. Notice which is which. Interest is cost on principal; APR includes certain additional fees when the loan is made. If you compare two loans, compare APR to APR. Stop. No lender chosen. No “best refinance.” No crypto pitch. No bank SKU.

Picture 6 — Dashboard change. Tile: “+12%.” Sticky: *old value* = ? Until the old value appears, the story is incomplete. If a chart truncates the axis, the leap can look theatrical — note it; deeper redraw practice lives in the statistics chapter.

Close the pictures. Try a parallel on your own stub, shelf, or invented isomorphic numbers.

Practice that actually builds it

Blocked first: several “name the base” items; then several percent-of computations; then percent-change with old value explicit. Mixed later: tip base, unit price, stacked discount, dashboard tile in one sit.

Talk-to-yourself box

Opening (locked): “What is the base — and am I finding part, whole, or rate?”

Follow-ups: 1. Is this percent-of or percent-change? 2. What is the old value (for change)? 3. Are the units matched for this unit price? 4. Am I looking at interest or APR — and can I compare like with like? 5. If someone added successive discount rates, what error is that?

How to wait alone. Cover the answer. Slow three. Wait again if you stop mid-reason. Look at the stub, shelf tag, or disclosure row — not at a photo-solver. Rescue at one second is still rescue.

Stuck silence usually means. Keyword hunt; wait-time one was zero; guessing what a worksheet wants; finished-looking chatbot steps taken as a check; tip-base confusion; anxiety — shorten, win one base-naming item, return tomorrow.

Named try-its

Try-it A — Pay stub or receipt base (commute / lunch)

Time: 10–15 minutes.

Materials: stub or receipt; note.

Safety: structure only; not tax advice; private.

The move: Underline one percent. Write $\text{*base} = \underline{\hspace{1cm}}^*$. Compute the part or verify the rate. One sentence: “I am finding $\underline{\hspace{1cm}}$ because $\underline{\hspace{1cm}}$.”

Try-it B — Unit price compare (commute / lunch or store aisle)

Time: 8–12 minutes.

Materials: two packages; unit-price tags if present.

Safety: shopping is not a sting; buy only what you planned or compute without

performing a scene.

The move: Match units. Compute or read cents per ounce (or per sheet, per count).

Sentence: “Per, costs less.”

Try-it C — Dashboard % with old value (desk or shift)

Time: 15–20 minutes.

Materials: blurred dashboard tile or composite numbers.

Safety: no proprietary paste into public tools.

The move: Write old, new, change, percent change. Glance: does the chart axis start at zero? Note only; full critique later.

Try-it D — Tip and tax bases (desk or lunch)

Time: 10 minutes.

Materials: receipt.

Safety: ordinary arithmetic; not tax advice.

The move: Name which total is the tip base in your context. Estimate 15% and 20%. Compute one. Separate tax as its own base story if present.

Try-it E — Double number line / ratio table (desk)

Time: 20–30 minutes.

Materials: paper.

Safety: ordinary.

The move: Build a table for a proportion you care about (scale a mix; map a simple “per” rate). Solve with the table before any cross-multiply chant. Then, if you use an equation, say what each symbol is.

Try-it F — Successive-discount autopsy (weekend deep sit)

Time: 30–45 minutes.

Materials: paper; wrong twin ready.

Safety: ordinary retail math; not a coupon scam tutorial.

The move: Show 20% then 10% off on a \$100 composite. Label the add-the-rates wrong twin. Write the factor multiplication. Invent one parallel (15% then 10%) unaided.

Try-it G — APR row literacy (weekend, optional)

Time: 20–30 minutes.

Materials: toy disclosure or a real disclosure you already have, greeked.

Safety: literacy only — not lending or investment advice; CFPB shape; compare APR to APR; stop.

The move: Point to interest rate; point to APR; write one sentence on the difference.

If an amortization row is present, notice interest vs principal portion as labels — do not choose a product.

Try-it H — Pharmacy label rate reading (weekend or desk, optional)

Time: 10–15 minutes.

Materials: labelled composite or household OTC label you already use.

Safety: **rate and units only — not medical advice.** Generic “the label says”; no brand chrome. If the real question is clinical, ask a pharmacist or clinician.

The move: Write the rate (for example mg per tablet $\times$ tablets as unit conversion, or mg per mL). Check that units cancel in a way that matches the question you framed as math, not as care.

Incorrect example to diagnose.

Claim: “20% off, then another 10% off — that’s 30% off, so \$70 on a \$100 item.”

Error: adding successive discount rates; second discount applies to a new base.

Repair: $0.80 \times 0.90 = 0.72 \rightarrow \72 ; 28% off. Say it aloud.

Second incorrect twin.

Claim: “The dashboard is up 12%, so we’re fine” — with no old value and a truncated axis.

Error: change rate without base; possible visual theater.

Repair: demand old value; note axis start; postpone drama.

Mixed pair. After blocked wins: one tip-base item + one unit price + one stacked-discount twin in the same sit. Near transfer: stubs *and* shelves *and* dashboards in the same week. Vary jackets; keep structure.

Spacing week seed (not a 180-day plan). Mon micro: base/retrieve. Tue micro: unit price. Wed desk: double number line. Fri micro: dashboard % with old value. Optional Sun: successive discount + APR sentence + checklist. Miss a day $\rightarrow$ resume from last sticky, not a punitive dump.

Tiny record. Dated line: base named; one computation; wrong twin repaired if any. Work product a colleague could follow — never a fake certificate.

Deeper schema practice (desk or weekend)

Use this section when the micros feel easy but mixed jackets still trip you. It is still not a 180-day dump. It is a deeper sit menu.

Schema card 1 — Part-of-whole. Cue: “what percent of,” “how much is 15%

of,” withholding as a percent of gross in a structure sketch. Move: identify whole; convert percent to decimal or fraction; multiply; estimate first. Wrong twin: multiplying by 15 instead of 0.15. Repair: percent is per hundred.

Schema card 2 — Change-from-old. Cue: “increased by,” “down 12%,” “growth rate.” Move: find old value; find change (new — old); divide by old; sign tells increase or decrease. Wrong twin: dividing by the new value. Repair: the base for change is the old value unless a defined index says otherwise — and if someone changes the rule, they must say so.

Schema card 3 — Unit rate. Cue: “per,” unit-price tags, speed, cost per hour. Move: match units; divide; sentence with units. Wrong twin: comparing \$3.00/12 oz to \$4.00/1 lb without converting. Repair: convert to the same unit before crowning a winner.

Schema card 4 — Scale factor. Cue: recipe scale, map scale (light touch — geometry owns deeper scale reasons later), “make it 150% of original size.” Move: factor as new/old; multiply each measured amount; keep area vs length distinction in mind if a later geometry chapter will need it — for now, do not scale a length factor and call it an area factor.

Schema card 5 — Successive multipliers. Cue: stacked discounts, successive raises, compound jackets without needing full finance theory. Move: write each remaining or growing factor; multiply; interpret. Wrong twin: add the percents. Repair: dollars on the page beat slogans in the air.

Copy one card onto a sticky. Run three isomorphic items. Then mix cards.

Percent points versus percent change (one careful page)

Suppose a rate moves from 8% to 10%. That is a rise of 2 percentage points. Relative to the old rate, the change is $2/8 = 0.25 = 25\%$ increase of the rate. Both sentences can be true. Confusion between them is a common adult wrong twin in meetings and headlines. Your move: ask which jacket is being used. If only one number is offered — “up 2%” — ask: points or relative change? Old value? This is ordinary stakes clarity, not a politics example. Keep the illustration numeric and dry.

Paycheck structure without tax advice

A composite stub might show gross pay, a withholding percent, and a net. Your math job: name the base for the withholding percent (often gross, depending on the line — read the line you have). Compute the part. Check whether the listed dollar amount matches $\text{rate} \times \text{base}$. If a second percent uses a different base, write a second $\text{base} = ___$ sticky. You are practicing structure. You are not filing anything. You are not optimizing withholding. If you need tax help, that is a human with the right role — not this chapter.

APR amortization glance (literacy only)

A toy amortization row might show payment total, interest portion, principal portion. Notice that early in many loans the interest portion can be large relative to principal portion — as a reading observation on a composite row, not as advice to refinance. Connect to percent: $\text{interest portion} \div \text{remaining principal}$ relates to how interest is applied for a period — details vary by product, so do not invent a universal statute from a blog. Your done-enough is: I can label the columns and not confuse interest rate with APR. Stop there.

More incorrect twins to keep handy

1. **“20% more than 50” treated as 20% of 50 only.** 20% of 50 is 10; 20% more than 50 is $50 + 10 = 60$. The phrase “more than” changes the story.
2. **Sales tax added before discount or after — without checking the receipt policy.** The math move is to know which base each percent uses on *this* receipt. Policies vary; read the ticket. Do not globalize one store’s order as a law of nature.
3. **Comparing APRs to interest rates across two offers.** Mixed comparison. Repair: APR to APR (CFPB).
4. **Treating a relative risk sentence as complete without a base rate.** Preview only: the statistics chapter owns risk representation. Here, notice the missing base and write “base missing” on a sticky. Do not diagnose a disease. Do not advise a screen.

Weekend deep sit script (optional, ~60 minutes)

1. Diagnostic wrong: write the stacked $20\%+10\%=30\%$ claim; repair with factors. (10 min)
2. Longer solved: unit price with a unit conversion included; talk structure — what stayed matched? (15 min)
3. Alternative strategies: solve one proportion with a table and with an equation; say why both legal. (15 min)
4. Mixed four: tip base; dashboard change; successive discount; interest $\neq$ APR sentence. (15 min)
5. Done-enough checklist + schedule three micro cues. (5 min)

If sixty minutes is too long, split across two evenings. Spacing still counts.

Anxiety and identity on this strip

Percent shame is common because the topic looks “basic” while feeling slippery. Social permission to say “I’m not a numbers person” is real; cheerful “I’m not a reading person” is rarer. Refuse the binary without a pep talk that denies the feeling. Short sessions. Early base-naming wins. No countdown timer as default rigor. If working memory floods, you are not proving low ability — you are meeting a known load pattern.³⁹ Shorten. Return.

Transfer design for this chapter

Near transfer is the honest hope: percent on stubs *and* shelves *and* dashboards in the same week. Far transfer to an unseen domain without knowledge is not promised. Implication: vary jackets, keep structure. When you only ever practice tip percents, a dashboard change will feel like a new subject. It is not. It is change-from-old in a new jacket.

³⁹Maloney and Beilock; Beilock and Willingham on math anxiety: loads working memory; short sessions and early wins. Not a clinical protocol in this book.

Closing encouragement for Chapter 2

You now have language for the adult spine: base, part, whole, rate; of versus change; successive factors; unit rates; interest $\neq$ APR as literacy. On the go, write $\text{*base} = __\text{*}$. At a deeper sit, autopsy the stacked discount and keep the wrong twin labelled. Point tools back to the refresh hour. Done-enough is a skill strip, not a certificate and not a lender.

When this checklist clears, algebra refresh will ask what stays constant in a table — a cousin question to “what is the base?” Keep percent warm with maintenance micros while you open Chapter 3.

Five parallel items (try first, then uncover)

Work these on paper. Do not scroll to the answers until you have written your own.

1. 12% of \$250 — estimate, then compute. What is the base?
2. A price rises from \$40 to \$46. Percent change?
3. 25% off, then 10% off, on \$80. Final price?
4. \$5.00 for 16 oz vs \$7.00 for 24 oz — which is lower unit price?
5. Toy disclosure: interest 5.9%, APR 6.3%. Write one sentence on which to compare across lenders.

Answers after you wait. (1) Base \$250; $0.12 \times 250 = \$30$; estimate near \$30. (2) Change \$6; $6/40 = 0.15 = 15\%$ increase. (3) $0.75 \times 80 = 60$; then $0.90 \times 60 = \$54$ (not 35% off $\rightarrow$ \$52). (4) $5/16 = 0.3125$ \$/oz; $7/24 \approx 0.2917$ \$/oz — second is lower per ounce. (5) Compare APR to APR; interest $\neq$ APR (CFPB literacy).

If you peeked early, mark that honestly on your sticky and rerun one item tomorrow unaided. Peeking is data, not a verdict on your worth. Struggle before rescue still applies when the rescuer is the bottom of the page.

If it isn't clicking

Diagnostic 1 — You grab “of means multiply” and hope.

Likely meaning: schema not named; keyword theater.

Next move: force the triad on paper — part / whole / rate — before any operation.

Redo a parallel item. If a chatbot hands you steps mid-attempt, close the window and restart unaided.

Diagnostic 2 — Stacked percents still feel like addition.

Likely meaning: the base is silently changing and you are not tracking it.

Next move: draw two stages with dollars, not only percents. Circle the new base before the second percent. Keep the wrong twin labelled beside the repair.

Diagnostic 3 — Interest and APR blur every time.

Likely meaning: the words were taught as synonyms in casual talk.

Next move: rewrite the CFPB distinction in your own one sentence; mark a toy row; practice comparing APR to APR only. Do not escalate into product shopping as “practice.”

Extra signals. Tip on the wrong total → write the base in words before digits. Unit price chaos → match units first, always. Anxiety spike → one base-naming win, untimed, stop. Marathon PDF highlight → return to micro shape.

When to slow down. When wrong twins stop feeling wrong. Return to blocked base-naming.

When to go ahead. When you can name base/rate/part on a new surface, compute percent change with old value explicit, catch a keyword error, compare two unit prices with matched units, and say interest $\neq$ APR in one sentence.

When to ask a human. Colleague, tutor, or class — bring a wrong twin and a stub photocopy with numbers you may share, or isomorphic inventions. Correct the reason without crushing the attempt.

Tools, including AI

Short box. Point back to the refresh-hour AI box.

Calculator: after estimate, to check arithmetic — not to choose the base for you.

Spreadsheet: fine for a ratio table you built; ban pasting an employer sheet into a

public chat for “cleanup.”

After you tried: explain the idea to yourself from a named lesson; isomorphic practice with *your* key; hint after attempt; diagnose work already done; labelled calculator or CAS *after* the try.

Ban during attempt: photo-to-key; “what’s the answer”; paste worksheet/employer sheet; unsupervised chatbot as only partner; invent disclosure numbers; proprietary data in public tools; agent as first-attempt partner; fluency certificate from a chat.

ON-AGE-adjacent reminder: unguarded chatbot practice can look better and leave you worse when the window closes (high-school field experiment — not an adult RCT).⁴⁰ The model may prepare you; it may not do your problem during the attempt.

What “done enough” looks like

Skill-strip checklist. Placement by skill, not diploma year. No fake certificate. No bank SKU as a trophy.

- ☐ I can name base, rate, and part on a new surface (stub, receipt, or shelf story).
- ☐ I can compute percent change with the old value explicit.
- ☐ I can catch one keyword error in a wrong worked example and repair it.
- ☐ I can compare two unit prices with matched units in one sentence.
- ☐ I can show why 20% then 10% off is not 30% off and label the wrong twin.
- ☐ I can say interest $\neq$ APR in one plain sentence and know to compare APR to APR.
- ☐ My exit sticky names a next cue — not a lender, not a certificate.

Pharmacy rate reading, if you used it, stays in the “units I can read” column — not in a self-care protocol column.

Revisit percent next week in three micros even as you open algebra. Maintenance keeps the spine warm. Life-of-the-habit lives in the refresh hour, this checklist, and keeping track — not in a tenth teaching chapter.

⁴⁰Bastani et al. (2025), *PNAS*: high-school math field experiment; unguarded chatbot crutch pattern. **ON-AGE-adjacent** — **not an adult RCT**. Lunch-break rule: the model may prepare you; it may not do your attempt.

On the commute, name the base on one percent you already see. At the desk, talk through one solved twin before three cold problems. This weekend, prove the stacked-discount repair once on paper. You do not need to be a mathematician. You do need to hear “of means multiply” as a keyword grab, not a schema — and to put the base back in the sentence.

Ordinary stakes, kept ordinary

Money, shelves, and dashboards motivate. They do not replace the ratio that means. When a conversation turns toward live controversy, stock sermons, or medical culture-war, leave that conversation outside this chapter. Safe objects only: stub structure, shelf tags, cooking scale, toy disclosure, composite label rates, sports box-score rates if you like rates in plain clothes.

Sports rate glance (optional micro). Points per game is a unit rate. Say the units. Estimate before you divide. This is play, not a gambling pitch.

Cooking scale as ratio. Tripling a recipe is a scale factor of 3. Halving is $1/2$. A factor of $3/2$ for $4 \rightarrow 6$ returns from Chapter 1 with a new jacket: the factor is a rate relating new amount to old amount. Table it once.

What not to overclaim. No bank SKU. No “best refinance.” No medical advice. No promise that a named study will raise your paycheck fluency by a guaranteed margin. This is a useful practice design, not a promise that every adult will see the same result.

Representation before the slogan

Before any formula, show two rows of a payment table or two stages of a discount in dollars. Before unit-price arguments, write both “per” sentences. Before a dashboard argument, write old and new. Star-style structure moves (from a school algebra practice guide, portable here as self-study grain, labelled school-origin) apply to these representations too: what stays visible in the table before the percent sign does any theater?⁴¹

⁴¹Jon R. Star and colleagues, *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (WWC practice guide, 2015): Recommendation 2 on structure (minimal evidence tier); Recommendation 3 on alternative strategies (moderate). Grades 6–12 origin — portable self-study grain for representation-before-slogan on percent tables; not an adult refresher RCT.

When two strategies are legal — ratio table versus equation; decimal rate versus fraction rate — try both once on the same item and say why both are legal. Flexibility is not cheating. Rigid one-method identity is the rust signal for returners who memorized a single chant.

A full desk sit, narrated (composite)

You have thirty-five minutes between tasks. Warm-up retrieval: name last night's tip base. Representation: sketch a double number line for "\$12 for 4 lb → cost for 10 lb." Solve on the line. Short model: a solved unit-price twin with matched ounces. Your tries: two shelf pairs; one interloper stacked-discount question slipped in. Self-question: locked opening on the interloper. Wait. Write: *base for the second discount is the reduced price*. Exit sticky: *matched units + new base*. You did not need a textbook chapter. You needed a base, a representation, and a wait.

If your job is full of percents

You may already compute in pockets of work and still feel shaky when the jacket changes. That is normal. Adults do not become all-purpose numerate because they were hired. Use isomorphic invented numbers when proprietary data cannot leave the building. Practice the schema on the invented twin; apply the calm schema at work without pasting internal sheets into consumer AI.

Done-enough at work looks like: you can explain a percent change with old value to a colleague in one breath; you catch a stacked-percent claim; you refuse an APR-to-interest comparison in a meeting without needing a sermon. Still not tax advice. Still not lending advice. Structure and sense.

Quick reference you may copy into a note (then hide during attempts)

- Percent of: $\text{part} = (\text{percent}/100) \times \text{whole}$
- Percent change: $(\text{new} - \text{old}) / \text{old}$
- Successive discounts: multiply remaining factors $(1 - p)$

- Unit price: money $\div$ size with matched units
- Interest vs APR: interest on principal; APR includes certain fees; compare APR to APR (CFPB)
- Always: base = ____ ; role = part / whole / rate

Hide this card during the attempt. Retrieve it after. Retrieval builds the strip more than rereading the card.

Three micro prompts for next week (copy to calendar)

1. Name the base on one receipt percent; estimate; compute.
2. Two unit prices; matched units sentence.
3. One wrong twin: added successive discounts; repair with factors.

That is maintenance while Chapter 3 opens — or rebuild, if the checklist is not yet honest.

You do not need to be a mathematician. You do need to hear “of means multiply” as a keyword grab when a schema was asked for, and to put the base back where it belongs. That is the spine. Hold it on the train. Hold it at the shelf. Hold it when a dashboard tile shouts a percent without an old value. Then move toward algebra with the same respect for structure you just practiced on rates and bases. Chapter 3 will ask what stays constant in a linear story. That question is kin to this one: what is fixed, what is the rate, and what are you solving for — without grabbing a slogan. Keep one percent micro alive each week even after you turn the page. Name the base first. Role next. Always.

Chapter 3 — Algebra refresh

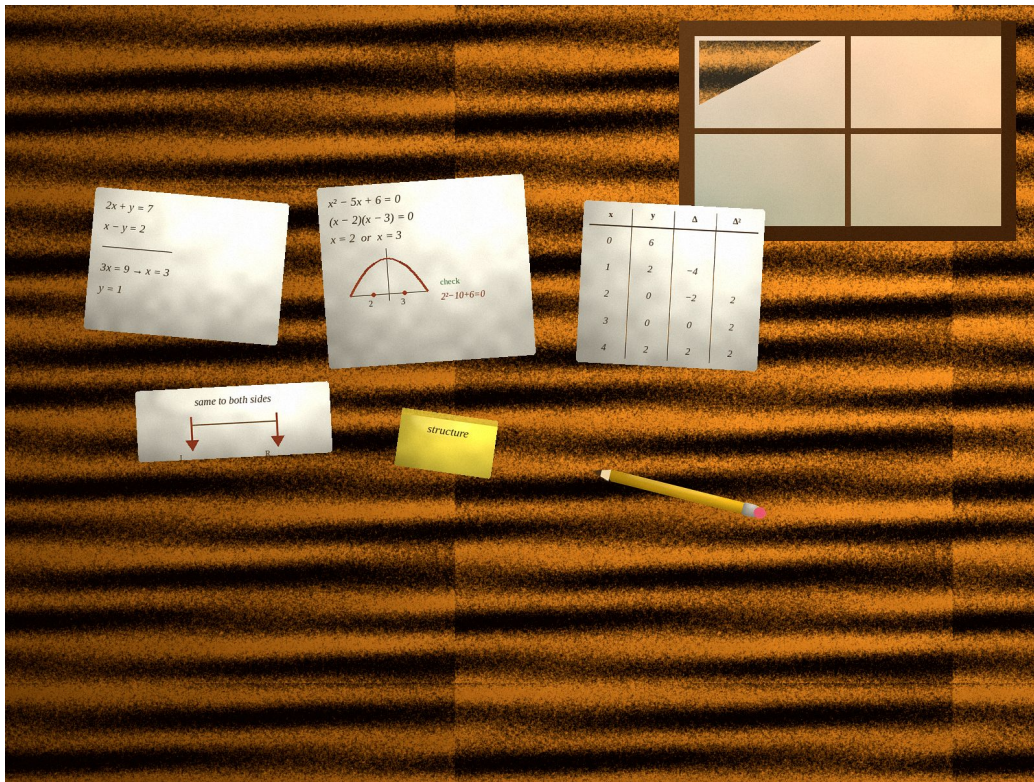


Figure 6: A still-life: solved linear twin with structure marks; tiny differences table; labelled wrong twin sticky. No people. No logos. No readable UI.

Why this matters

Algebra is where structure gets a voice. An expression is not a pile of letters. An equation is not a permission slip to move symbols until something looks finished. Linear stories keep a constant rate. Quadratic stories keep a constant second difference. When you can say which is which from a small table — before any regression button, before any formula chant — you are refreshing algebra as meaning, not as costume jewelry from a diploma year.

Many adults arrive with one of two scars. Procedure-only: “I memorized steps, then forgot.” Discovery-frustrated: “we talked, I never got fluent.” Conceptual understanding, computational fluency, and problem solving belong together.⁴² In this paperback, conceptual means: What quantity? What stays constant? Why is this rewrite legal? Does the size make sense? Procedural means accurate, flexible execution. Worked examples glue both.

This chapter rebuilds expressions as structure; equations that balance for a reason; linear and quadratic patterns adults meet (cost models, break-even, simple growth); and refuses letter-moving as identity. Same-to-both-sides is the default equation story — balance reason, not “move and flip” magic. An operational equal sign (“do the next step”) versus a relational equal sign (“these are the same”) is a named wrong-turn type for returners.

If fractions and percent bases still feel soft, point back to Chapters 1 and 2. Foundations still gate algebra. Having “taken Calc once” does not prove linear structure is present. Place by skill strip, not birthday and not ambition-alone. Adults can learn algebraic structure when prerequisites are in place — “too old” is the same myth as “too young” when the prior opportunities were missing or rusty.⁴³

The load-bearing portable move comes from a school algebra practice guide for grades 6–12 (Star and colleagues, 2015): analyze solved problems, attend to structure, and compare alternative strategies when more than one is legal.⁴⁴ Those

⁴²National Mathematics Advisory Panel, *Foundations for Success* (2008), Finding 10: conceptual understanding, computational fluency, and problem solving mutually reinforce. Access date: 5 September 2026.

⁴³NMAP Finding 15: developmental appropriateness is largely contingent on prior opportunities to learn — place by skill strip when foundations are in place; unfinished fractions still block. Finding 4 / Critical Foundations continuity if remediating.

⁴⁴Jon R. Star and colleagues, *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (WWC practice guide, 2015): Recommendation 1 solved problems (min-

evidence tiers are school-guide tiers — Rec 1 and Rec 2 labelled minimal, Rec 3 moderate — portable to self-study when you name the move and drop the classroom furniture. Minimal is not “ignore.” This is not an adult RCT and not a promise about a test score. It is a way to practice.

What this chapter unlocks is lawful rewriting and pattern sense you can use on ordinary stakes: $\text{hours} \times \text{rate} = \text{pay}$; fixed cost plus variable cost; two constraints on time and budget. No live controversy. No culture-war topics dressed as systems. Just structure you can check.

Understand it yourself

Everyday picture. A rideshare-style composite: you pay a fixed pickup fee plus a per-mile rate. That is a linear cost model — a constant rate of dollars per mile, plus an intercept. If you sketch total cost against miles, the first differences in a table stay roughly constant when the mile steps are equal. If instead a table’s first differences themselves grow by a constant, you are looking at a quadratic pattern — constant second differences. The everyday question is not “what is the vocabulary word?” It is: what stays constant?

Precise picture. An expression names a quantity with operations and symbols. Like terms and factor structure are properties, not decorations. An equation asserts that two expressions are the same magnitude. A legal rewrite preserves equality because you did the same operation to both sides, or because you replaced an expression with an identical twin (distribution, combining like terms, and so on).

Linear in one variable (for patterns we care about here): constant first differences when inputs are equally spaced; constant rate of change; graph as a straight line in the usual plane. Slope is the rate in units — dollars per hour, miles per gallon, points per game — not a naked letter m without a story.

Quadratic: constant second differences for equally spaced inputs; a squared degree in the usual polynomial jacket; graph as a parabola. Solution methods — factoring when friendly, square-root method when appropriate, completing the square, quadratic formula, graph/table read — can be alternative legal strategies. Flexibility is not cheating. Formula-before-table as a default order is the rust signal this

imal evidence); Recommendation 2 structure (minimal); Recommendation 3 alternative strategies (moderate). Grades 6–12 origin. Portable autopsy / structure / strategies grain for self-study; not an adult refresher RCT and not a WWC claim about adult commute apps.

book refuses.

Systems of two equations: two constraints on the same unknowns — hours available and budget ceiling, for example. Graphically, intersection; algebraically, substitution or elimination with reasons.

Wrong answers you should be able to hear (including from yourself).

1. **“Move the letter to the other side” — no property named.** Letter-moving as magic. Repair: same operation to both sides; say what you did.
2. **“Equal sign means do the next step only.”** Operational equal sign. Repair: relational reading — left and right are the same.
3. **“Quadratic formula before any table or graph.”** Order fetish. Repair: representation first; formula when it helps.
4. **“One method only; flexibility is cheating.”** Identity rigidity. Repair: two legal strategies, say why both legal.
5. **“I took Calc, so linear structure is present.”** Diploma year as placement. Repair: three-row table test; skill strip first.

Five-minute warm-up. Make a three-row table with $x = 0, 1, 2$ and a simple linear rule you invent ($y = 3x + 2$). Compute first differences. Then a quadratic toy ($y = x^2$) and second differences. Say which is linear and why. Cover any key. Warm-up done.

How to try it this week

Refresh-hour shapes apply. This week’s strip: linear vs quadratic from tables; same-to-both-sides; one solved-problem autopsy before cold problems (Star Rec 1 grain, labelled grades 6–12 origin, portable to you as discussant).

Session shape. Retrieve: what stayed constant last time? Solved autopsy: fully worked linear or quadratic on paper — first useful move; where someone goes wrong; what the table/graph showed before the algebra. Parallel try unaided. Self-question with locked opening. Mixed interloper after blocked wins. Exit sticky.

Commute / lunch (15–20 min). Materials: phone note; tiny table; one equation. Safety: private; no proprietary paste. Move: “is $x = 3$ a solution?” check by

substitution with a reason; or first differences on a three-row table; or one wrong solved twin with the illegal move labelled.

Desk or shift (25–40 min). Materials: paper; optional Desmos-class grapher *after* the try. Safety: invented isomorphic numbers if work data is sensitive. Move: representation first — cost model from story → table → equation → sketch; fade a step on a worked twin; two tries plus one interloper (a quadratic table slipped among linear ones).

Weekend deep sit (45–75 min when needed). Materials: solved correct/incorrect twins; quadratic table; calendar. Move: autopsy one solved problem; one parallel unaided; name the first useful move; solve one quadratic two ways and say why both legal (Star Rec 3 grain — moderate evidence in the school guide); optional system with hours and budget constraints — ordinary stakes. Schedule next micro.

Exact wording. “If this is linear, what stays constant — the rate or the second difference?” Wait. For a quadratic follow-up in the same box when needed: what is constant, and how do you know from the table?

Fade help. Autopsy with full solution visible → cover the middle steps → unaided parallel. Stop looking up when you can name the first useful move before peeking twice in a row.

First try-it (on the go). From a three-row table, say linear or quadratic and why. Deeper sit: talk through one solved problem, then one parallel unaided, then name the first useful move.

Sample micro (composite). Bus seat. Retrieval: slope last night was \$/hour. Solved glance: $2x + 5 = 11 \rightarrow$ subtract 5 both sides $\rightarrow 2x = 6 \rightarrow$ divide both sides by 2 $\rightarrow x = 3$. First useful move: same-to-both-sides, not “move 5.” Parallel: $2x + 5 = 15$; unaided. Exit sticky: *same to both sides*. Close.

Worked pictures you can steal (then put away)

Picture 1 — Is it a solution? Equation: $4x - 1 = 11$. Candidate $x = 3$. Substitute: $4(3) - 1 = 11$. Yes — left equals right. Reason: substitution checks the relational equal sign. Wrong twin: changing only one side until it “looks right.”

Picture 2 — Linear table.

x: 0, 1, 2, 3

y: 5, 8, 11, 14

First differences: +3, +3, +3 → constant rate 3 (y-units per x-unit). Slope story: +3 per step. Intercept visible at $x = 0$: $y = 5$. Equation jacket: $y = 3x + 5$.

Picture 3 — Quadratic table.

x: 0, 1, 2, 3

y: 0, 1, 4, 9

First differences: +1, +3, +5

Second differences: +2, +2 → constant second difference. Quadratic pattern. Graph sketch before formula.

Picture 4 — Same-to-both-sides. $3x + 4 = 19$. Subtract 4 from both sides: $3x = 15$. Divide both sides by 3: $x = 5$. Say the property each time. Wrong twin: “move 4 and flip to minus” without naming the operation on both sides.

Picture 5 — Cost model. Fixed cost \$20; variable \$4 per unit. Total $C = 20 + 4n$. Break-even against revenue $R = 9n$ (composite): $20 + 4n = 9n \rightarrow 20 = 5n \rightarrow n = 4$. Units: units sold. Ordinary stake; not a business plan.

Picture 6 — Two legal quadratic strategies. $x^2 - 5x + 6 = 0$. Factor: $(x - 2)(x - 3) = 0 \rightarrow x = 2$ or 3 . Graph/table: $y = x^2 - 5x + 6$ crosses zero at 2 and 3. Both legal because both find the same roots with reasons. Formula also legal when factoring is unfriendly — after you know what you are finding.

Picture 7 — Wrong twin (labelled). Someone rewrites $2x + 3 = 11$ as $2x = 11 + 3$. Illegal: added 3 to one side only while “moving.” Repair: subtract 3 from both sides $\rightarrow 2x = 8$. Mark the wrong twin clearly; do not study unmarked errors on a tired train (Star-style caution on incorrect solved problems).

Close the pictures. Invent a parallel table. Autopsy one twin yourself.

Practice that actually builds it

Blocked: several “is this a solution?” checks; then several first-difference tables; then several same-to-both-sides linear solves. Mixed: linear table, quadratic table, wrong twin, cost model in one sit.

Talk-to-yourself box

Opening (locked): “If this is linear, what stays constant — the rate or the second difference?”

Follow-ups: 1. What stayed the same when we rewrote this? 2. Which form makes the rate, intercept, or total visible? 3. Is there another legal strategy — and why is it legal? 4. Does a solved twin show a place someone goes wrong? 5. If this is quadratic, what is constant — and how do you know from the table?

How to wait alone. Cover the answer. Slow three. Look at the table or the balance of both sides — not at a photo-solver. Rescue at one second is still rescue.

Stuck silence usually means. Letter-moving story; wait-time one was zero; hunting for a formula name; chatbot steps pasted as a check; “I’m not a math person” stop-rule; anxiety — shorten, win one substitution check, return tomorrow.

Named try-its**Try-it A — Solution check (commute / lunch)**

Time: 8–12 minutes.

Materials: one equation; two candidate values.

Safety: private.

The move: Substitute; say yes/no with equal-sign reason. One wrong candidate on purpose.

Try-it B — First differences (commute / lunch)

Time: 10 minutes.

Materials: three-row table (pay per hour; miles; composite).

Safety: invent numbers if needed.

The move: Compute first differences; say linear or not; name the rate in units.

Try-it C — Wrong solved twin (commute / desk)

Time: 10–15 minutes.

Materials: incorrect twin with illegal move.

Safety: label WRONG clearly.

The move: Find the illegal step; rewrite with same-to-both-sides; check by substitution.

Try-it D — Linear cost / pay model (desk)

Time: 25–35 minutes.

Materials: paper.

Safety: ordinary composite business story — not investment advice.

The move: Story $\rightarrow$ table $\rightarrow$ equation $\rightarrow$ sketch. Interpret slope in units. Optional break-even against a second linear revenue.

Try-it E — Quadratic from table (desk or weekend)

Time: 25–40 minutes.

Materials: table; graph paper or after-try grapher.

Safety: ordinary.

The move: Second differences; sketch; solve by two legal methods; say why both legal. Delay formula until meaning of root is said aloud.

Try-it F — System as two constraints (weekend)

Time: 30–45 minutes.

Materials: paper.

Safety: hours and budget only — no culture-war topic.

The move: Write two equations; solve; check in both; interpret units.

Try-it G — Solved autopsy + parallel (weekend deep sit)

Time: 45–60 minutes.

Materials: fully worked linear or quadratic; parallel blank.

Safety: label incorrect twins.

The move: Star Rec 1 grain — you are the discussant. Ask: first useful move? Where could someone go wrong? What did table/graph show before algebra? Then parallel unaided. Name the first useful move on an exit sticky. Schedule three micros.

Incorrect example to diagnose.

Problem: $5x - 2 = 3x + 10$.

Wrong work: $5x = 3x + 10 - 2 \rightarrow 5x = 3x + 8 \rightarrow 5x - 3x = 8 \rightarrow 2x = 8 \rightarrow x = 4$ — wait, this one might accidentally be repairable; use a clearer illegal twin:

Wrong work: $5x - 2 = 3x + 10 \rightarrow 5x = 3x + 10 + 2 \rightarrow 5x = 3x + 12 \rightarrow$ then “cancel $3x$ ” by writing $5 = 12$.

Error: inconsistent operations; destroyed the variable unlawfully.

Repair: subtract $3x$ from both sides: $2x - 2 = 10$; add 2 to both sides: $2x = 12$; divide both sides by 2: $x = 6$. Check: $5(6) - 2 = 28$; $3(6) + 10 = 28$.

Mixed pair. After blocked wins: solution check + table family + wrong twin in one sit. Near transfer: pay model *and* break-even *and* a quadratic toy in the same week.

Spacing seed. Mon: solution check. Tue: first differences. Wed desk: cost model. Fri: wrong twin. Sun deep: autopsy + two quadratic methods + checklist.⁴⁵

Tiny record. Dated autopsy note; first useful move named; parallel result; wrong twin repaired. Colleague-readable. Never “Math Refresher I.” Never a fake certificate.

Same-to-both-sides, slowed down

Returners often remember a choreography: “move this, flip that.” The choreography sometimes lands on a correct answer and still teaches a false story — that symbols migrate by magic. The true story is smaller and stronger: whatever you do to one side, do to the other, because equality means sameness.

Try this drill once at a desk sit. Take $2x + 7 = 15$. On each line, write a verb:

1. Subtract 7 from both sides.
2. $2x = 8$.
3. Divide both sides by 2.
4. $x = 4$.
5. Check: $2(4) + 7 = 15$.

Now rewrite a wrong twin that “moves 7 and flips” without naming both sides. Label it WRONG. The contrast is the lesson. Star-style incorrect examples help when labelled; unmarked wrong methods on a commute are how rust learns the wrong dance.

Relational equal sign reading: left and right name the same magnitude. Operational reading: “the next step goes here.” You will still write steps. You will not let the equal sign become a vending machine for the next line without sameness.

⁴⁵Murray and colleagues (2025): spacing in mathematics $\sim g \approx 0.28$. Useful study grain for distributed micros; not a promise that every adult will see the same result.

Structure before letters (Star Rec 2 grain)

Before you chase x , ask what the representation shows. A table's differences. A graph's intercept. A word story's fixed fee and per-unit rate. Equivalent forms of an expression can make different features visible: factored form shows roots; expanded form shows coefficients; slope-intercept form shows rate and intercept. Choosing a form is strategy, not decoration.

Adult grain: before a break-even equation, show two columns — cost and revenue — for a few values of n . Before a quadratic formula, sketch where the graph might cross the axis. Before a system, say the two constraints in plain sentences. Letters come after the story can be heard.

Alternative strategies without chaos (Star Rec 3 grain)

When two methods are legal, try both on one item and articulate the choice. Factoring versus formula. Substitution versus elimination. Table read versus equation. The school guide rated this recommendation moderate evidence for grades 6–12 — still school-labelled, still portable as a self-study move. Flexibility is not cheating. One-method identity is often a scar from timed worksheets.

A rule of thumb for this book: if you only ever use one method, schedule one deep sit where a second legal method must appear. Say why it is legal. Then return to whichever method is efficient for you — by choice, not by fear.

Linear stories adults actually meet

Pay. $y = rt$ for simple hourly pay; or $y = rt + b$ if a bonus appears. Slope in \$/hour.

Cost. $C = f + vn$ — fixed plus variable. Interpret f and v .

Conversion. A linear map between units you already use (when the relationship is linear). Keep units visible.

Trend that is honestly linear. Some dashboard tiles are linear in a window. Others are not. First differences tell you; a slogan on a slide does not.

Refuse to force a culture-war story into a system. Hours and budget are enough.

Quadratic stories without formula worship

Toy projectile numbers, area relationships with a square, or a simple profit model that peaks — composites labelled composites. Second differences first. Sketch second. Roots as “where this quantity is zero” third. Formula fourth if needed. Power-rule calculus is not this chapter; do not drag it in early. If you are curious about rate language, Chapter 6 is optional and ready-only.

FOIL-without-graph as a required identity is refused. If you expand, know what you are expanding for. If you factor, know you are hunting structure that reveals roots or simplifies a rational expression later. Purpose first.

Systems as ordinary constraints

Example composite: you can work at most 40 hours; materials cost \$8 per unit and you have at most \$160; time costs 0.5 hour per unit. Write inequalities or equations as your skill allows — even two equations with a clear feasible story is enough for refresh. Solve; check; interpret. This is not linear programming class. It is two constraints, one brain, one check.

More incorrect twins

1. Distributing only to the first term: $3(x + 4) \rightarrow 3x + 4$.
2. Canceling terms across a sum as if it were a factor: $(x + 2)/(x + 5) \rightarrow 2/5$ by “canceling x .”
3. Treating slope as a naked number with no units in a pay story.
4. Declaring “not factorable” before trying integer pairs on a friendly trinomial — or the reverse, forcing factors that do not multiply back.
5. Using a grapher’s regression to crown “quadratic” on four noisy points without asking whether second differences are roughly constant or whether the model is even appropriate — shared wrong turn with the functions chapter.

Repair each with a reason, not a scold.

Parallel items (try first)

1. Is $x = -2$ a solution of $3x + 5 = -1$? Why?
2. Table y : 4, 7, 10, 13 for $x = 0..3$ — linear? Rate? Equation?
3. Table y : 2, 3, 6, 11 for $x = 0..3$ — what do second differences say?
4. Solve $4x - 9 = 2x + 5$ with verbs on each line; check.
5. $x^2 - 9 = 0$ two ways; say why both legal.

Answers after wait. (1) $3(-2)+5 = -1$; yes. (2) First differences +3; $y = 3x + 4$. (3) First +1,+3,+5; second +2,+2; quadratic pattern. (4) Subtract $2x$ both sides: $2x - 9 = 5$; add 9: $2x = 14$; $x = 7$; check $28 - 9 = 14 + 5$. (5) Square root: $x = \pm 3$; factor $(x - 3)(x + 3) = 0$; same roots.

Anxiety on algebra return

Letter fear is common. Short sessions. Early substitution wins. Untimed. If a deep sit becomes a shame spiral, stop and schedule a micro tomorrow. Anxiety loads working memory; it is not a math gene.⁴⁶ This book is not therapy; it is dose design.

What success feels like this week

You hear an illegal move in your own draft and repair it without panic. You look at a tiny table and say what stays constant. You autopsy a solved problem and can name the first useful move before you touch a cold twin. That is refresh. That is enough to walk toward geometry with reasons.

If it isn't clicking**Diagnostic 1 — Letter-moving without properties.**

Likely meaning: missing same-to-both-sides story; operational equal sign.

⁴⁶Maloney and Beilock; Beilock and Willingham on math anxiety — loads working memory; short sessions, early wins. Not a clinical protocol in this book.

Next move: rewrite one solution with a verb on each line (“subtract 4 from both sides”). Autopsy a wrong twin. Ban photo-to-key mid-attempt.

Diagnostic 2 — Cannot tell linear from quadratic.

Likely meaning: representation skipped; regression-button habit or formula-first habit.

Next move: only tables for two days — first and second differences — no formula. Say the constant aloud in units when linear.

Diagnostic 3 — FOIL or formula flex without meaning.

Likely meaning: procedure-only scar.

Next move: roots on a sketch first; then one algebraic method; then optional second method. Point back to Chapter 1 if arithmetic inside the algebra fails — foundations still gate.

Extra signals. Chatbot steps pasted → closed window; Bastani ON-AGE-adjacent reminder from refresh hour. Marathon sit → return to micro. “I took Calc” skip urge → three-row diagnostic. Anxiety → one substitution win, untimed, stop.

When to slow down. When illegal moves stop looking illegal. Return to labelled wrong twins.

When to go ahead. When you interpret slope in units; decide linear vs quadratic from a table without regression; solve one quadratic two ways and say why both legal; repair one illegal move on a wrong twin.

When to ask a human. Tutor, class, or colleague — bring the wrong twin and the table. Correct the reason without crushing the attempt. Community-college developmental pathways exist for enrolled students with supports; they are not proof that a solo commute app is enough, and they are not required to open this chapter.

Tools, including AI

Short. Point back to the refresh-hour AI box.

Labelled calculator / Desmos / GeoGebra dynamic tools / Wolfram / CAS: after the try, to check or explore — not to finish the attempt. Graphers are excellent for seeing a parabola after you predicted second differences; they replace thinking

when they become the only family ID.

After you tried: explain to yourself from a named lesson; isomorphic practice with your key; labelled SCRIPT for you to say; hint after attempt; diagnose work already done (crop to paper).

Ban during attempt: photo-to-key; “just give me the steps”; paste worksheet; unsupervised chatbot as only partner; invent table values mid-check; proprietary data in public chat; agent as first-attempt partner; fluency certificate; detector-grading your own unaided write-up.

Star note again: the school guide’s solved-problem recommendation is a move you turn on yourself — not a vendor crown for a kitchen algebra app, and not a WWC claim about adult commute refresh.⁴⁷

ON-AGE-adjacent: unguarded chatbot practice can inflate practice scores and hurt closed-window performance in a high-school field experiment — not an adult RCT.⁴⁸ Model prepares; you attempt.

What “done enough” looks like

Skill-strip checklist. Placement by skill. No fake certificate. Calculus edges later are optional for ready adults — not gated behind finishing every quadratic method in existence.

- ☐ I can interpret slope in units from a table or equation story.
- ☐ I can decide linear vs quadratic from a table without a regression button.
- ☐ I can solve a linear equation with same-to-both-sides reasons and check by substitution.
- ☐ I can solve one quadratic two legal ways and say why both are legal.
- ☐ I can repair one illegal move on a wrong twin and label the error.

⁴⁷Jon R. Star and colleagues, *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (WWC practice guide, 2015): Recommendation 1 solved problems (minimal evidence); Recommendation 2 structure (minimal); Recommendation 3 alternative strategies (moderate). Grades 6–12 origin. Portable autopsy / structure / strategies grain for self-study; not an adult refresher RCT and not a WWC claim about adult commute apps.

⁴⁸Bastani et al. (2025), *PNAS*: high-school math field experiment (grades 9–11); unguarded chatbot crutch. **ON-AGE-adjacent — not an adult RCT.** Same lunch-break rule as the refresh hour.

- ☐ I can autopsy a solved problem and name the first useful move before a parallel try.
- ☐ My exit sticky names a next cue — not a diploma year, not a certificate.

If fraction arithmetic inside these problems fails, return to Chapter 1 without shame. If percent bases in word jackets fail, return to Chapter 2. Algebra does not replace foundations; it sits on them.

Maintenance: two micros a week of a table check and a wrong-twin repair while you open geometry. Rebuild when letter-moving returns. Life-of-the-habit lives in the refresh hour, this checklist, and keeping track — no tenth teaching chapter.

Expressions as structure (desk micro-extension)

Before equations, spend one desk sit on expressions alone. Simplify $3(2x - 4) + 5x$ two ways: distribute first, or factor a common structure if you see one later. Combine like terms with a reason (“like terms name the same unit of x ”). Evaluate for $x = 2$ before and after simplifying — same output is a check that rewriting preserved meaning. This is relational thinking without yet solving for x .

Wrong twin: $3(2x - 4) + 5x \rightarrow 6x - 4 + 5x$. Distribution incomplete. Repair: $6x - 12 + 5x = 11x - 12$. Check at $x = 2$: original $3(4 - 4) + 10 = 10$; simplified $22 - 12 = 10$.

Why “move the letter” feels good and still misleads

It feels good because it is fast and sometimes lucky. It misleads because it hides the property, and hidden properties do not transfer when the equation gets a fraction, a parenthesis, or a second variable. Same-to-both-sides transfers. Magic migration does not. When you catch yourself saying “move,” translate: “I will subtract this from both sides” or “I will divide both sides by this.” Translation is the practice.

Graph as reason, not decoration

A quick sketch is a representation, not art class. For linear: two points or a point and a slope. For quadratic: vertex sense optional; axis crossings matter for roots. If a grapher is used after the try, predict first: “I expect a straight line with slope about 3” or “I expect a U-shape crossing near 2 and 3.” Prediction makes the tool a check. Without prediction, the tool is a spectacle.

Connecting back to percent and number sense

A slope of 0.15 in a tip model is 15% of the base when the story is $\text{tip} = 0.15 \times \text{bill}$. Algebra and percent meet. If 0.15 still feels like a stranger, Chapter 1's rename and Chapter 2's base sticky are the repair — not more FOIL. Likewise, if $3/2$ as a scale factor inside an expression panics you, return to the line. Algebra does not forgive missing magnitude; it amplifies it.

Optional CLEP or course pointer (not required)

If credit is your goal, optional exams and courses exist with institution-set credit policies. Sitting an exam is not the same as finishing this skill strip. This book never invents a certificate titled Math Refresher I or Emerging Adults Math I. Keep dated autopsy notes and worked checks a stranger can read. Placement tools from vendors are labelled vendor when you use them; they are not this chapter's spine.

Week plan seed (rebuild week)

Mon micro: substitution check.

Tue micro: first differences on a pay table.

Wed desk: cost model story $\rightarrow$ equation $\rightarrow$ sketch.

Thu off or light sticky retrieval.

Fri micro: labelled wrong twin repair.

Sat/Sun deep: quadratic two methods + system constraints + done-enough.

Miss a day $\rightarrow$ resume from last sticky. Do not double-dump punitively. Spacing research in mathematics supports distributing practice; it is not a promise that every adult will see the same result.

Maintenance week after done-enough

Three micros: one table family ID; one same-to-both-sides solve; one wrong-twin repair. One interloper from percent (name a base) to keep the spine linked. No heroics. No new textbook binge.

If you also help a teenager

This chapter still addresses *you* as the learner. Parent talk-boxes and transcript machinery live in a different book. Shared surfaces (a cost model, a table) can

dual-use if roles stay clear: you model self-questioning; you do not photo-to-key anyone's attempt, including your own.

Final walk-through sentence bank

Use these until they become your own:

- “First differences are constant, so the rate is constant — linear.”
- “Second differences are constant — quadratic pattern.”
- “I subtract ___ from both sides because equality must stay true.”
- “The first useful move was ___.”
- “This other method is legal because ___.”
- “The wrong twin failed at step ___ when it ___.”

Say them aloud on a walk. Silent fluency is fine too — but if anxiety eats working memory, speech can slow the panic and keep the reason visible.

Star-labelled practice grain (keep forever)

You are the discussant. Put a fully worked linear or quadratic on paper or phone. Ask: first useful move? Where could someone go wrong? What did the table or graph show before the algebra? Then one parallel item unaided. Incorrect solved problems: label the error; do not leave a wrong method looking authoritative when tired on a train. Structure of the representation before letter-chasing. Alternative strategies when legal — say why. Evidence tiers stay labelled school-origin; portable does not mean “proven for every adult lunch break.”

Ordinary stakes gallery (composites)

- $\text{Hours} \times \text{rate} = \text{pay}$; overtime as a second linear piece if your stub uses it — structure only.
- Fixed shop fee + per-unit materials.

- Break-even sketch for a small event budget.
- Two constraints: at most 40 hours; at most \$200 materials — find feasible pairs.
- Sports cumulative points that grow linearly vs a toy that accelerates — table first.

No live controversy. No investment pitch. No “beat the market with algebra.”

A full weekend deep sit narrated (composite)

Diagnostic wrong twin on letter-moving (10). Longer solved linear with structure talk — circle the same-to-both-sides verbs (15). Quadratic table: second differences; sketch; factor and formula; say why both legal (20). Mixed four including one system of hours/budget (15). Done-enough checklist; calendar Mon/Wed/Fri micros (5). You stop. You do not “finish the PDF.” That is a deep sit.

Closing encouragement for Chapter 3

On the go, decide linear vs quadratic from a three-row table. At the desk, talk through one solved twin before three cold problems. This weekend, autopsy one worked example, try a parallel unaided, and name the first useful move. You do not need to be a mathematician. You do need to hear letter-moving with no property as a wrong twin — and to put balance back in the equation.

When this strip is honestly done-enough, geometry with reasons is next: givens, show, bridge. The equal sign’s relational meaning will travel with you. Keep one algebra micro alive weekly so the rate stays constant in your practice life, not only in your tables.

One more solved autopsy you can copy as a model

Worked (correct): Solve $(1/2)x + 3 = 7$.

Multiply both sides by 2: $x + 6 = 14$.

Subtract 6 from both sides: $x = 8$.

Check: $(1/2)(8) + 3 = 4 + 3 = 7$.

Talk-through: First useful move cleared the fraction by multiplying both sides by 2 — same-to-both-sides, not “move the $1/2$.” The table was not needed here; the balance was. Where someone goes wrong: multiplying only the left half, or converting to decimals midstream without care.

Parallel blank for you: $(1/3)x + 2 = 6$. Autopsy first if you write a full solution; then hide it and retry tomorrow.

Inequality glance (optional light edge)

If your life uses “at most” and “at least,” a short inequality refresh helps: same-to-both-sides still holds for adding/subtracting; multiplying or dividing by a negative reverses the inequality sign — say it when you do it. Check with a test point. This is optional light edge, not a full inequality chapter. If it floods you, skip and stay with equations until done-enough is honest.

Closing reminder

Algebra refresh is lawful rewriting plus pattern sense. Star solved problems, structure, and strategies — labelled grades 6–12 origin — are the portable practice grain. Ordinary stakes only. Tools after the try. Bases and magnitudes still matter underneath. On the commute, read a table. At the desk, balance an equation with verbs. On the weekend, autopsy and compare legal methods. That is the work. That is done enough when the checklist says so — by skill, not by birthday.

If a problem set from an old textbook appears in your life, do not binge it. Pick three items that match this week’s strip: one substitution check, one table, one solve with verbs. Autopsy one worked example from the answer key as a twin — first useful move, wrong-turn hunt — then close the key and try your parallel. Struggle before rescue. Representation before procedure. Schemas not keywords. You already practiced that spine on percent; keep it here on letters and rates.

When you turn to Chapter 4, take the relational equal sign with you. Geometry will ask what you are given, what you must show, and what bridge gets you there. The habit of naming a reason — not only a shortcut label — starts in the balance of an equation and continues into a marked diagram. You are ready when this chapter’s checklist is honest, not when a podcast says algebra is “easy now.”

Chapter 4 — Geometry with reasons

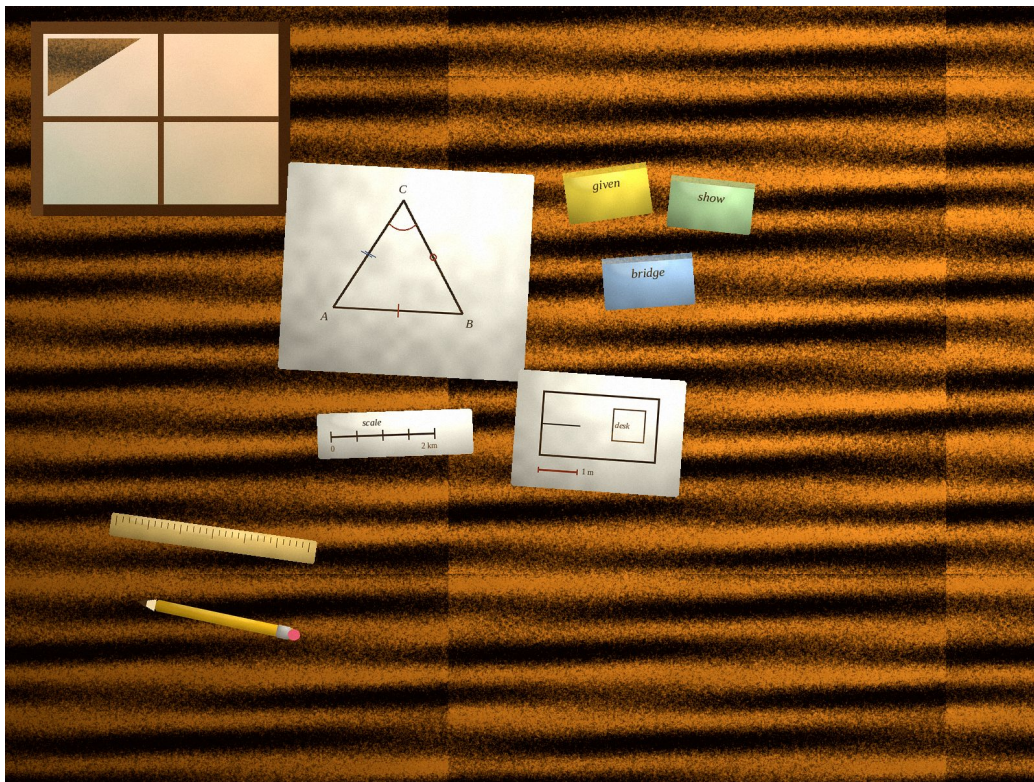


Figure 7: A still-life: marked triangle diagram with sticky notes labelled given, show, and bridge; a map scale bar scrap; a tape measure beside a simple room sketch. No people. No logos. No readable UI.

Why this matters

What are we given? What must we show? What bridge gets us there?

That triad is the whole chapter in one breath. Geometry with meaning is not a laminated formula sheet you last saw in school. It is a habit of saying what is already true on the figure, what must follow, and which definition, property, or theorem carries you across. When those three are clear, the format — two-column, paragraph, or flow — is a costume for the argument, not a religion about what counts as real mathematics.

Many adults remember high-school geometry as “memorize SAS, then fill two columns until the teacher is satisfied,” or as “plug into area and volume until the quiz ends.” Formulas can appear later as summaries of reasoned measurement. Two-column can be a useful scaffold. Neither is the first story. The first story is: mark the figure; name the givens; name what must be shown; find a bridge; say a reason out loud; then pick a format once the logic is clear.

You need this because ordinary adult life asks you to trust a measurement. A map scale claims a trail is three miles. A cut list claims two panels will fit. A room sketch claims you need enough paint for the walls and enough trim for the baseboards — and those are different units. A ramp or a stair rise-and-run claims a slope. Without a reason, you are hoping the sticker is right. With a short reason, you can check yourself before you buy, cut, or walk.

What this idea unlocks sits next door to algebra and next door to functions. Similar triangles are one of the Critical Foundations that support slope thinking later.⁴⁹ Coordinate distance reuses structure you already practiced when you balanced equations. Right-triangle ratios need a real triangle before they become a chant. If geometry is only a formula card, every new figure is a hunt for the right blank. If given, show, and bridge are spoken first, you can change formats without losing the argument — on a commute sticky, at a desk, or in a weekend sit with a trail map.

The struggle is worth it because finished-looking numbers are everywhere. A list-

⁴⁹National Mathematics Advisory Panel, *Foundations for Success* (2008): Critical Foundations include particular aspects of geometry and measurement; similar triangles support slope. Finding 10: conceptual understanding, fluency, and problem solving mutually reinforce. Finding 15: effort and opportunity matter; place by skill, not “too old.” Access date for URLs in these notes: 5 September 2026.

ing says “1,200 square feet.” A box says “covers 400 square feet.” A GPS thumb-nail shows a blue line with no scale bar in view. Fluent numbers are not yet a check. The move is to treat the figure as a claim: what is marked, what must follow, and why.

This book is not a reprint of *Math for Emerging Adults*, *Math for Young Minds*, *Math for Little Thinkers*, or *Mathematics for Homeschooling*. One pointer each: those books keep a parent in the room and a student section on the page. This book does not. You are the learner. Reasons still matter. Then we teach adult stakes — map, room, cut list, hardware-store measure — not teen field-trip dogma. One pointer to *Critical Thinking for Adults*, then math: a short written reason a colleague could follow is a neighbor habit; here the object is a diagram and a measurement claim.

You do not need to be a mathematician. You do need to hear a formula without a reason as a missing-argument move, not a cute slip, and to ask what is given, what must be shown, and what bridge gets there, without grabbing a photo-solver.

The adult who last saw two-column proof in tenth grade is not broken. Custom faded. Meaning can return. You rebuild trust in measurement the same way you rebuild anything usable: one clear claim, one honest wait, one parallel try.

This week you can learn to hear “two-column is the only real proof,” “reasons are for teenagers; adults just measure,” and “paint coverage from length alone.” Today you can mark givens on a simple diagram, write what must be shown, and leave a five-sentence reason for a scale or distance claim.

Understand it yourself

Here is the everyday picture.

You are at a hardware store or at home with a tape measure. The wall is twelve feet by eight feet. Someone says, “That’s ninety-six square feet of paint.” That sentence can be true as area. It is not yet trim length. Baseboard needs feet of length, not square feet. Mixing length units with area units is a classic adult wrong turn — buying trim by area, buying paint by length. The diagram (even a rough rectangle on scrap) marks what is measured. The reason names the unit that matches the job.

A second everyday picture, still ordinary. A trail map shows a scale bar: one inch

equals one mile. You measure two and a half inches of path with a scrap of paper. The real distance is about two and a half miles. At three miles per hour walking, that is roughly fifty minutes — an estimate, not a promise about terrain. Scale factor is the bridge. The map distance is given. The real distance is what you show. Time is optional after the distance is honest.

A third picture, still ordinary stakes. You need to know whether a rectangular panel will fit through a doorway diagonally. The doorway opening and the panel's face make a right-triangle story if you tilt the panel. Pythagoras is not a school souvenir. It is distance with a reason when a right angle is present.

Here is the precise picture.

A **reasoned claim** in geometry is a short chain: given facts on the figure (marked lengths, right angles, parallel marks, similar-tick marks) → a bridge (shared property, congruence or similarity criterion, dissection of area, distance meaning of Pythagoras, scale factor) → what must be shown. A **proof format** is how you write that chain. Two-column pairs statement with reason. Paragraph embeds reasons in sentences. Flow shows premise → conclusion arrows. Formats are optional. Reasons are required.⁵⁰

Think of the format as clothing. The person is the reason. A paragraph that says “because the scale bar says one inch is one mile, and I measured two inches of path, the real distance is two miles” is mathematics. A two-column page that fills rows without connecting givens to the goal is busywork wearing proof clothes.

Similarity and scale. Two figures are similar when corresponding angles match and corresponding lengths stay in a constant ratio — the scale factor. Lengths scale by that factor. Areas scale by the square of that factor. Volumes (when you need them) scale by the cube. Adults who enlarge a cut pattern by “two” and then wonder why material use jumped by four have met the area-scale rule the hard way. Say the rule out loud before you buy sheet goods: if lengths multiply by k , areas multiply by k^2 .

Congruence, lightly. Congruent figures match in size and shape; corresponding parts match. A congruence criterion (such as SAS: two sides and the included angle) is a bridge only when you name which parts correspond. Saying “SAS”

⁵⁰Continuity labelled from Cirillo/Herbst-style proof-teaching arguments and NCTM geometry essential concepts: focus on logical arguments; proofs may be communicated in a variety of formats (two-column, paragraph, and more). School-origin grain, portable as a habit — not a promise that every adult will see the same result, and not a format war as this chapter's identity.

while pointing at the wrong angle is like saying “because algebra” without a step. For adult refresh, you need enough congruence language to trust a matching claim on a diagram. You do not need a semester of contest proofs.

Pythagoras as distance. In a right triangle, the square on the hypotenuse equals the sum of the squares on the legs. On a coordinate sketch, the distance between two points is the hypotenuse of a right triangle whose legs are the horizontal and vertical runs. The formula is not a blank. It is distance meaning with a reason: right angle $\rightarrow$ Pythagoras $\rightarrow$ length. If there is no right angle marked or constructed, stop before you square and add.

Coordinate reading. Axes are a grid you already trust from graphs. Place a simple figure, compute a length or a slope, conclude with algebra you already have from Chapter 3. The algebra carries the reason. The figure still gets marked. Example grain: two opposite sides of a sketched parallelogram have equal slopes, so those sides are parallel — slope equality is the bridge.

Slope language on a ramp (optional out-of-home). Rise over run is slope. A wheelchair ramp, a driveway, or a loading lip is ordinary noticing — not a building-code lecture unless you already have a code in hand as *your* artifact. The math is the ratio and the unit match. Noticing is not a sting. You are not auditing a store; you are practicing slope language on an object you chose.

Dissection of area. An L-shaped room is a rectangle plus a rectangle, or a big rectangle minus a cutout. The bridge is dissection: break the figure into pieces whose areas you trust, then add or subtract. The formula card for “L-shape” is optional; the dissection reason is the work.

Three formats, honest tradeoffs:

Format	Helps	Watch for
Two-column	Forces statement + reason pairing	Filling columns without a global argument
Paragraph	Closer to how adults write a check	Omitting reasons; jumping to the conclusion
Flow	Shows premise $\rightarrow$ conclusion; good planning	Less familiar; may need a rewrite into paragraph

Start a hard claim with a know / need / bridge outline or a flow. Then pick two-column or paragraph for the write-up once the logic is clear. Demand a reason either way. Once this week, optionally rewrite the same reason in a second format so your ear separates costume from person.

Wrong answers you should be able to hear

These are illustrations, not portraits of real workplaces or shops.

1. **Formula name without knowing what is measured.** Someone says “area equals length times width” while pointing at a circle, or quotes Pythagoras with no right angle marked. Ask: what must be true about this figure first? How do you know?
2. **Two-column is the only real proof.** Format fetish. Ask whether another format still carries the same reason. Cirillo/Herbst-style continuity in proof teaching presses logical arguments over form fetish — labelled school-origin grain, not a commute RCT.⁵¹ Two-column proving became a sticky American school custom; custom is not mathematical necessity.
3. **Scale changes lengths and areas by the same factor always (unchecked).** Enlarging lengths by 2 does not enlarge area by 2. Area scales by $2^2 = 4$. Catch it on a cut list, a fabric estimate, or a photo enlargement that must cover a wall.
4. **Paint coverage estimated with length units alone.** Or trim bought in square feet. Unit mismatch. Ask: are you mixing length units with area units?
5. **Reasons are for teenagers; adults just measure.** Adults still need trust in a measurement. A tape can lie if you measured the wrong thing. A map can mislead if the scale bar is ignored. The reason is how you know — including how you know you measured the right quantity.

A sixth you will also hear from yourself on a tired day: “The app already calculated it.” Fluent output is not a check. Estimate from the scale bar or the tape first; then compare.

⁵¹Continuity labelled from Cirillo/Herbst-style proof-teaching arguments and NCTM geometry essential concepts: focus on logical arguments; proofs may be communicated in a variety of formats (two-column, paragraph, and more). School-origin grain, portable as a habit — not a promise that every adult will see the same result, and not a format war as this chapter’s identity.

Five-minute warm-up

Before the try, take a scrap of paper. Draw a simple rectangle. Mark length and width. Write: *Given*: ____ . *Show*: *area (or perimeter)*. *Bridge*: ____ . Say one sentence out loud that is not only a formula name. Then glance at a second object — a map snippet with a scale bar, or a room sketch — and ask yourself the locked opening once. Cover any answer key. Place one sticky: “show: ____ / bridge: ____.” You are installing the ear this week needs.

A worked grain you can autopsy (composite).

Given: a rectangular closet floor measures 4 ft by 6 ft on your sketch; you need vinyl for the floor and a threshold strip for the 4 ft doorway edge only.

Show: (a) floor area; (b) threshold length.

Bridge for (a): rectangle area after stating the shape. *Bridge* for (b): length of one side — not area.

Paragraph reason: “The floor is a rectangle 4 ft by 6 ft, so the area is $4 \times 6 = 24$ square feet; that is what a covering needs. The threshold is a length along the 4 ft edge, so I need 4 feet of strip, not 24 feet and not 24 square feet.”

Wrong twin to hear: “I need 24 feet of vinyl.” That sentence mashes area into length language. Cross it out. Keep the unit in the noun.

Similarity grain you can say aloud.

A photo of a rectangular table is 2 in by 3 in on paper. The real table is 4 ft on the short side. Scale factor from photo to real: 4 ft / 2 in requires matched units first — convert, then divide. Corresponding long side follows the same factor. Area of the real tabletop is not “photo area times the length factor”; it is photo area times the length factor squared, after units match. If unit conversion feels rusty, point back to Chapter 1 and rename before you scale.

How to try it this week

The Refresh Hour in the front of this book is the full version: micro-session, desk sit, weekend deep sit, talk-to-yourself shape, and the AI box once for adults. Here is the shape scaled to this idea.

Session shape across bands

Commute / lunch (~15–20 minutes). Retrieve (2–3 min): cover notes; ask “What are we given? What must we show?” about yesterday’s diagram; one sentence. Solved-problem autopsy (5–7 min): look at a marked figure already worked; name the first useful move (usually marking); name the bridge; where could someone go wrong (unit mismatch, formula with no right angle, scale applied to area as if it were length). Parallel try (5–7 min): a new simple diagram — mark givens, write what must be shown — unaided. Exit sticky (1 min): I can / I almost / I cannot yet — next cue is ____.

Desk-or-shift (~25–40 minutes). Warm-up retrieval mixed with one length-vs.-area glance. Representation first (8–12 min): mark a diagram before any formula; tick marks, right-angle box, scale bar copied onto scrap. Short model / fade a step (5 min): know / need / bridge on one figure, then stop before finishing the last sentence for yourself. Two tries + interloper (10–12 min): one scale claim, one Pythagoras-as-distance, and one interloper that looks like area but asks for perimeter (or the reverse). Self-question + wait + written reason (3 min). Exit ticket: name the bridge in one sentence.

Weekend deep sit (~45–75 minutes when needed). Diagnostic wrong: a worked twin that bought paint with length units, or enlarged a pattern by 2 and multiplied area by 2. Longer solved with structure: map scale $\rightarrow$ real distance $\rightarrow$ optional walking time at a stated speed. Alternative strategies when legal: same reason in paragraph and in two-column (optional swap); dissection versus “L-shape formula” for an L-room. Mixed 4–6 with interleaving: similarity scale factor, coordinate distance, room area, trim length, one ramp rise/run if you want slope language. Done-enough checklist. Schedule next week’s three micro cues on a calendar — space across days; do not dump the chapter into one Sunday marathon.

Timing honesty: commute noise and fatigue are real. A micro-session that becomes scrolling is not a session. A deep sit that becomes “finish every theorem in a PDF” is a textbook dump wearing a weekend hat. Three to five micros plus one desk sit in a week, with an occasional deep sit, is the honest ambition for this strip.

Exact wording you can say to yourself

- “What are we given? What must we show? What bridge gets us there?”
- “What does the diagram already mark?”
- “Is my reason only a shortcut name?”
- “Are the units length or area — and which does this job need?”
- “Would another format still carry the same reason?”

- “If lengths scale by k , what happens to area?”
- “You may change your mind. That is not losing.”

Ask one. Wait. A slow three is a convention, not a sacrament (Stahl think-time, labelled classroom-origin; Rowe’s waits were science class — not an adult math RCT).⁵² After you stop mid-reason, wait again before peeking. Look at the diagram, the map, or the tape marks — not at a phone’s answer key — if the silence is hard. Rescue at one second is still rescue. Abandonment for forty minutes of scrolling is not a micro-session.

First try-it (on the go)

Mark givens on a simple diagram; write what must be shown. Time: 5–10 minutes. Materials: scrap paper or phone note; a printed diagram or a quick sketch of a rectangle or right triangle. Safety: ordinary noticing — not an undercover audit of a workplace floor plan; do not paste proprietary drawings into a public tool. The move: highlighter or underline for givens; one line that starts “Show: ____”; do not compute until those two are written. That is the whole first try. Computation waits for the next sit.

How to fade help

First time: you copy a know / need / bridge from a solved model, then fill the reason. Second time: you write know / need / bridge yourself from a new figure. Third time: you write a short paragraph reason unaided, then optionally swap into two-column to see the same reason wearing different clothes. If a dynamic geometry sketch helps you *see* corresponding parts after you tried, use it as a check — not as the first attempt. If a tool is in the room, it waits. Point back to the refresh-hour AI box for the full allow and ban list. This chapter is not an AI policy paper.

When to stop looking up

Stop when you can name givens, state what must be shown, and give a reason that is not only a shortcut label. Looking up the name of a theorem after you have stated the property in plain language is fine. Looking up a finished proof to paste as yours during the attempt is not the work. Looking up a coverage rate on a paint can is a product label; converting your measured area with that rate is arithmetic on top of a reason — still not a product pitch from this book.

⁵²Stahl (1994) think-time; Rowe (1972) wait-time in science class. Classroom-origin conventions for pausing before rescue. Do not mash as an adult math RCT.

Practice that actually builds it

Named try-its live **on the go and in deeper sits**. Maps, rooms, and cut lists *are* the practice. They do not replace the reason.

Talk-to-yourself box

Opening question (locked): “What are we given? What must we show? What bridge gets us there?”

Follow-ups:

1. What does the diagram already mark?
2. Is your “reason” only a shortcut name?
3. What bridge connects givens to what must be shown?
4. Would another format still carry the same reason?
5. Are you mixing length units with area units?

How to wait alone: After you ask, cover the answer. Count a slow three. If you are mid-reason, wait again before peeking. Look at the figure, not the key. If the silence is hard, put your finger on one marked given and say only that given out loud — then return to the opening question.

Stuck silence usually means: the question was vague; wait-time was zero; you are hunting for a theorem name instead of a property; you offered a formula where a unit match was asked for; you think “adults just measure” means skip the reason; a finished-looking chatbot paragraph looks like a check; anxiety is eating working memory — shorten the session, mark one given successfully, come back tomorrow. Your job is to pose, wait, and point back at the object — not to fill the silence with a photo-solver’s proof.

On the go — commute / lunch

Try-it A — Scale factor glance. Time: 8–12 minutes. Materials: map snippet, floor-plan thumbnail, or textbook diagram with a scale bar (composite is fine; greek any brand). Safety: public map or your own print — not photographing proprietary employer plans into a public tool. The move: measure map distance; apply scale; write real distance with units; optional one-line time estimate at a stated walking speed you choose. Exit: sticky with “given / show / bridge.”

Try-it B — Square vs linear units. Time: 5–8 minutes. Materials: mental room or a phone photo of a wall you already know. Safety: ordinary home noticing. The move: name one job that needs area (paint, flooring) and one that needs length (trim, cable). Write both units. Catch the mismatch on purpose once — write a wrong twin that says “22 ft of paint” for a 12-by-10 wall, then cross it out and repair.

Try-it C — Mark-only micro. Time: 5 minutes. Materials: any simple figure on a flyer or in a book. The move: mark givens only; write “Show: ____”; stop. No calculation. This installs the triad before arithmetic steals the session.

Desk-or-shift

Try-it D — Room measure with reason. Time: 20–30 minutes. Materials: tape measure or known dimensions; scrap paper. Safety: measure your own space; do not climb unsafely; this is math practice, not a contractor bid and not a landlord dispute script. The move: sketch the floor; mark givens; show area for a covering; show perimeter or baseboard length separately; five-sentence reason that names the bridge (rectangle area formula *after* stating the shape, or dissection if the room is L-shaped). Unit check ends the page.

Try-it E — Similarity cut-list. Time: 15–25 minutes. Materials: a simple shape to enlarge by a scale factor (2 or 3). The move: scale all lengths; compute original and new area; say why area did not scale by the same factor as length. If you cut paper models, safety is ordinary scissors sense — no workplace machine practice unless that is already your job and your training.

Try-it F — Coordinate distance. Time: 15–20 minutes. Materials: grid scrap. The move: plot two points; draw the right triangle; compute distance; speak the Pythagoras reason before the calculator’s square root.

Weekend deep sit

Try-it G — Map or trail with five-sentence reason. Time: 30–45 minutes. Materials: trail map or city map with scale bar; ruler or marked scrap. Safety: desk practice first; if you walk the trail later, ordinary outdoor sense — this try-it is the math on paper. The move: scale bar → measured path → real distance → optional time; write five sentences: given, show, bridge, calculation, unit check. Optionally estimate elevation change only if the map provides a clear contour story you

can trust — skip contour estimates you cannot see on the map.

Try-it H — Format swap. Time: 15–20 minutes. Same reason written as paragraph, then as two-column or flow. Read both aloud. Ask: did the reason change, or only the costume?

Try-it I — Ramp rise/run (optional out-of-home). Time: 15 minutes on site or from a photo you take of a ramp you already use. Safety: noticing, not a code inspection; do not block access; do not climb on structures. The move: estimate or measure rise and run; write slope as a ratio; one sentence of slope language. If you hold an actual code as your artifact, compare — that comparison is *your* document, not this book’s legal advice.

Blocked practice for a new move: three figures in a row where you only mark givens and write “show,” no computation yet. Then three where you only name the bridge. Then three full short reasons.

Mixed practice: after the move is familiar, interleave scale, area-vs-length, distance, and similarity so you must choose *when* to use each. Mixing is how you learn the trigger, not only the steps.

One incorrect example to diagnose

A worked twin (composite illustration) says: “Room is 12 ft by 10 ft. Paint needed: 22 ft.” The error is treating a sum of sides (or a perimeter fragment) as coverage. Diagnose with the locked opening: what was given? What must be shown for paint? What bridge (area) was skipped? Repair: $\text{area} = 120 \text{ ft}^2$; then, if a can claims a coverage rate, convert — still structure, not a brand pitch. Leave the wrong twin labelled wrong so it does not look authoritative when you are tired on a train.

A second wrong twin (scale): “Pattern enlarged by 2, so fabric area doubles.” Repair: $\text{lengths} \times 2$; $\text{area} \times 4$. Say it in one sentence you could tell a colleague at a cutting table.

Solved-problem autopsy template (use once this week).

Take any short geometry solution — from OpenStax, a worksheet, or a dated note you already wrote. Cover the final number. Ask: What was marked first? What was the bridge? Where could someone go wrong? Then try a parallel figure with different numbers, unaided. Star-style solved-problem habits travel from school guides into self-study when you are the discussant; the evidence tiers are school-

origin — portable, not an adult RCT.⁵³ This is the same autopsy shape as algebra in Chapter 3, moved onto a figure.

Spacing across the week (seed, not a 180-day plan).

Mon micro: mark givens only. Tue micro: scale bar distance. Wed desk: room area vs trim length. Fri micro: Pythagoras distance on a tiny coordinate pair. Optional Sun: five-sentence map reason + checklist. Murray-style spacing in math is a useful study grain for distributed practice; it is not a promise that every adult will see the same result.⁵⁴ Mix one interloper from Chapter 2 (unit price) or Chapter 3 (slope) so you practice *choosing* the strip.

If it isn't clicking

Diagnostic 1 — You freeze when the figure is rotated or reflected. Next move: redraw the figure in a familiar orientation; rematch corresponding parts with your finger; then rotate it back and rematch again. Panic at rotation is ordinary rust, not a verdict on your intelligence. Corresponding parts are a matching claim, not a vibe.

Diagnostic 2 — Your “reason” is only a name. You write “SAS” or “Pythagorean theorem” with no pointing and no plain-language property. Next move: say the property in ordinary words first (“two sides and the included angle match,” or “right triangle, so distance is the hypotenuse”), then optionally attach the name. Shortcut labels without meaning are formula-card geometry wearing proof clothes.

Diagnostic 3 — Units keep sliding. Paint, trim, cable, and flooring blur together. Next move: two-column sticky on your desk — left column “length jobs,” right column “area jobs” — with one example each from *your* life. Every try-it this week ends with “unit check: ____.” If scale factors confuse area, draw a 1-by-1 square and a 2-by-2 square and count the unit squares by hand once.

⁵³National Mathematics Advisory Panel, *Foundations for Success* (2008): Critical Foundations include particular aspects of geometry and measurement; similar triangles support slope. Finding 10: conceptual understanding, fluency, and problem solving mutually reinforce. Finding 15: effort and opportunity matter; place by skill, not “too old.” Access date for URLs in these notes: 5 September 2026.

⁵⁴Murray and colleagues (2025) mathematics spacing meta-analysis: spacing versus massing about $g \approx 0.28$ — labelled encouragement for distributing geometry micros across days, not an adult commute RCT of this chapter’s objects.

When to slow down: if similar triangles and scale factors are still mush, stay on scale and unit match before congruence criteria. Point back to Chapter 1 if fraction scale factors feel shaky, and to Chapter 3 if coordinate slope language is still letter-moving without structure. Unfinished fractions still block; diploma year does not clear the gate.⁵⁵

When to go ahead: if you can mark givens, state show, and write a five-sentence reason on a map or room, add optional format swap and a similarity cut-list. You do not need a full high-school proof course to be done enough for adult measurement trust. Place by skill strip, not by how many geometry theorems your school once listed.

When to ask a human: a colleague in a trade, a tutor, or a community-college desk hour — especially if a workplace drawing has conventions you do not yet read. No shame. Correct a wrong reason without crushing the attempt — applied to yourself. “Never correct yourself” is not this book’s rule. Hearing a wrong reason, asking a better question, naming a better reason if needed, is the work.

Anxiety note for this strip: short sessions, early wins (one clean given/show pair), no countdown timer as default rigor. That is not a therapy manual; it is session design that respects working memory under math anxiety.⁵⁶ If a deep sit turns into dread, cut it to one map scale and stop. Return tomorrow.

Tools, including AI

Optional helpers. Keep this short. Point back to the refresh-hour AI box.

You may, **after** you have tried: check a distance with a calculator; explore a dynamic figure in GeoGebra or Desmos Geometry *after* the reason is spoken; ask

⁵⁵National Mathematics Advisory Panel, *Foundations for Success* (2008): Critical Foundations include particular aspects of geometry and measurement; similar triangles support slope. Finding 10: conceptual understanding, fluency, and problem solving mutually reinforce. Finding 15: effort and opportunity matter; place by skill, not “too old.” Access date for URLs in these notes: 5 September 2026.

⁵⁶Erin A. Maloney and Sian L. Beilock, “Math Anxiety: Who Has It, Why It Develops, and How to Guard Against It,” *Trends in Cognitive Sciences* 16, no. 8 (2012): 404–6; Sian L. Beilock and Daniel T. Willingham, “Math Anxiety: Can Teachers Help Students Reduce It?,” *American Educator* 38, no. 2 (Summer 2014). Anxiety loads working memory. Not a clinical protocol in this book.

a tool for a hint level (question → name the idea → next move still not the finished proof); diagnose a photo of *your* marked diagram (crop to paper — no face); generate isomorphic practice with answers on **your** page.

You may not: photo-to-key a textbook proof during the attempt (Photomath-class loops — the loop is the ban); paste an employer floor plan into a public consumer chat; ask “just give me the steps” before your know / need / bridge exists; treat a finished chatbot proof as your work; invent a measurement the tape did not give.

Labelled dynamic geometry tools replace thinking when they finish the attempt. They check or explore when they wait. A calculator’s square root is fine after the Pythagoras reason is said. The model may prepare you and the next problem; it may not do your problem during the attempt — the same lunch-break rule as the refresh hour (Bastani ON-AGE-adjacent mechanism, not an adult RCT).⁵⁷

Illustration cue (for you, on scrap). Marked diagram with givens highlighted; sticky that says “show: ____ / bridge: ____.” Map scale bar next to a measured scrap. Crossed-out “22 ft of paint” sticky beside a corrected “120 ft².” No portraits. No brand logos. The picture teaches the triad and the unit catch — money and DIY scenes motivate; they do not replace the reason in the picture.

What “done enough” looks like

Place by skill, not diploma year. A “geometry badge” from a chatbot is a receipt for nothing. Optional credit paths (including exams named later in Keeping track) are bridges when credit is the goal — not the spine of this chapter. Sitting an exam is not finishing this book.

Skill-strip checklist — Geometry with reasons

- ☐ I can mark givens on a simple diagram and state what must be shown.
- ☐ I can name a bridge that is not only a shortcut label.
- ☐ I can catch length-vs-area unit mismatch on an ordinary stake (paint vs trim).
- ☐ I can turn a scale bar into a real distance with units (and optional time).

⁵⁷Bastani et al. (2025), *PNAS*: high-school math field experiment (grades 9–11, Turkey); un-guarded chatbot helped practice look better and left closed-exam performance worse; guarded tutor withheld answers. **Not an adult RCT.** Mechanism sentence for the lunch break: the model may prepare you; it may not do your attempt.

- ☐ I can use Pythagoras as distance on a right triangle or coordinate sketch with a reason.
- ☐ I can scale lengths by k and explain why area scales by k^2 .
- ☐ I can choose a format (paragraph / two-column / flow) without treating format as the mathematics.
- ☐ I can diagnose a wrong twin that used a formula with no figure reason.

Life of the habit (plain lines): Revisit a map or room check next Tuesday for ten minutes. Space a similarity scale-factor item across the week. Mix last week's percent or algebra sticky with this week's diagram once, so strips talk to each other. Stop when the next life demand (a cut list, a trip plan, a furniture fit) is met — not when a fake certificate appears.

If this strip is solid and you need function families or a thin trig module next, Chapter 5 waits. If calculus edges are not your need, you are not behind. Measurement trust was the promise here. Reasons required. Formats optional. Ordinary stakes were enough.

Chapter 5 — Functions and trigonometry

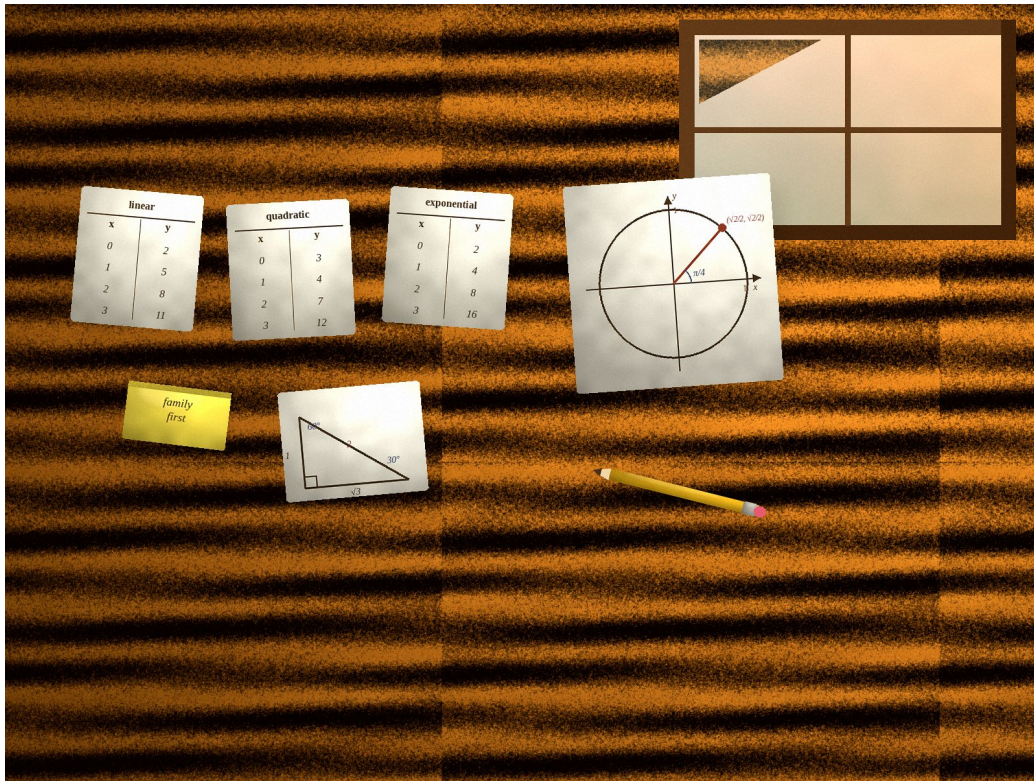


Figure 8: A still-life: three anonymous function-family table cards; a unit-circle sketch with quadrant signs; a right-triangle height scrap. No people. No logos. No readable UI.

Why this matters

Is this table linear, quadratic, or exponential — how do I know?

That question is the spine of the first half of this chapter. A table is not a logo to memorize. It is a story about what stays constant: a common first difference (linear rate), a common second difference (quadratic), or a common ratio (exponential). When you can say which family and why — without a regression button as the only decision — you can read a sports rate, a cost model, a savings story, or a growth sketch with structure instead of letter-chasing.

Rust on function families is ordinary. Many adults were taught to recognize a parabola by shape and to press a button when the homework said “regression.” Recognition and buttons are not nothing. They are not yet “how do I know?” This chapter rebuilds the how.

The second half is a thin trigonometry module **by need**. Right-triangle trig for height and distance when life or work asks. Unit circle for signs and special angles when further study or a technical task asks. Many adults never need a full precalculus course to be numerate at work and at home. Some need a thin trig strip for trade, design, navigation-adjacent tasks, or a STEM bridge. Place by need and by skill. Do not treat “everyone must finish the unit circle” as a moral finish line.⁵⁸

What this idea unlocks is cleaner algebra behind you and optional calculus edges ahead. Covariation language — as this changes, that changes — is the bridge toward rate talk in Chapter 6 without forcing calculus on every reader. Function families without structure become graph-logo memorization. Trig without a triangle becomes a SOHCAHTOA chant at empty air. Structure first. Chant later, if at all.

You need this because ordinary adult surfaces already behave like functions. Hours map to pay when the rate is steady. Time maps to distance when speed is roughly constant. A dosage schedule maps time to amount remaining in a labelled toy story (rate language — not medical advice). A dashboard tile maps week number to a count. The question is not “did I take precalc?” The question is whether you can say what kind of relationship you are looking at, and why.

⁵⁸College Board AP Precalculus Course and Exam Description is a labelled map when a course uses it — not a kitchen RCT and not imported wholesale into this how-to. Unit circle and trig by need. Access date: 5 September 2026.

This book is not a reprint of *Math for Emerging Adults* or the other school-age manuals in this house. One pointer: content names overlap — families, unit circle, right-triangle trig. Parent talk-boxes and dinner-table unit-circle scripts do not transfer here. You are the learner. Talk to yourself. Then teach the strip you actually need this month.

You do not need to be a mathematician. You do need to hear regression-button family ID, SOHCAHTOA with no triangle in view, and “signs on the unit circle are arbitrary” as wrong turns — and to ask a better question before a tool finishes the attempt.

You already use function thinking when you say “depends on.” This chapter only makes the depends-on honest: which family, which triangle, which sign — and how you know.

This week, on the go, you can sort one table into a family with a why. In a deeper sit, if this strip is yours, you can place three special angles on a unit-circle card and justify one sign.

Understand it yourself

Here is the everyday picture.

A three-row table of hours worked and dollars earned:

Hours	Pay (\$)
2	40
3	60
4	80

First differences in pay: +20, +20. The rate stays constant. That is a linear story: twenty dollars per hour in this composite. You did not need a curve-fit button. You needed “what stays constant?”

A second table, still ordinary:

Week	Balance (toy)
0	100
1	110
2	121

Ratios: $110/100 = 1.10$, $121/110 = 1.10$. Common ratio. Exponential growth at 10% per week in this invented toy — labelled invented, not a product pitch. Linear would have added the same dollar amount each week; this multiplies.

A third table:

x	y
0	0
1	1
2	4
3	9

First differences of y: +1, +3, +5. Second differences: +2, +2. Common second difference points to quadratic. The graph would be a parabola shape — but the table reason came first, not a logo.

Here is the precise picture.

A **function** assigns to each allowable input exactly one output. You can say “pay as a function of hours” when each hour-total maps to one pay amount under the rule. **Covariation** is the plain habit: as the input changes, the output changes — and you can describe how. Linear: constant rate of change. Quadratic: rate of change itself changes in a patterned way (constant second differences on equally spaced inputs). Exponential: constant multiplicative factor per step.

Family sort without logos. Tables and “what stays constant?” beat memorizing graph shapes as brand logos. Exponential: common ratio. Linear: common first difference. Quadratic: common second difference. Other families exist (absolute value, rational, periodic trig, piecewise). Meet them by need. Do not pretend a kitchen must finish a catalogue.

Right-triangle trigonometry (thin module). In a right triangle, for an acute angle θ :

- $\sin \theta = \text{opposite} / \text{hypotenuse}$
- $\cos \theta = \text{adjacent} / \text{hypotenuse}$
- $\tan \theta = \text{opposite} / \text{adjacent}$

SOHCAHTOA is a memory aid **after** the right triangle is identified and the sides are named relative to the angle. Chant without a triangle is incomplete. Height across a gap, ladder lean (ordinary safety — do not climb beyond your training), or a ramp angle from rise/run are adult objects when you need them.

Unit circle (by need). The unit circle is a circle of radius 1 centered at the origin. An angle in standard position lands at a point $(\cos \theta, \sin \theta)$. Signs follow quadrants: both positive in QI; sine positive and cosine negative in QII; both negative in QIII; sine negative and cosine positive in QIV. Signs are not arbitrary; they follow coordinates. Special angles ($0, \pi/6, \pi/4, \pi/3, \pi/2$ and their degree twins) have coordinates you can rebuild with 30-60-90 and 45-45-90 triangles, not only flashcards.

Degree / radian mode (name once). Calculators and CAS tools have a mode switch. Wrong mode is a tool trap: a cosine in degree mode when the problem is in radians (or the reverse) produces confident nonsense. Check mode before you trust a button. Then return to meaning.

AP Precalculus CED? If you meet a College Board map in a course, label it a map. It is not a kitchen RCT, and this book does not import it wholesale.⁵⁹ Place by need.

Linear, quadratic, exponential — a longer look at “what stays constant.”

Equally spaced inputs matter for the difference tests. If your x-values jump unevenly, first differences can mislead; either resample at equal steps or go to a rate-per-unit calculation. On equal steps:

- Constant first difference $\rightarrow$ linear. The common difference divided by the x-step is the rate (slope) in output units per input unit.
- Constant second difference $\rightarrow$ quadratic pattern on that grid. The parabola story is waiting; the table already told you the family.

⁵⁹College Board AP Precalculus Course and Exam Description is a labelled map when a course uses it — not a kitchen RCT and not imported wholesale into this how-to. Unit circle and trig by need. Access date: 5 September 2026.

- Constant ratio (each y multiplied by the same factor) $\rightarrow$ exponential on that grid. Growth factor 1.05 is “up 5% per step”; decay factor 0.9 is “down 10% per step.” Successive percents from Chapter 2 are the same multiplicative idea wearing finance clothes — here the clothes are a function family.

Piecewise rules show up at work: overtime after 40 hours; shipping tiers; tax brackets as a neighbouring idea (structure, not tax advice). A piecewise function is still a function if each input lands in one piece. Say which piece before you compute.

Covariation toward rate language (without forcing calculus).

Ask: as x changes by a little, what happens to y ? If the change in y per change in x stays steady, you are in linear rate language — Chapter 3’s slope in units. If that rate itself trends up or down, you are hearing the door to Chapter 6’s “what is the average rate becoming?” You do not have to walk through that door. You do need the sentence: as this changes, that changes — how?

Right-triangle module — worked grain (composite).

A baseline across a pond is measured as 40 m (composite). A sight angle to the opposite bank tree is 35° from horizontal in a textbook-style problem. Draw: right triangle; adjacent = 40 m for that angle if the baseline is adjacent; opposite = height difference you seek if that is the setup — or rearrange which side is opposite based on your diagram. The point is not one memorized word problem. The point is: diagram first, names second, ratio third, calculator fourth. Mode: degrees. Estimate: $\tan 35^\circ$ is a bit less than $\tan 45^\circ = 1$, so opposite is a bit less than 40 m — sense check before you trust seven decimals.

Unit circle — worked grain.

Angle $\pi/4$ sits on the line $y = x$ in QI. The point is (22, 22). Both coordinates positive — because both x and y are positive in QI. Angle $2\pi/3$ is in QII: cosine negative, sine positive. You can know the sign before you recall the exact value. That order — location, then sign, then exact value — fights “signs are arbitrary.”

Reference angles pull a QII/QIII/QIV angle back to a QI acute angle for magnitude, then reattach the sign from the quadrant. If that sentence is new, stay on QI specials until they are solid; then add one QII angle with sign justification only.

Wrong answers you should be able to hear

1. **Family ID by graph logo memorization only.** “It looks like a U, so quadratic.” Sometimes yes; sometimes a U-shaped sketch is something

else, or the window is lying. Ask what stays constant in the table.

2. **Regression button as the only decision.** The tool fits a curve; you still need a reason. Ask: can you decide without the button?
3. **SOHCAHTOA with no triangle in view.** Chant aimed at empty air. Ask: where is the right angle? Which side is opposite?
4. **Signs on the unit circle are arbitrary.** “I just memorize All Students Take Calculus.” Mnemonics can help after quadrant meaning; they are not a substitute for (x, y) signs. Ask: where is this angle — and why is the sign what it is?
5. **Everyone must finish precalc to be “numerate.”** False as a status claim. Many adults are done enough without a full precalc course. Place by need and by skill strip.

A sixth: “As x doubles, y doubles” said about an exponential as if it were linear. Covariation sentence needed: for exponential, equal input steps multiply y by a constant; doubling x is not the same as one step unless the step size is designed that way.

Common ratio vs common difference — finger test.

Put your finger on y_1, y_2, y_3 . Ask: did I add the same number, or multiply by the same number? If neither, check second differences for quadratic, or admit “other / need graph / need context.” Not every table in life is one of the three families. A messy operations log may be piecewise or noisy. Honest “not a clean family” is better than forcing a logo.

Law of sines / cosines?

Named only so you know they exist for non-right triangles when a course or trade task requires them. Not required for done-enough in this chapter’s thin module. If you need them, rebuild from a labelled diagram and a named course resource — after right-triangle fluency.

Radians as distance on the unit circle.

One radian is the central angle that cuts an arc equal in length to the radius. On the unit circle, arc length equals the angle in radians. That meaning helps why calculus later likes radians — labelled forward pointer, not a requirement here. If you only work in degrees for a trade task, say so and keep mode matched. Conversion:

π radians = 180° . Keep that pair on a sticky if both languages appear in your materials.

Five-minute warm-up

Cover any key. Write three tiny tables (or copy the three above without labels). For each, ask: first differences, second differences, or ratios — what stays constant? Circle the family name. Then, if trig is in your plan this week, sketch a right triangle, mark one acute angle, and label opposite, adjacent, hypotenuse once. Stop. That is the warm-up.

Understand it further — exponential vs linear in one adult story.

Two composite savings sketches (invented round numbers, not advice). Plan A: add \$50 each month to a jar — linear in months if we ignore interest. Plan B: a balance that multiplies by 1.01 each month in a toy model — exponential. After many months the multiplicative plan pulls away; early on they can look similar if you only glance. The table tells them apart: constant difference vs constant ratio. Adults who say “it grows steadily” may mean either story. Ask which steady — add or multiply.

Periodic edges (optional mention).

Temperature over a day, tide tables, or a rotating machine cycle can look periodic. Sine and cosine as waves live here. If your life never asks for a wave model, you may skip. If a course asks, connect the unit-circle point $(\cos \theta, \sin \theta)$ as θ advances — the wave is the coordinate travelling. One weekend sit is enough to meet the idea; a full trig identity course is a different project.

Inverse talk, lightly.

Sometimes you know the output and need the input: given pay, find hours at a known rate; given a ratio, find an angle (arctan) in a right-triangle setting. Inverse trig buttons need the same triangle discipline and mode check. Sense-check the angle size: an elevation of 80° is steep; 10° is gentle. If the button returns a quadrant you did not expect, ask whether the application needs a different quadrant than the calculator’s principal value.

Incorrect solved twin — family.

A worked page shows points (1,2), (2,4), (3,8), (4,16) labelled “linear because it increases.” Repair: ratios 2,2,2 $\rightarrow$ exponential. Increasing is not a family. What stays constant is the family test.

Incorrect solved twin — trig.

A page computes hypotenuse with sine using an opposite side but labels the angle at the wrong vertex. Repair: redraw; mark the angle with an arc; rebuild opposite/adjacent. The error is correspondence, the same disease as swapped corresponding parts in geometry.

Desk script you can say (after you tried, labelled SCRIPT).

“First I look for equal x-steps. Then I compute first differences. If those are constant, I stop and call it linear. If not, I compute second differences and ratios. I name the family and write because _____. Only then may I open a graph check.”

For trig: “First I draw. Then I mark the right angle. Then I name opposite and adjacent for *this* angle. Then I choose sine, cosine, or tangent. Then I check mode. Then I estimate. Then I compute.”

How to try it this week

The Refresh Hour holds the full session shapes and the AI box. Here is the shape scaled to families and thin trig.

Session shape across bands

Commute / lunch (~15–20 minutes). Retrieve: “What stayed constant in yesterday’s table?” Solved autopsy: one worked family sort; name the first useful move (differences or ratios before graph). Parallel try: sort one new anonymous table with a why — unaided. Exit sticky: family + one-line why.

Desk-or-shift (~25–40 minutes). Representation first: table before equation before graph logo. Short model: show first differences on a linear, second on a quadratic, ratios on an exponential; fade the last column for yourself. Two tries + interloper: two family sorts and one table that looks curved in a sketch but is linear in the numbers (window trick). If trig is active: one right-triangle height with sides named before the ratio. Self-question + wait. Exit: “I know because _____.”

Weekend deep sit (~45–75 minutes when needed). Diagnostic wrong: regression-only decision or SOHCAHTOA with no triangle. Longer solved: three-table family sort; match one graph to a family with a table reason. Alternative strategies: equation, table, and graph for the same linear story — say which

form makes the rate visible. If this strip includes unit circle: card with 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$; place points; justify one sign by quadrant. Mixed: one covariation sentence (“as x increases by 1, y ...”). Done-enough checklist. Schedule micros — space; do not mass the unit circle into one panicked Sunday unless that is truly your need this month.

Exact wording you can say to yourself

- “Is this table linear, quadratic, or exponential — how do I know?”
- “What stays constant — first difference, second difference, or common ratio?”
- “Can I decide without a regression button?”
- “Where is this angle — and why is the sign what it is?”
- “Is SOHCAHTOA attached to a real triangle here?”
- “As this changes, that changes — how?”
- “You may change your mind.”

Ask one. Wait a slow three. Look at the table or the circle sketch, not the key. Rescue at one second is still rescue.⁶⁰

First try-it (on the go)

Sort one table into a family with a why. Time: 8–12 minutes. Materials: scrap or phone note; a three- or four-row table (pay, distance, or invented). Safety: ordinary; do not paste proprietary employer datasets into a public tool. The move: compute differences or ratios; write “linear / quadratic / exponential because ____.” No regression button on the first try.

How to fade help

First: differences/ratios columns filled in a model. Second: you add the columns. Third: you sort unaided and only then optionally check with a graphing tool. For trig: name sides before pressing sin/cos. Fade the chant until the triangle is labelled.

When to stop looking up

Stop when you can sort three tables with reasons and, if trig is in plan, justify one unit-circle sign and one right-triangle ratio. Looking up a special-angle coordinate after you tried to rebuild it from a triangle is fine. Looking up a finished identity proof to paste during the attempt is not.

⁶⁰Stahl think-time / Rowe wait-time: classroom-origin pause habits. Not an adult math RCT.

A full micro-session example (composite times).

Retrieve (3 min): Yesterday’s table was exponential because the ratio was 1.1 each week — say it without notes. Autopsy (6 min): A solved linear pay table; first useful move was subtracting successive pays; wrong turn would be calling it exponential because “pay grows.” Parallel try (6 min): New table 5, 8, 11, 14 → first differences +3 → linear. Exit sticky: “I can spot constant first differences. Next cue: second differences on a curved-looking list.”

That is a complete commute session. No unit circle required that day. Trig waits for the week you need it.

Weekend deep-sit example when trig is in plan (composite).

Diagnostic (10 min): SOHCAHTOA written with no figure — cross out; redraw. Solved (15 min): height problem with baseline and angle; full labels. Alternative (10 min): same height using tangent vs using sine with hypotenuse if hypotenuse given in a twin problem — say why both can be legal when the figure supports them. Unit-circle card (20 min): three angles, one sign justified. Mixed (15 min): one family table interloper among trig items. Checklist + calendar next micros (5 min). Stop. Do not “finish the PDF.”

Practice that actually builds it**Talk-to-yourself box**

Opening question (locked): “Is this table linear, quadratic, or exponential — how do I know?”

Unit-circle follow-up (in this box, when that strip is active): “Where is this angle — and why is the sign what it is?”

Follow-ups:

1. What stays constant — first difference, second difference, or common ratio?
2. Can you decide without a regression button?
3. Where is this angle — and why is the sign what it is? (unit-circle)
4. What would covariation say in one sentence?
5. Is SOHCAHTOA attached to a real triangle here?

How to wait alone: Cover the key. Slow three. If stuck on signs, point to the

quadrant first — positive/negative axes — then name sine/cosine as y/x on the unit circle.

Stuck silence usually means: hunting a vocabulary word; waiting zero seconds; treating graph shape as a brand logo; chanting SOHCAHTOA at no triangle; thinking a regression R^2 is a reason; chatbot steps looking like a check; anxiety — shorten to one table sort, return tomorrow.

On the go — commute / lunch

Try-it A — One-table family sort. Time: 8–12 minutes. Materials: one table. Move: differences/ratios; family; why. Exit sticky.

Try-it B — Sports rate as function of time glance. Time: 5–10 minutes. Materials: box score or split times (public). Move: yards per carry, pace per mile, or points per game as a rate — name input and output; say whether a short list looks roughly linear over that window. Safety: ordinary fan math, not gambling advice.

Try-it C — Covariation sentence. Time: 5 minutes. Move: write “As ___ increases, ___.” for one real surface (hours $\rightarrow$ pay; miles $\rightarrow$ fuel on a rough average). One sentence is enough.

Desk-or-shift

Try-it D — Three-table sort. Time: 20–30 minutes. Materials: three anonymous tables (copy composites from this chapter or OpenStax-style sets). Move: sort all three with written whys; one interloper that is linear but sketched with a curved-looking freehand line to catch logo thinking.

Try-it E — Graph match. Time: 15–20 minutes. Materials: three sketch cards (line, parabola, rising curve). Move: match each to a family; then demand a table reason, not only the sketch.

Try-it F — Right-triangle height/distance. Time: 15–25 minutes. Materials: composite measures (a known baseline across a gap; an angle of elevation stated in a problem). Move: draw; mark right angle; label opposite/adjacent/hypotenuse; write the ratio; solve; unit check. Safety: desk geometry; if you ever measure outdoors, ordinary sense — no unsafe climbing.

Weekend deep sit

Try-it G — Unit-circle card (if this strip is yours). Time: 30–45 minutes. Materials: blank circle; special angles $0, \pi/6, \pi/4, \pi/3, \pi/2$ (and degree twins if you prefer). Move: place three angles; write coordinates you can justify; pick one angle in QII or QIII and explain the sign from axes, not from a mystic mnemonic alone. Rebuild one coordinate from a 45-45-90 or 30-60-90 triangle drawn in the circle.

Try-it H — One sine/cosine with reason. Time: 15 minutes. Move: for $\pi/3$ or 60° , say cos and sin with a triangle reason or unit-circle point reason.

Try-it I — Structure across forms. Time: 20–30 minutes. Move: same linear relationship as table, equation $y = mx + b$, and graph; ask which form makes the rate visible (Chapter 3 echo). Star-style structure grain applied to tables/graphs — school-origin port, you as discussant.⁶¹

Blocked practice: five tables where you only compute first differences; then five where you only compute ratios; then mixed family decisions.

Mixed practice: interleave family sort with one right-triangle item so you choose the strip.

One incorrect example to diagnose

Wrong twin: a calculator regression reports “quadratic” for four points that are exactly linear plus one typo. The human twin trusted the button and never checked first differences. Diagnose: what stays constant if we fix the typo? Repair: first differences constant $\rightarrow$ linear; the button was not the reason.

Second wrong twin: “ $\sin = O/H$ ” written beside a non-right triangle with no altitude drawn. Repair: build a right triangle or stop.

More on-the-go and desk texture.

Try-it J — Paycheck hours $\rightarrow$ pay (structure). Time: 10 minutes. Materials: a stub with hours and gross (greek amounts if sharing a space). Move: treat hours as input, gross pay as output for the hourly portion; ask if overtime makes it piecewise.

⁶¹Star (2015) algebra guide: solved problems / structure / strategies — grades 6–12 origin, portable to self-study when you are the discussant. Structure grain applied here to tables and graphs. Not an adult RCT.

Safety: your stub; structure only — not tax advice. Chapter 2 percent may sit beside this; here the family question owns the page.

Try-it K — Cooking scale as linear function. Time: 8 minutes. Materials: recipe yields. Move: servings $\rightarrow$ ingredient amount as roughly linear if the recipe scales that way; write $y = kx$ for one ingredient; catch that some recipes do not scale yeast or spices linearly — real cooking judgment sits beside the math, and the math does not replace it.

Try-it L — Exponential toy decay. Time: 15 minutes desk. Materials: invented table: 800, 400, 200, 100. Move: common ratio $1/2$; name exponential decay; one sentence covariation. Label invented. Not a medication half-life protocol.

Try-it M — Mode trap drill. Time: 5 minutes. Materials: calculator. Move: compute $\cos(\pi/3)$ in radian mode and $\cos(60)$ in degree mode — same angle, matched modes. Then deliberately mismatch once and see the nonsense. Write “mode check” on your sticky this week.

Solved autopsy template. Cover the family label on a worked example. Ask: what did they compute first — differences or ratios? Where could a regression button have misled? Then parallel table, unaided. You are the discussant.

Spacing seed: Mon family micro; Tue covariation sentence; Wed desk three-table sort; Fri right-triangle or unit-circle card if in plan; optional Sun mixed + checklist. Spacing helps memory in math studies on average; this is a useful study grain, not a promise that every adult will see the same result.⁶²

If it isn’t clicking

Diagnostic 1 — Differences and ratios blur. Next move: two-column sticky — “add story” vs “multiply story.” Practice only linear vs exponential for three micros before reintroducing quadratic second differences.

Diagnostic 2 — Trig chant without picture. Next move: every trig try-it starts with a drawn right triangle and labelled sides. No calculator until labels exist. Mode check once (degrees vs radians).

⁶²Murray and colleagues (2025): spacing effect in mathematics meta-analysis around $g \approx 0.28$; retrieval may be weaker with CI crossing zero — hedge. Useful study, not a promise that every adult will see the same result.

Diagnostic 3 — Unit-circle signs feel arbitrary. Next move: ignore mnemonics for two sints. Only axes: is x positive? is y positive? Sine is y ; cosine is x on the unit circle. Place four angles, one per quadrant, and say signs aloud.

When to slow down: if Chapter 3 linear/quadratic table sense is shaky, stay there before exponential ratios. If geometry reasons for similar right triangles are mush, point back to Chapter 4 before height-across-gap trig.

When to go ahead: three tables sorted with whys; one covariation sentence; if needed, one justified unit-circle sign and one right-triangle ratio. Skip the rest of precalc without shame if life does not ask.

When to ask a human: tutor, colleague in a technical role, or class — especially for radian fluency before a required course. No shame. Anxiety: short sessions; one clean family win; no timer as default rigor.⁶³

If a course requires more than this chapter.

OpenStax Algebra and Trigonometry or Precalculus, or a local class, can extend identities, laws of sines/cosines, and polar edges. This book's job is usable family sense and a thin trig module with meaning. Depth beyond that is fit, not failure. Resources (Chapter 9) names fit without ranking. Khan and OpenStax remain \$0-first spines when you want more practice away from a photo-solver.

Workplace caution, repeated once.

If a table lives in an employer sheet, practice on a redacted copy or on invented isomorphic numbers. Do not paste proprietary data into a public consumer tool. Your acceptable-use page is local; this book does not invent a federal statute for your workplace.

Tools, including AI

Optional. Point back to the refresh-hour AI box.

You may, after you tried: Desmos or a graphing calculator to *check* a family after table reasons; GeoGebra for a unit-circle exploration after you placed angles by hand; a hint ladder; isomorphic tables with your own key; Wolfram/CAS to check a value after meaning.

⁶³Maloney/Beilock math anxiety and working memory — short sessions, early wins. Not a clinical protocol.

You may not: photo-to-key; regression-first as the attempt; “give me all the steps” during the try; paste employer data; treat chatbot identities as your work; invent coordinates.

Degree/radian mode: check once every session you use a button. Dynamic tools replace thinking when they finish the attempt; they help when they wait.

What “almost” looks like — honest middle.

Almost done enough: you can sort linear vs exponential reliably, but quadratic second differences still need a written column every time. That is fine. Keep the column. Fluency grows. Almost on trig: you always draw the triangle, but you still swap opposite and adjacent under time pressure. Slow down; point with a finger; say the angle’s name before the ratio.

Not done enough yet: family decisions only from a software dropdown; trig only as a memorized button sequence; unit-circle signs only from a song with no quadrant pointing. Those are rust signals, not character flaws. Return to table columns and axis signs.

Connection backward and forward.

Chapter 3 gave you linear and quadratic meaning and same-to-both-sides. This chapter keeps those families and adds exponential as a multiplicative cousin, then optional trig. Chapter 4’s similar right triangles support the trig thin module — if similar triangles are shaky, repair geometry reasons before height-across-gap. Chapter 6 will ask what average rates become as intervals shrink — only if you are ready and need it. Chapter 7 will question sample vs population on dashboards; a dashboard tile can be a function of time *and* a statistical claim — different chapters, same adult Tuesday.

What “done enough” looks like

Place by skill and by need — not by diploma year and not by precalc as status. A chatbot “precalc certificate” counts for nothing here. Optional CLEP Precalculus is a bridge when credit is the goal (fee and credit rules live in Keeping track / Resources) — sitting $\neq$ finishing this book.⁶⁴

⁶⁴CLEP Precalculus fee \$97 plus administration/remote fees as listed by College Board at publication check; institution sets credit. Optional. Confirm live pages when you register.

Skill-strip checklist — Functions and trigonometry

- ☐ I can sort linear / quadratic / exponential tables by what stays constant, with a why.
- ☐ I can decide without a regression button (button optional as check only).
- ☐ I can match a graph to a family and still give a table or rate reason.
- ☐ I can write one covariation sentence for an ordinary surface.
- ☐ If trig is in my plan: I attach SOHCAHTOA to a real right triangle with sides named.
- ☐ If unit circle is in my plan: I place common angles and justify at least one sign by quadrant.
- ☐ I check degree/radian mode before trusting a trig button.
- ☐ I can diagnose logo-only or chant-only wrong twins.

Illustration cue. Function-family table cards side by side; unit-circle sketch with QI–QIV signs; right-triangle with opposite/adjacent/hypotenuse labelled for one acute angle. Still-life only. The cards teach “what stays constant”; the circle teaches signs from coordinates; the triangle teaches that the chant needs a home.

Encouragement you can use on a tired Thursday.

You do not need to finish every special angle this week. You need one clean family why, or one clean triangle with labels. Tomorrow’s micro can be three minutes. Rust loosens in sessions you actually finish, not in chapters you only admire.

Life of the habit: Revisit one family-sort micro next Tuesday. If trig is yours, space three angles across the week — not one cram. Mix a Chapter 4 diagram reason once with a trig height so geometry and ratios talk. Stop when the next demand is met. Chapter 6 is optional edges for the ready — not a requirement to be a serious adult.

Many adults never need a full precalc course. Some need this thin module. You get to place yourself by need.

If functions are solid and trig is not your need, skip the circle without apology. If both are solid and a technical path asks for rates that shrink toward an instant, Chapter 6 is optional and ready-gated — not a climax the whole book was secretly aiming at.

Chapter 6 — Calculus edges

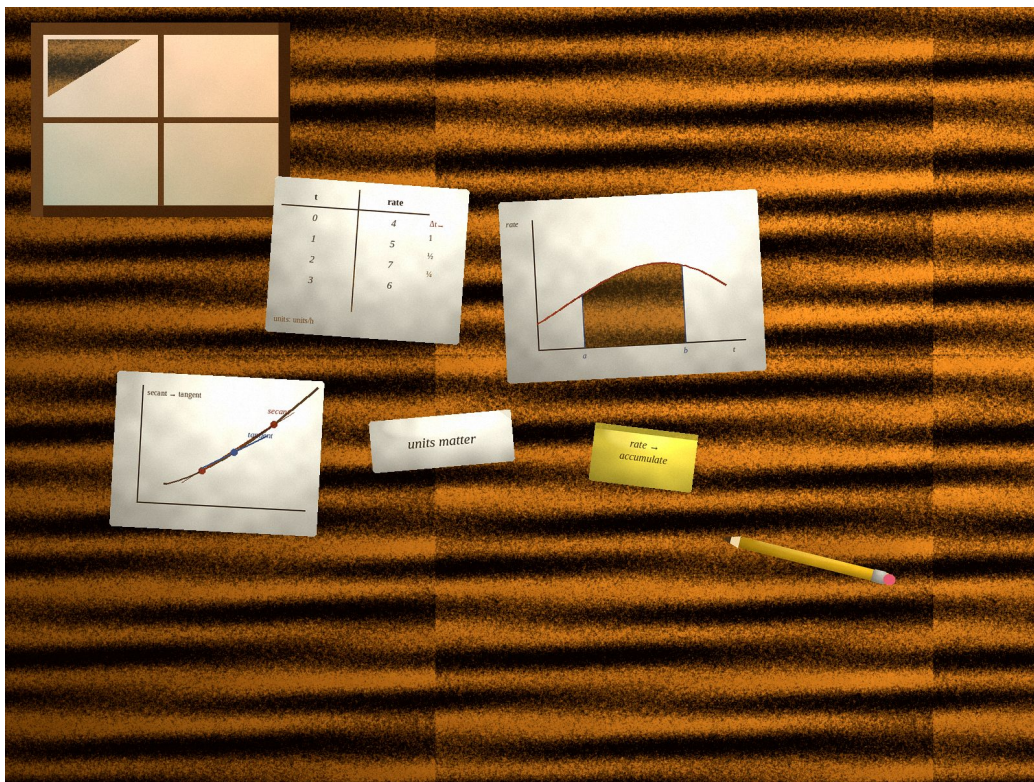


Figure 9: A still-life: rate table with shrinking intervals noted in the margin; accumulation sketch shaded under a rate graph; units written beside both. No people. No logos. No readable UI.

Why this matters

As the interval shrinks, what is this average rate becoming — in what units?

That question is the heart of the derivative as a meaningful idea. The follow-up sits beside it: what is piling up — and what are the units of the pile? Together they are rate and accumulation. Calculus edges, in this book, means those two ideas and their relationship in plain language — when work or further study requires them.

This chapter is **optional**. It is for ready adults. It is **not for everyone**. It is not a moral summit. It is not the climax the rest of the book was secretly aiming at. PIAAC adult numeracy does not require Calculus I. Many excellent quantitative lives never take a derivative. Skipping this chapter is not failure.⁶⁵

Ready means: linear and quadratic meaning in place; function rate language from Chapter 5 in place; a real need — further study, technical work, or sustained curiosity with time. Ambition alone is not readiness. “Too old” is the same myth as “too young” when prerequisites exist; the flip is also true — birthday and desire do not replace prerequisites.⁶⁶

Many adults open a calculus video, hear power rules in the first three minutes, and conclude they are behind. They are not behind a birthday. They may be ahead of a meaning gap the video never named. Start with average rate and units. The rules will wait. The units will not teach themselves.

When calculus *is* the work, refuse power-rule-first with no rate story. Refuse “integral is only an antiderivative recipe” with no pile. Procedures after meaning. Meaning is not a license to skip procedures forever if a course requires them — but this how-to will not pretend flashcards of nx^{n-1} are the first definition of the derivative.

What this idea unlocks, for those who need it, is honest talk about instantaneous rate (how fast something is changing right now) and accumulation (how much piles up when a rate runs over an interval). A speedometer story. A fill-rate story.

⁶⁵PIAAC / NCES adult skills context does not require Calculus I for adult numeracy levels discussed in this book’s welcome. Optional edges only. Access date: 5 September 2026.

⁶⁶National Mathematics Advisory Panel (2008), Finding 15: effort and opportunity; place by skill and prerequisites, not age myths. Finding 10: understanding, fluency, and problem solving together.

A savings-rate sketch. Units on every sentence. The Fundamental Theorem, when you are ready, in one plain sentence: rate and pile undo each other.

This book is not a reprint of *Math for Emerging Adults* calculus-with-meaning chapters. One pointer: rate + accumulation continuity (Thompson line) is shared grain; parent scripts and AP-as-default are not.⁶⁷ You are the learner. ϵ - δ as default belongs in a different course; meaning first here.

You do not need to be a mathematician. You do need to hear power-rule-first, calculus-as-status, instantaneous rate with no units, integral-as-recipe-only, and “skipping this chapter is failure” as wrong turns — and to ask what the average rate is becoming before any rule flex.

This week, if ready: on the go, state one average rate in units from a short table. In a deeper sit: sketch accumulation from a rate story and name the pile’s units.

If you are not ready or not in need, keep Chapters 1–5 and 7. Return later if a path asks. No shame.

Understand it yourself

Here is the everyday picture.

A car’s trip computer shows average speed for the last few minutes. As you shrink the window — last minute, last ten seconds, last second — that average hunts a number for how fast you are going *right now*. Instantaneous rate is what average rate becomes as the interval shrinks. Units matter: miles per hour, not “miles” and not “per hour” alone.

Flip the story. Speed over time draws a graph. The region under that graph piles up distance. Finger-trace from time a to time b: that accumulation is how far you went, in miles, when the rate was in miles per hour and time in hours. Integral language begins as “running total of a rate,” not as “find the antiderivative.”

A second everyday picture: a tank filling at gallons per minute. Average fill rate over ten minutes might be 2 gal/min. As you shrink the interval around a mo-

⁶⁷Thompson line continuity: rate, accumulation, FTC as relationship — labelled pedagogical continuity from calculus-education research traditions. Tallman & Carlson sampling of Calculus I finals: few items beyond procedures — labelled continuity / sobriety about procedure-only culture; independent open details not re-audited here as a kitchen claim. Useful grain, not a promise that every adult will see the same result. ϵ - δ not default in this how-to.

ment when someone opened a valve wider, the average may rise toward a higher instantaneous rate. The pile is gallons. Rate $\times$ time, added up, builds the pile.

A third: a pharmacy label rate reading (mg per mL, or tablets per day) as **rate language warm-up** — units and rate only, **not medical advice**, not calculus itself.⁶⁸ If the real question is clinical, ask a pharmacist or clinician. Here you only practice saying “amount per time” or “amount per volume” out loud.

Here is the precise picture.

Average rate of change of $y = f(x)$ on an interval from a to $a+h$ is

$$\left[\frac{f(a+h) - f(a)}{h} \right]$$

in units of y per units of x . As h shrinks toward zero (from both sides when the context allows), that quotient approaches the **derivative** $f'(a)$ when the limit exists — the instantaneous rate at $x = a$. On a graph, secant lines through $(a, f(a))$ and a nearby point tilt toward the tangent line.

Limit (lightly). A limit asks what value we approach. Tables of closer inputs, graphs you trace with a finger, and algebra after the question is real — three representations. Continuity, roughly: the limit matches the function value so the pencil need not lift. You do not need a topology course. You need to stop saying “just plug in” as the only strategy when plugging in fails.

Integral as accumulation. If f is a rate (gal/min, mi/h, \$/day), then the definite integral from a to b accumulates total change in the underlying quantity. Riemann sums are the honest numerical story: many thin slices of (rate $\times$ tiny time). The definite integral is what those sums approach. Area is the geometric shadow when the axes mean rate and time; prefer pile language when the integrand is a rate.

Fundamental Theorem (words first). If $F(x) = \int_a^x f(t)dt$, then under usual conditions $F'(x) = f(x)$ — the rate of change of the accumulation is the integrand. And $\int_a^b f = F(b) - F(a)$ when $F' = f$. Say: rate and pile undo each other. Then, if a course requires antiderivative recipes, learn them as fluent tools — after the sentence.

Meaning order locked for this how-to: average rate in units $\rightarrow$ shrinking interval $\rightarrow$ instantaneous rate talk $\rightarrow$ accumulation sketch $\rightarrow$ total change $\rightarrow$ FTC in one plain sentence when ready. Delay power-rule flex until meaning is said aloud.⁶⁹

⁶⁸Pharmacy try-its: rate and units only — not medical advice, not dosing protocol.

⁶⁹Thompson line continuity: rate, accumulation, FTC as relationship — labelled pedagogical

Thompson grain (labelled continuity): rate, accumulation, and FTC as relationship — not procedure-only culture. Tallman and Carlson’s sampling of Calculus I finals found few items that asked students to go beyond procedures — labelled continuity / a reminder that rule-only kitchens match a weak exam culture, not a promise about every classroom.⁷⁰ This is a useful study grain, not a promise that every adult will see the same result.

AP Calculus CED? Labelled map only if your path uses it (change, limits, analysis of functions; accumulation unit). Not a kitchen RCT. Not required to use this chapter.⁷¹

Understand further — why units are non-negotiable.

A derivative without units is a trivia number. In motion, f' in miles per hour answers “how fast.” In a cost model (ordinary stakes, not investment advice), if $C(q)$ is dollars to produce q items, $C'(q)$ is dollars per item — marginal language lightly, no sermon. In a tank problem, gal/min. Write the unit every time you write the rate. Adults who skip units often skip meaning; the two failures travel together.

Net change and signed rates.

If a rate dips below zero — velocity in reverse, a tank draining — regions below the axis subtract from the pile if you want net change. Total distance travelled may use absolute speed. Say which question you are answering: net displacement or total distance. One weekend sentence is enough for the distinction; a full particle-motion unit can wait for a course.

Numerical derivative by hand.

You do not need a symbolic f to talk instantaneous rate. A table of stock levels, temperatures, or odometer readings (public or personal) supports average rates on

continuity from calculus-education research traditions. Tallman & Carlson sampling of Calculus I finals: few items beyond procedures — labelled continuity / sobriety about procedure-only culture; independent open details not re-audited here as a kitchen claim. Useful grain, not a promise that every adult will see the same result. ϵ - δ not default in this how-to.

⁷⁰Thompson line continuity: rate, accumulation, FTC as relationship — labelled pedagogical continuity from calculus-education research traditions. Tallman & Carlson sampling of Calculus I finals: few items beyond procedures — labelled continuity / sobriety about procedure-only culture; independent open details not re-audited here as a kitchen claim. Useful grain, not a promise that every adult will see the same result. ϵ - δ not default in this how-to.

⁷¹College Board AP Calculus CED (e.g., Effective Fall 2020 maps): labelled course map when used — not a kitchen RCT. Not required for this chapter.

small intervals. Choose intervals small enough to be interesting and large enough that measurement noise does not dominate — judgment, not a ritual $h=0.0001$. Sense comes first.

Antiderivatives as reverse rates.

Once FTC's sentence is honest, "find a function whose derivative is $2x$ " means "find an accumulation story whose rate is $2x$." The family of answers differs by a constant — different starting piles, same rate. That constant is not a nuisance; it is the initial amount in the story.

Wrong answers you should be able to hear

1. **Power rules first; meaning later — or never.** Chant nx^{n-1} but cannot say what $f'(2)$ means in a distance–time story. Ask the locked opening.
2. **Calculus is required to be a serious adult.** Status cosplay. Refuse it. Done-enough lives in earlier chapters for most readers.
3. **Instantaneous rate with no units talk.** A derivative that is a naked number. Ask: in what units?
4. **Integral is only an antiderivative recipe.** Can compute x^2, dx but cannot say the pile's units from a rate graph. Ask: what is piling up?
5. **Skipping this chapter is failure.** It is not. Optional edges. Ready only.

A sixth: confusing zeros of f' with zeros of f — hilltop height is not zero when slope is zero. A seventh: calculator integral button before any hand Riemann sketch.

Riemann sum in one kitchen paragraph.

Suppose a rate is roughly 10, 12, 14, 16 gallons per minute at the start of four successive minutes. Using left rectangles of width 1 minute: pile $\approx 10+12+14+16 = 52$ gallons. Right rectangles: $12+14+16+$ next — if you only have four rates, be honest about the missing piece or use trapezoids: average the tops. The number is an estimate. The idea is the definition's spirit: add rate $\times$ time slices. Formal integral notation is a later name for the limit of such sums.

Derivative notation variants.

$f'(x)$, dy/dx , df/dx — same idea when they mean the instantaneous rate. Leibniz form dy/dx reminds you of a quotient of small changes. Say the English first; pick

a notation your course or notes use consistently.

When the limit fails.

Corners and jumps can block a derivative. A V-shaped absolute-value graph has no single tangent at the tip. Average rates from left and right disagree as h shrinks. Naming “does not exist here” is honest calculus talk — not a failure of the student. Sketch once; move on.

Done-enough vs course-done.

Done enough in this book’s strip is meaning. A college Calculus I final may demand fluent procedures, related rates, and exam timing. Those are legitimate course goals. They are not secretly required to finish *Math Refresher for Adults*. Keep the scopes separate so neither shame nor false completion creeps in.

One more incorrect twin.

“Integral of velocity is acceleration.” Repair: integral of velocity accumulates displacement (pile); derivative of velocity is acceleration (rate of a rate). Rate vs pile mix-up wearing physics nouns. Ask the accumulation follow-up.

Five-minute warm-up (if ready)

Write a three-row distance–time table. Compute one average rate with units. Then imagine the time step cut in half and say one sentence: “As the interval shrinks, this average is becoming ____ in units ____.” Cover any key. If that sentence will not come, you may need more Chapter 5 rate language — or this chapter can wait.

A longer shrinking-interval walkthrough (composite).

Let $f(t)$ be miles from a start marker at time t hours. Suppose:

t (h)	$f(t)$ (mi)
2.0	100
2.5	130
2.1	106
2.01	100.6

Average rate from 2.0 to 2.5: $(130-100)/(0.5) = 60$ mi/h.

From 2.0 to 2.1: $(106-100)/0.1 = 60$ mi/h.

From 2.0 to 2.01: $(100.6-100)/0.01 = 60$ mi/h.

In this composite, averages sit at 60 mi/h — a constant-rate story, which is the gentlest door into “becoming.” Next weekend, use $f(t)=t^2$ (in invented units) so the averages change as h shrinks; watch the settling number; name it as the instantaneous rate at that t , still with units.

Constant rate vs varying rate — pile sketches.

If rate is constant 60 mi/h for 2 hours, the pile is a rectangle: height 60, width 2, area 120 miles. If rate rises from 40 to 80 over those 2 hours roughly linearly, a trapezoid approximates the pile. Hand estimates beat a symbolic integral on day one of accumulation. The button can check later.

Related rates and max/min?

Named as later course topics. Schema if you meet them: draw, label rates with units, write a relation, differentiate, interpret. Not required for done-enough here. Keyword hunt (“maximum means set derivative to zero” with no sketch) is a wrong twin — same disease as keyword percent in Chapter 2.

Limit table habit.

When someone says “the limit as x approaches 2,” write $x = 2.1, 2.01, 2.001$ and $1.9, 1.99, 1.999$ with $f(x)$ beside them before algebra. The table makes “approaching” visible. Algebra simplifies when the table has already made the question real.

What this chapter refuses (plain list).

- ε - δ as the default first week for this how-to
- Calculus for everyone as status
- CLEP Calculus as moral finish
- Shame for skipping
- Power-rule-first as the only meaning
- Gatekeeping Chapters 1–5 or 7 behind this module

Micro-session example (ready adult, composite).

Retrieve: “Yesterday’s average rate was 60 mi/h over a half hour.” Autopsy: shrinking table; units written first. Parallel: new table $\rightarrow$ average rate. Exit: “I

can attach units without prompting. Next cue: one pile sketch Sunday.”

Connection to life after the page.

If your job never needs instantaneous rates, maintain average rates and unit honesty from sports, pay, and fill stories — still valuable without derivative notation. If a degree plan lists Calculus I, use this chapter as meaning on-ramp, then a full text or class for procedures and exams. OpenStax Calculus is a \$0-adjacent backbone when you want more; photo-to-key remains banned during attempts.

How to try it this week

Only if ready. Otherwise skip to Chapter 7 or maintain earlier strips.

The Refresh Hour holds full shapes and the AI box. Point back. Bastani crutch warning still applies if AI is used here — model may prepare you; it may not do your attempt.⁷²

Session shape across bands

Commute / lunch (~15–20 minutes). Retrieve: state yesterday’s average rate with units. Autopsy: a short table of shrinking intervals already worked; first useful move was writing units. Parallel try: new three-row table → one average rate unaided. Exit sticky: rate + units. Optional warm-up: pharmacy label rate reading — units only, not calculus, not advice.

Desk-or-shift (~25–40 minutes). Representation first: table of average rates as h shrinks; sketch secants. Short model: one shrinking story; fade the last h . Two tries + interloper: one motion rate, one fill rate, one interloper that asks for total distance when you only have a single instantaneous speed (insufficient — say what is missing). Self-question + wait. Exit: “becoming ____ in ____.”

Weekend deep sit (~45–75 minutes when needed). Diagnostic wrong: power-rule page with no rate sentence. Longer solved: average → shrink → instant talk. Accumulation sketch from a rate graph (speedometer or savings-rate story); name pile units; rough rectangle or trapezoid estimate before formal integral notation. Alternative: constant-rate rectangle vs varying-rate sketch. FTC one-sentence

⁷²Bastani et al. (2025), *PNAS*: HS math field experiment; unguarded chatbot crutch pattern. **Not an adult RCT.** Same lunch-break rule as the refresh hour.

meaning when ready. Mixed: one units-only rate warm-up interlaced. Done-enough checklist for ready adults. Schedule micros. Do not mass “all of Calc I” into one Sunday.

Exact wording you can say to yourself

- “As the interval shrinks, what is this average rate becoming — in what units?”
- “What are the units of this average rate?”
- “What is piling up — and what are the units of the pile?”
- “Can I say the meaning before any power rule?”
- “If the rate were constant, what would the pile sketch look like?”
- “You may change your mind.”
- “I can pause this chapter without failing.”

Ask one. Wait a slow three.⁷³ Look at the table or sketch. Rescue at one second is still rescue.

First try-it (on the go, if ready)

State one average rate in units from a short table. Time: 8–12 minutes. Materials: scrap; three pairs (time, distance) or (time, volume). Safety: ordinary. The move: subtract, divide, write units. No derivative symbol required on day one.

How to fade help

First: units filled for you in a model. Second: you write units. Third: shrinking h table unaided; only then optional CAS check. Power rules stay covered until you can say the locked opening aloud on a fresh story.

When to stop looking up

Stop when you can state average rate in units, describe what shrinking aims at, sketch a pile with units, and (when ready) say FTC in one plain sentence. Looking up a power rule after meaning is fine for fluency. Looking up a finished solution to paste during the attempt is not.

A full desk-sit skeleton for this strip (ready).

Warm-up retrieval (3 min): write one average rate from memory with units. Representation first (10 min): build a shrinking-h table for a simple f; no rule sheet

⁷³Stahl think-time; Rowe wait-time — classroom-origin pauses. Not an adult math RCT.

visible. Short model (5 min): speak the locked opening over the table; fade the last row for yourself. Two tries + interloper (12 min): (1) motion average rates; (2) fill-rate averages; (3) interloper that gives only an instantaneous speed and asks for distance over an hour without a rate history — answer “not enough; I need the rate over the interval or a pile sketch.” Self-question + wait (3 min). Exit ticket (2 min): units of today’s instantaneous estimate.

Weekend deep-sit skeleton (ready).

Diagnostic wrong (10 min): power-rule-only page — rewrite with a rate story first. Longer solved (20 min): speed graph $\rightarrow$ rectangle/trapezoid pile $\rightarrow$ miles. Alternative (10 min): same pile with more rectangles; notice the estimate change. FTC sentence (10 min). Mixed (15 min): one family-sort interloper from Chapter 5 so you still choose when edges apply. Checklist + calendar (5 min). Stop even if energy remains — depth without dump.

Why refuse power-rule-first as only meaning (one more beat).

Rules are compressed intelligence. Compression without the experience it compresses becomes karaoke. Karaoke can pass a quiz. Karaoke fails when a new surface asks what the number means in units. Adults returning to calculus often have karaoke residue. This chapter scrapes residue gently: tables, sketches, units, then rules as fluent shorthand.

Continuity with Chapter 5 covariation.

“As x changes, y changes” becomes “as x changes by a little, y changes by about $f'(x)$ times that little — when f' exists.” That sentence is the bridge. You practiced covariation without calculus. Here the rate gets a local linear story. If the covariation sentence was never fluent, build it in Chapter 5 before living here.

Continuity with Chapter 3 slope.

Slope of a line is a constant rate — average and instantaneous agree everywhere. Calculus edges matter when the rate itself changes. A linear cost model does not need derivatives. A curved cost or a varying speed does — when your path asks.

Safety and stakes again.

Sports rates: ordinary. Pharmacy labels: units only, not advice. Savings toys: labelled invented, not investing pitches. Employer data: redact or isomorphic invent; no public paste. No live controversy examples. Ordinary stakes keep the brain on units and meaning.

What a stuck silence is not.

It is not proof you lack a math gene. It is often wait-time zero, vague question, or status fear. Pose the locked opening again. Write only the units blank. If still stuck, stop the sit early and return tomorrow — that is discipline, not quitting.

Practice that actually builds it**Talk-to-yourself box**

Opening question (locked): “As the interval shrinks, what is this average rate becoming — in what units?”

Accumulation follow-up (in this box): “What is piling up — and what are the units of the pile?”

Follow-ups:

1. What are the units of this average rate?
2. What happens as the interval shrinks?
3. What is piling up — and what are the units of the pile?
4. Can you say the meaning before any power rule?
5. If the rate were constant, what would the pile sketch look like?

How to wait alone: Cover the key. Slow three. If stuck, write only the units blank first — “___ per ___” — then return to the number.

Stuck silence usually means: hunting a formula name; wait-time zero; power-rule reflex; pile units wrong (saying mi/h for area under a speed graph); status anxiety (“I should know calculus”); chatbot steps as check; working-memory load — shorten, one average rate win, stop.⁷⁴

On the go — commute / lunch (ready)

Try-it A — Average rate from a short table. Time: 8–12 minutes. Materials: three-row table. Move: one average rate; units; sticky.

Try-it B — Sports rate units. Time: 5–10 minutes. Materials: public split times or pace. Move: distance/time with units; say this is average over that split, not yet

⁷⁴Maloney/Beilock math anxiety — short sessions, early wins. Not a clinical protocol.

instantaneous. Ordinary fan math — not gambling advice.

Try-it C — Pharmacy label rate warm-up. Time: 5 minutes. Materials: household OTC label you already use, or a labelled composite. Move: read amount per volume or per time; write units. **Safety: not medical advice; not dosing; if clinical, ask a professional.** Not calculus — rate language only.

Desk-or-shift (ready)

Try-it D — Shrinking interval talk. Time: 20–30 minutes. Materials: function values at a , $a+1$, $a+0.1$, $a+0.01$ (from a simple formula you choose, e.g. $f(x)=x^2$ at $a=3$). Move: compute average rates; watch them settle; say the locked opening; still no power rule required — though you may notice a pattern.

Try-it E — Secant to tangent sketch. Time: 15 minutes. Materials: graph scrap. Move: draw two secants getting closer; write units on the slope.

Weekend deep sit (ready)

Try-it F — Speedometer → distance pile. Time: 30–40 minutes. Materials: sketched speed vs time (piecewise constant or simple curve). Move: approximate area with rectangles or trapezoids; name miles; compare constant-rate rectangle to varying-rate sketch.

Try-it G — Savings-rate accumulation sketch. Time: 20–30 minutes. Materials: invented \$/week rate over weeks (labelled invented — not advice). Move: pile = dollars; sketch; units of pile.

Try-it H — FTC one sentence. Time: 10–15 minutes when ready. Move: write: “The derivative of the accumulation function recovers the rate; the net pile from a to b equals $F(b)-F(a)$ when $F'=f$.” Then invent one story that sentence fits.

Try-it I — Power rule only after meaning. Time: 15 minutes, delayed until F–H feel honest. Move: now compute $f'(x)$ for $f(x)=x^2$ two ways — shrinking numerical rates near a point, and power rule — and say they agree. Meaning first was the point.

Blocked practice: five average rates with units only. Then three shrinking tables. Then one pile sketch.

Mixed practice: interleave a Chapter 5 family table so you still choose when calculus edges are even relevant.

One incorrect example to diagnose

Wrong twin: “Derivative of position is velocity” written, then a numerical answer with no units, then a power-rule line with no story. Diagnose with the locked opening. Repair: average rates on a table with mi/h; shrink; then optionally power rule; units on the final instantaneous rate.

Second wrong twin: shaded region under a speed graph labelled “mi/h.” Repair: $\text{rate} \times \text{time} \rightarrow \text{miles}$ for the pile.

Third wrong twin: “I failed because I skipped calculus.” Repair: optional edges; return to Chapter 7 or maintenance.

More practice texture.

Try-it J — Fill-rate story. Time: 20 minutes desk. Materials: invented gal/min over 10 minutes (piecewise). Move: estimate gallons; units; compare to “average rate $\times$ time” shortcut and say when the shortcut is exact (constant rate).

Try-it K — Wrong units autopsy. Time: 10 minutes. Materials: a crossed-out twin that labels a speed-graph area as mi/h. Move: rewrite as miles; explain with $\text{rate} \times \text{time}$.

Try-it L — Say it before symbols. Time: 10 minutes. Move: explain instantaneous rate to yourself in two sentences with no f' symbol; then rewrite with the symbol. If the English was empty, the symbol will not save you.

Try-it M — Optional CLEP glance (not required). Time: 15 minutes. Materials: College Board Calculus exam description page (public). Move: notice that limits/differential and integral both appear; ask whether *meaning* on this chapter’s checklist is in place before you spend registration money. Sitting without transfer credit checked wastes money — say plainly. Fee **\$97** plus fees; confirm live page.⁷⁵

Spacing seed for ready adults: Mon average-rate micro; Wed shrinking table; Fri pile sketch; optional Sun FTC sentence + checklist. Do not mass. Murray-style

⁷⁵CLEP Calculus: about 44 questions / about 90 minutes per College Board listing opened 5 September 2026; fee **\$97** plus administration or remote-proctor fees; credit institution-set. Optional. Confirm live pages when registering. Sitting $\neq$ finishing this book. AP not adult default.

spacing is useful study grain — not a promise of identical gains.⁷⁶

Solved autopsy template. Cover the final derivative. Ask: were units present? Was there a shrinking story or only a rule? Where could someone confuse $f=0$ with $f'=0$? Parallel numerical table, unaided.

If it isn't clicking

Diagnostic 1 — Units keep vanishing. Next move: every line ends with units in parentheses. No naked numbers for a week of micros.

Diagnostic 2 — Power-rule reflex. Next move: cover the rule sheet. Only tables and sketches for two desk sits. Speak the locked opening before uncovering rules.

Diagnostic 3 — Pile vs rate confusion. Next move: two-column sticky — left “rate units (per),” right “pile units (total).” Every sketch labels both axes and the region’s unit.

When to slow down / exit: if Chapter 5 covariation and linear rates are shaky, pause this chapter. If anxiety spikes around “calculus,” stop — that spike is information about status pressure, not about your worth. Maintain earlier strips. Try Chapter 7.

When to go ahead: average rate in units fluent; shrinking story speakable; pile sketch with units; FTC sentence when ready. A full Calc I course, CLEP Calculus, or AP path is a different project — optional, institution-ruled, not this book’s moral finish.⁷⁷

When to ask a human: tutor, community-college instructor, or colleague in a technical role. No shame. Correct your wrong reason without crushing the attempt. Short sessions beat dread marathons.⁷⁸

Entry gate reminder: ambition alone is not readiness. Prerequisites first. Finding 15 adult flip cuts both ways.⁷⁹

⁷⁶Murray and colleagues (2025): spacing in mathematics $\sim g \approx 0.28$; retrieval hedge if CI crossed zero. Useful study, not a promise that every adult will see the same result.

⁷⁷CLEP Calculus: about 44 questions / about 90 minutes per College Board listing opened 5 September 2026; fee \$97 plus administration or remote-proctor fees; credit institution-set. Optional. Confirm live pages when registering. Sitting $\neq$ finishing this book. AP not adult default.

⁷⁸Maloney/Beilock math anxiety — short sessions, early wins. Not a clinical protocol.

⁷⁹National Mathematics Advisory Panel (2008), Finding 15: effort and opportunity; place by

You try this week (seed reminder). On the go, if ready: state one average rate in units from a short table. Deeper sit, if ready: sketch accumulation from a rate story and name the pile’s units. If not ready: leave this page marked “later,” open Chapter 7 or revisit Chapter 5 covariation, and treat that choice as placement — not as a grade.

Tools, including AI

Optional. Point back to the refresh-hour AI box.

You may, after you tried: calculator for arithmetic on average rates; Desmos to graph secants after hand estimates; Wolfram/CAS to check a derivative after you stated meaning; Riemann-sum exploration after a hand sketch; hints; isomorphic tables with your key.

You may not: photo-to-key; “give me the steps” during the attempt; paste proprietary data; treat chatbot solutions as your work; print a calculus fluency certificate from a model; skip meaning because a CAS differentiates instantly.

Dynamic tools replace thinking when they finish the attempt. The lunch-break rule still holds.⁸⁰

Honest middle — “almost ready.”

You can compute average rates with units, but shrinking still feels mystical. Stay on numerical tables for another week. Do not rush symbols. Almost on accumulation: you shade a region but still say the wrong unit once per sit — keep the two-column rate/pile sticky until it is boring. Boring sticky is success.

If a manager or podcast says “everyone should learn calculus.”

Translate: some paths need rate and accumulation meaning; many do not. Your skill strip and your next demand decide. This book will not argue with a podcast in the margins. It will keep the gate honest.

Forward and side doors.

skill and prerequisites, not age myths. Finding 10: understanding, fluency, and problem solving together.

⁸⁰Bastani et al. (2025), *PNAS*: HS math field experiment; unguarded chatbot crutch pattern. **Not an adult RCT.** Same lunch-break rule as the refresh hour.

Chapter 7 statistics does not require this chapter. Functions in Chapter 5 help both. If you need differential equations someday, they begin with rate sentences you practiced here — labelled far door, not this week's homework.

Encouragement.

Ready adults can rebuild calculus edges with meaning. Rust on derivatives you last saw at nineteen is ordinary. Status pressure is optional; decline it. One average rate with units today is a real session. One pile sketch this weekend is a real deep sit. You do not owe anyone a power-rule flex on the train.

What “done enough” looks like

For ready adults only. Everyone else: done enough is recognizing that this chapter is optional and returning to strips you need. That recognition is success, not a gap.

Skill-strip checklist — Calculus edges (ready)

- ☐ I can state an average rate from a table with correct units.
- ☐ I can describe what a shrinking interval aims at in plain language.
- ☐ I can speak instantaneous rate as “what average rate is becoming,” with units.
- ☐ I can sketch accumulation from a rate story and name the pile's units.
- ☐ I can approximate a pile with rectangles or trapezoids before formal notation.
- ☐ I can say FTC in one plain sentence when ready (rate and pile undo each other).
- ☐ I delay power-rule flex until meaning is spoken — then use rules as tools.
- ☐ I refuse calculus-as-status and refuse shame for pausing or skipping.

Life of the habit: If this strip is yours, revisit one average-rate micro next week; space a pile sketch; do not cram ϵ – δ . If you sat optional CLEP Calculus (\$97 plus fees; institution sets credit), that score report is a bridge — sitting $\neq$ finishing this book.⁸¹ Mix maintenance from Chapters 2–5 so edges do not float free of ordinary numeracy.

⁸¹CLEP Calculus: about 44 questions / about 90 minutes per College Board listing opened 5 September 2026; fee **\$97** plus administration or remote-proctor fees; credit institution-set. Optional. Confirm live pages when registering. Sitting $\neq$ finishing this book. AP not adult default.

Illustration cue. Rate table with shrinking Δt ; accumulation sketch from a rate graph; units in the margin. Still-life. The picture teaches rate $\rightarrow$ pile, not a trophy.

You can be a fully numerate adult in the sense of this book without this chapter. If you are ready and need these edges, you can learn them with meaning first. Either way, on purpose: the path through ordinary quantitative life was the promise all along. A fake certificate was not. Calculus for everyone was not. Power-rule-first as the only meaning was not.

When you finish — or when you wisely stop — Chapter 7 waits with sample vs population and dashboard critique for life and work. That chapter is for more readers than this one. Go there when you need it. Keep a dated note of what you can say about rates and piles. That dated note is the real work product on your desk. Not a badge and not a trophy.

Closing beat before the notes.

Calculus edges are a doorway some adults walk through and many do not need. If you walked through with units on your rates and names on your piles, you did the work this chapter asked. If you stood at the door and turned toward statistics or toward maintaining percent and algebra, you also did adult placement correctly. Place by skill. Need by life. Meaning before karaoke. That is enough.

Chapter 7

Statistics for life and work

A fluent slide is not a check. A dashboard tile with a crisp percent can still hide the base, truncate the axis, or treat twelve people as “everyone.” This chapter is about reading tables and charts the way you already meet them — at work, in a leaflet, in a news summary — and asking one honest question before you treat the number as settled.

The locked opening for this strip is: **“Is this number about the sample, or a claim about the population?”**

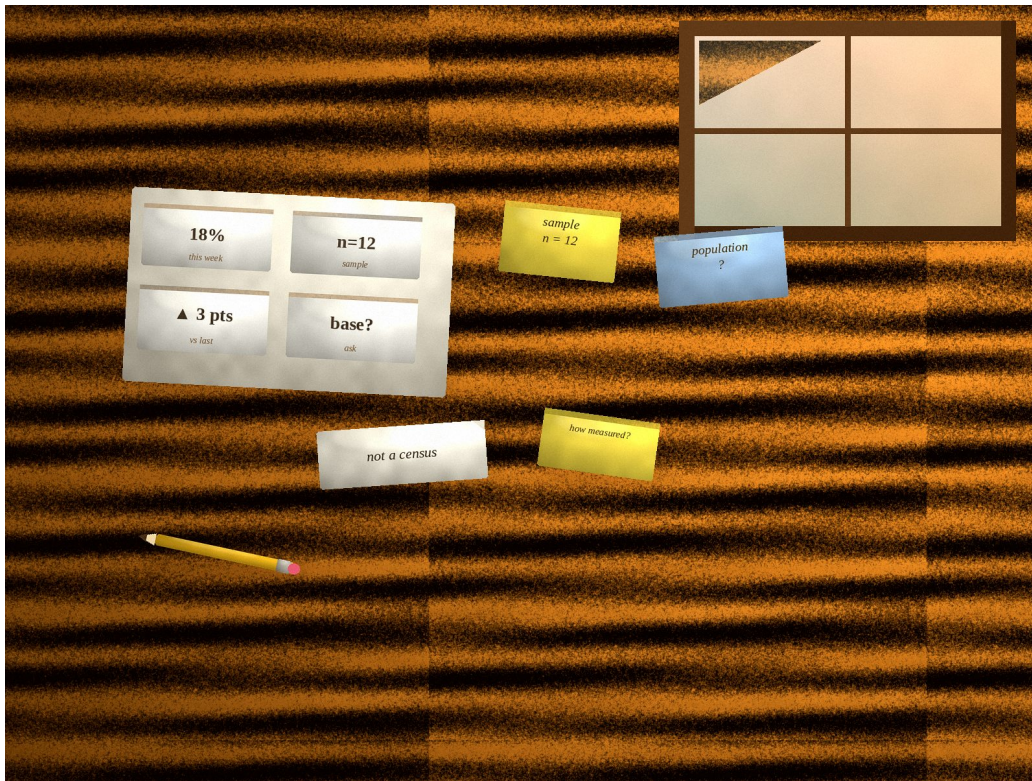


Figure 10: Dashboard tiles and sample-vs-population stickies — no product chrome

Why this matters

Ordinary adult life is full of claims that look like facts. A shift dashboard says errors are down. A deck chart leaps. A headline says risk was cut in half. A coworker says “our sample shows customers prefer X.” None of those sentences is automatically wrong. None is automatically finished.

What this idea unlocks is quiet judgment. You learn to label a sample as a sample. You learn to name axes, units, a time window, and a comparison before you nod at a title. You learn to write both an absolute change and a relative change when a base is known — and to write “base missing” when it is not. You learn that two

numbers moving together is not yet a causal story, and that a margin of error is a kind of honesty, not a confession that the study failed.

This is worth the struggle because the alternative is trusting the chrome of the slide. At work, the graph is often the argument. Supervisors rarely ask for a proof. They ask whether the number on the screen matches the number on the floor — and whether the leap on the chart is a real leap or a truncated axis. At home, a health leaflet or a news graphic can shrink or inflate a risk by how it is framed. Numeracy research on adults finds that how numbers are framed matters for risky decisions, and that lower numeracy goes with more susceptibility to framing.⁸² This chapter does not turn you into a statistician. It does not require a full introductory statistics course. It makes you fluent enough to hear five wrong turns: a sample mean offered as a population fact; a truncated axis leap trusted as magnitude; a relative risk without a base treated as complete; a dashboard title trusted over axes and filters; and spreadsheet fluency mistaken for statistical judgment.

You already practiced percent bases in an earlier chapter. Paychecks, unit prices, and APR structure stay there. Here the object is claim critique: sample versus population, dashboard reading, and risk as **representation** — not medical advice, not a product pitch, not a live fight. Politics, identity fights, and medical culture-war stay off this table. Safe math objects only: a sample-vs-population sticky, a described dashboard, a composite bar pair, an invented risk toy labelled invented.

A path through ordinary quantitative life includes this strip. A fake certificate does not. Optional credit exams live later, if credit is your goal. For now: one good question, one chart, one cautious sentence.

Understand it yourself

Everyday picture. Imagine two sticky notes on the edge of a monitor. Sticky A says: “Among the 40 invoices we checked this week, 6 had a coding error.” That is a sample statement. Sticky B says: “Six of every forty invoices in the company

⁸²Valerie F. Reyna and colleagues, and Ellen Peters / Edward Cokely lines of work on numeracy and risky decision-making: numeracy relates to better decisions and to framing susceptibility when numeracy is low — cited in stream research as adult-useful. This is a useful body of work, not a promise that every adult will see the same result. Access date for sources in these notes: 5 September 2026.

have coding errors.” That is a population claim dressed in sample clothes. Same digits. Different jobs. Your job is to notice which sticky you are holding — and to refuse B when only A was earned.

Another everyday picture: a slide with two bars. The left bar is a little shorter than the right. The axis starts near the top of the shorter bar, so the leap looks huge. Someone says, “Look how far we jumped.” You redraw from zero on a scrap. The leap shrinks. The numbers did not change. The picture did.

Precise picture. A **population** is the whole group the claim is about — every invoice this quarter, every customer in the region, every shift in the plant. A **sample** is the part you actually observed — forty invoices, twenty-eight Tuesday shoppers, one week of tickets. A number from a sample can be useful. It becomes overreach when someone treats it as the whole without saying what would have to be true for the leap: a clear population definition, a fair method, enough observations for the decision at hand, honesty about what was excluded.

A **dashboard** is a set of summary numbers and charts for a process or team. Fluent tiles do not guarantee that the metric definition, the filter, or the time window match what a listener hears. When was the number computed? What rows did the filter drop? Is “errors” the same as “errors caught,” or is it “errors shipped”? The math move includes asking those questions in plain language.

A **relative** change compares a new value to an old one as a percent of the old. An **absolute** change names the difference in the original units or in percentage points. Both can be true at once. “Cut in half” is relative. “Fell by one percentage point” or “fell by 1 in 1,000” is absolute. When only the relative line is offered and the starting risk is missing, the picture is incomplete. Health leaflets and news summaries often lean relative because the number sounds larger. Your job is representation literacy — **not** deciding whether to take a test or a treatment.⁸³

Association means two things move together in the data you have. **Causation** is a stronger claim: that one produces the other. On ordinary stakes, say “together in this table” before you say “because.” A margin of **error** (when a careful survey reports one) is a way of saying the sample estimate is uncertain within a range

⁸³Absolute vs relative risk as representation: health-numeracy teaching shape. Gerd Gigerenzer and Ulrich Hoffrage, “How to Improve Bayesian Reasoning Without Instruction: Frequency Formats,” *Psychological Review* 102, no. 4 (1995): 684–704 — labelled neighbour for representation (counts out of a fixed population), not a clinic protocol and not medical advice. Toy trees in this chapter use invented round numbers labelled invented.

under stated assumptions. It is honesty. It is not “the study is worthless.”

Association, causation, and margin of error (plain language)

You will hear “linked to,” “associated with,” and “leads to” used as if they were the same sentence. They are not. On a dashboard, two lines can rise together because both follow the season, the staffing plan, or a third factor you have not named. The adult move is modest: say what the table shows, then say what it does not show. “Defects and overtime hours rose in the same weeks” is an association statement. “Overtime caused defects” is a causal claim that needs a different kind of evidence than a dual-axis chart alone.

Margin of error, when a careful survey reports one, is a range around an estimate under stated assumptions. Hearing “plus or minus three points” should make you *more* willing to treat the estimate as honest about uncertainty — not less willing to read the study. Absence of a margin on a tiny convenience sample is also information: the number may still be useful locally, and it is still a sample.

Neither idea requires a full probability course this week. Both ideas protect you from two opposite mistakes: treating every chart as a causal sermon, and treating every uncertain estimate as worthless.

Wrong answers you should be able to hear

1. **“The average for our sample is 12.4, so the population average is 12.4.”**
A sample mean is about the sample unless you have a careful design and a clear population definition. On ordinary stakes, label the sample and say what would support a broader claim.
2. **“Look how far the bar jumped — the change must be huge.”** A truncated axis can make a small leap look dramatic. Redraw from zero (or from a fair baseline) and see whether the takeaway shrinks. Journalists have named “Gee-Whiz” graph habits for decades; treat that as a neighbour for your eye, not as a trial of every workplace tool.⁸⁴

⁸⁴Darrell Huff, *How to Lie with Statistics* (1954): Gee-Whiz graphs and truncated bars as journalistic neighbours — not a randomised trial of dashboards. Yang (2021) adult graph-comprehension work is a useful study on how presentation affects reading — not a guarantee about every 2026 workplace tool.

3. **“Risk was cut in half — that settles it.”** Relative “half” needs the starting risk. From 2 in 1,000 to 1 in 1,000 is a 50% relative reduction and a 1-in-1,000 absolute reduction. From 200 in 1,000 to 100 in 1,000 is also 50% relative and a much larger absolute change. Adults deserve both framings when both are known.
4. **“The tile says ‘On track,’ so the number is fine.”** Titles are rhetoric. Ask what the axes, units, time window, and comparison actually show — and when the number was computed.
5. **“I can make the chart in the spreadsheet, so I understand the claim.”** Buttons are fluency of a tool. Statistical judgment is naming what was counted, what was left out, and what the picture does not justify.

One precise worked shape (composite illustration)

Here is an illustration, labelled as such — not a reported workplace and not medical advice.

Slide title: “Defects down 50% — big win.”

Small print (if you leave the title): last month 4 defects in a checked batch of 200 units; this month 2 defects in a checked batch of 200 units. Axis on the bar chart starts at 1.5.

Sample or population? The counts are from checked batches — a sample of units, not necessarily every unit shipped, unless the note says so.

Relative change: $4 \rightarrow 2$ is a 50% relative reduction.

Absolute change: 2 defects fewer per 200, or 1 percentage point if you speak in percent defective ($2\% \rightarrow 1\%$).

Truncation: bars from 1.5 make a small drop look tall; redraw from zero and the leap shrinks.

Cautious sentence: “In the batches we checked, defects fell from 4 to 2 per 200 (50% relative; 1 percentage point absolute). We have not yet shown that every production line matches these batches.”

That paragraph is the work. Fluent slide $\neq$ check.

Five-minute warm-up

Before any try-it, cover this page and answer aloud:

- Name one number you saw this week that came from a *sample* (a check of some rows, a survey of some people, a week's worth of tickets).
- Name one claim you heard that sounded like it was about *everyone* or *the whole process*.
- For a percent you remember (“down 50%,” “up 12%”), can you name the old value? If not, say “base missing.”
- Glance at any chart on your phone or desk: where does the vertical axis start?

Then open one ordinary object: a blurred dashboard screenshot copied onto *your* practice sheet (not pasted into a public tool), a printed deck page, or a news-stat sticky you invent with round numbers labelled invented. Do not open a live controversy chart. Safe objects only.

How to try it this week

Use the refresh-hour shapes you already have. This week's idea is sample-versus-population and dashboard critique. Struggle before rescue. Try unaided first. Knowledge first: know the sticky labels well enough to hear overreach, then practice on surfaces you already meet.

Session shape across bands

Commute / lunch (about 15–20 minutes).

Warm-up retrieval (3–4 minutes): one sample sticky, one population-claim sticky, one “base missing” line.

Short model (3–4 minutes): a solved autopsy of a slide that says “Errors down 50%” without counts — you write the two counts if known, or “base missing.”

Your attempt (5–6 minutes): label one real or composite news/work number as sample or population claim.

One good self-question: the locked opening; wait a slow three.

Mixed glance (2 minutes): a described chart — where does the axis start?

Exit sticky (1 minute): one cautious sentence you could almost show a colleague.

Desk or shift (about 25–40 minutes).

Warm-up mixed from last week's percent strip plus one sample label (5 minutes).

Representation first (8–12 minutes): write axes, units, time window, comparison *before* any conclusion sentence.

Short model / fade a step (5 minutes): Curcio/Friel-shaped three questions on a workplace chart — what does it show, what comparison does it support, what does it not justify.⁸⁵ Cover the “beyond” answer and try it yourself.

Two tries plus one interloper (10–12 minutes): one clean base check; one truncated bar pair; one interloper that mixes a keyword percent with a sample claim.

Self-question + wait + written reason (3 minutes).

Exit ticket: “sample / population / base / axis start.”

Weekend deep sit (about 45–75 minutes when needed).

Diagnostic wrong twin (10 minutes): sample mean offered as population fact — repair in writing.

Longer solved with structure (15–20 minutes): redraw a truncated comparison from zero with composite numbers; write what changed in the takeaway.

Alternative strategies when legal (10 minutes): absolute and relative both written; optional natural-frequency toy with invented round numbers labelled invented — representation habit, not advice.⁸⁶

Mixed set of four to six (15–20 minutes), interleaved with one percent-base item from chapter 2.

Done-enough checklist.

Schedule three micro cues for next week on a calendar (sample label; truncation glance; base check). Spacing across days beats a Sunday dump.⁸⁷

⁸⁵Susan N. Friel, Frances Rae Curcio, and George W. Bright, “Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications,” *Journal for Research in Mathematics Education* 32, no. 2 (2001): 124–58. Read the data / between / beyond — classroom-origin framework used here as **shape** for adults. Do not claim the paper measured modern dashboard products.

⁸⁶Absolute vs relative risk as representation: health-numeracy teaching shape. Gerd Gigerenzer and Ulrich Hoffrage, “How to Improve Bayesian Reasoning Without Instruction: Frequency Formats,” *Psychological Review* 102, no. 4 (1995): 684–704 — labelled neighbour for representation (counts out of a fixed population), not a clinic protocol and not medical advice. Toy trees in this chapter use invented round numbers labelled invented.

⁸⁷Murray and colleagues (2025) math meta-analysis: spacing versus massed practice about $g \approx 0.28$ in math contexts — labelled encouragement for spacing across days, not an adult commute RCT of this book’s objects. Retrieval practice hedged where confidence intervals crossed zero in that synthesis. Dose honesty: there is no IES practice guide titled “Adult Commute Stats Hour.”

Exact wording you can say to yourself

When you open a chart, a tile, or a claim:

“Is this number about the sample, or a claim about the population?”

Then wait. Cover the answer key if you wrote one. Look at the axes or the sticky, not at a phone solver. After you answer, ask one follow-up from the box below. Then try a parallel item with new numbers.

First try-it this week

On the go — news-stat sticky (15 minutes).

Materials: one sticky, a pen, one number from a workplace summary or a news summary you already opened (ordinary stakes — sports rate, operations count, weather average, product return rate, library checkout total). No live controversy.

Safety: do not paste proprietary employer rows into a public consumer tool; rewrite isomorphic numbers on *your* sheet if you practice with a helper later. No under-cover audit. No public quiz of a colleague. Noticing is not a sting.

The move: Write the number. Under it write either “sample: ____ (who/what was counted)” or “population claim: ____ (who/what is asserted).” If you cannot tell, write “unclear — need sample size and population definition.” Exit: one sentence that refuses overreach.

Deeper sit — truncated bar redraw (30–40 minutes).

Materials: paper, pencil, composite counts.

Safety: composite numbers labelled as such; no real patient or colleague data; no portraits.

The move: Draw two bars that start well above zero so a small difference looks huge (example: counts 48 and 52, axis starting at 45). Then redraw from zero. Write what changed in the takeaway. Numbers unchanged; leap shrinks. Add one line: “Would I still say ‘huge jump’ after the redraw?”

How to fade help

First week of this strip: full sticky templates (“sample / population / base / axis”).

Next: blank sticky — you supply the four labels.

Then: one spoken sentence only, no template.

Stop looking up when you can refuse an overreach sentence without peeking at

this chapter's wording. When a tool is tempting mid-attempt, point back to the refresh-hour rule: unaided first.

Talk-to-yourself box

Opening question (locked):

“Is this number about the sample, or a claim about the population?”

Follow-ups:

1. What exactly was sampled? (*meaning*)
2. What population claim is someone tempted to make? (*and may I revise*)
3. Where does the axis start — and what changes if it starts at zero? (*representation*)
4. Is a base count missing from a percent claim? (*check / size*)
5. What do the axes, units, and time window actually show? (*alternative: what else could the same chart be saying*)

How to wait alone. After you ask, cover any answer you drafted. Count a slow three.⁸⁸ If you stop mid-reason, wait again before peeking. Look at the chart or the sticky. Rescue at one second is still rescue. Abandonment for forty minutes of scrolling is not a micro-session.

What a stuck silence usually means. The question was vague; wait-time 1 was zero; you are hunting for a vocabulary word; you are guessing what a textbook wants; you offered a keyword where a schema was asked for; a finished-looking chatbot paragraph feels like a check; anxiety is eating working memory — shorten the session, win one sample-label sticky, come back tomorrow. Your job is to pose, wait, and point back at the object — not to fill the silence with a photo-solver's steps.

⁸⁸Stahl (1994) think-time; Rowe (1972) science-class wait — classroom origin. Adult translation: pause before lookup. Not an adult math RCT.

Practice that actually builds it

Paychecks and shelves taught percent. Here, **dashboards and claims are the practice** — they still do not replace the structure: sample labelled, axes named, base present or marked missing. Tips and tricks below are teachable moves, not slogans.

Named try-its

1. Dashboard percent with base check — commute / lunch

Time: 15–20 minutes.

Materials: one dashboard tile or printed KPI line (blur names); practice sheet.

Safety: rewrite numbers onto your sheet before any public tool; no proprietary paste.

The move: Copy the percent claim. Write the old value and the new value if shown. If not shown, write “base missing.” Compute percent change only when both values exist. Exit sticky: “base = ____” or “base missing.”

2. Truncation glance — commute / lunch or desk

Time: 10–15 minutes.

Materials: a described bar pair or sparkline (no brand chrome, no readable product UI).

Safety: ordinary stakes only.

The move: Say aloud where the vertical axis starts. If it does not start at zero (or at an honest baseline for the metric), sketch a from-zero version. Ask: did the leap shrink? Adult graph-comprehension work finds presentation shape matters; treat useful studies as useful, not as a promise that every tool will behave the same.⁸⁹

3. Sample-vs-population overreach catch — desk

Time: 20–25 minutes.

Materials: two stickies; one composite workplace claim.

Illustration (labelled as illustration, not a reported workplace): “We surveyed 28 shoppers on Tuesday afternoon. 18 preferred the blue display. So customers prefer blue.”

The move: Rewrite as a sample sentence. Rewrite the population claim someone

⁸⁹Darrell Huff, *How to Lie with Statistics* (1954): Gee-Whiz graphs and truncated bars as journalistic neighbours — not a randomised trial of dashboards. Yang (2021) adult graph-comprehension work is a useful study on how presentation affects reading — not a guarantee about every 2026 workplace tool.

is tempted to make. Add one line: what would support a broader claim (clearer population, more days, larger sample, honest method note). Refuse the overreach without scorning the survey. Twenty-eight Tuesday shoppers are still twenty-eight Tuesday shoppers.

4. Curcio/Friel three questions on a workplace chart — desk or weekend

Time: 25–35 minutes.

Materials: one chart from a deck you already have access to, or a composite chart you draw.

Safety: blur proprietary titles for personal notes; no public humiliation of a colleague who built the slide.

The move: Answer in writing:

- *Read the data:* What exact values or categories does the chart show?
- *Read between:* What comparison does it fairly support?
- *Read beyond:* What story does it *not* justify?

Name axes, units, time window, and comparison in four short lines.⁹⁰ This framework was born in school graph instruction. Use it as **shape**. Do not claim the paper measured your 2026 dashboard product.

5. Absolute vs relative risk toy — weekend deep sit

Time: 30–40 minutes.

Materials: paper; invented round numbers labelled **invented**.

Safety: **not medical advice**; not a real disease; not a recommendation to screen, accept, or refuse anything. Representation only. Clinical questions belong to a clinician or pharmacist — this book does not dose and does not diagnose.

The move: Invent starting risk A → new risk B with round numbers. Example shape (invented): 2% → 1%, and separately 20% → 10%. For each, write: relative reduction; absolute percentage-point change; “1 in N” if you can. Notice that “50% less” can sit beside very different absolute stories. Stop.

6. Natural-frequency toy screening tree — weekend (optional)

Time: 30–45 minutes.

Materials: paper; invented counts out of 1,000 labelled **invented**.

Safety: cognitive/communication habit from natural-frequency research — **not a**

⁹⁰Susan N. Friel, Frances Rae Curcio, and George W. Bright, “Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications,” *Journal for Research in Mathematics Education* 32, no. 2 (2001): 124–58. Read the data / between / beyond — classroom-origin framework used here as **shape** for adults. Do not claim the paper measured modern dashboard products.

clinic guideline, not advice about any real test.⁹¹

Illustration shape (invented): Out of 1,000 people in a pamphlet toy, 10 have Condition Z; of those 10, 9 test positive; of the 990 without Z, 90 test positive. Positive tests = $9 + 90 = 99$; of which 9 have Z $\approx 9\%$ of positive tests in this toy.

The move: Build the tree in counts. Compare to a stack of conditional percents. Say in one sentence why counts can be easier to hold. Do not apply the toy to a personal medical decision.

7. Metric definition vs slide rhetoric — desk

Time: 15–20 minutes.

Materials: slide title vs methods footnote, filter note, or appendix line (composite OK).

The move: Leave the title. Open the definition. Write one mismatch if any (time window, excluded rows, before/after vs parallel comparison). Neighbor habit to reading two sources — applied to a metric.⁹² Structure travels; proprietary row does not.

8. Sports or operations rate sticky — commute (optional mix)

Time: 10–15 minutes.

Materials: a box score or a shift count you already see.

The move: Name the rate (per game, per hour, per hundred). Ask: is this one game's sample or a season claim? Label it. Ordinary stakes; no gambling pitch.

9. Stale refresh and filter check — desk

Time: 15 minutes.

Materials: one live dashboard you already use, or a composite description.

Safety: blur names in personal notes; rewrite isomorphic numbers before any public tool.

The move: Write the time stamp or “refresh time unknown.” Write one filter that might exclude rows (site, shift, status). Ask: if the excluded rows came back, could the percent flip direction? One sentence answer is enough.

⁹¹Absolute vs relative risk as representation: health-numeracy teaching shape. Gerd Gigerenzer and Ulrich Hoffrage, “How to Improve Bayesian Reasoning Without Instruction: Frequency Formats,” *Psychological Review* 102, no. 4 (1995): 684–704 — labelled neighbour for representation (counts out of a fixed population), not a clinic protocol and not medical advice. Toy trees in this chapter use invented round numbers labelled invented.

⁹²Neighbouring habit from *Critical Thinking for Adults*: two named sources and a check. Here applied to metric definition versus slide rhetoric — one pointer, then math. Do not mash PIAAC adult numeracy with thinking-test brands.

10. Parallel item after autopsy — any band

Time: 10 minutes after any solved wrong twin.

Materials: new numbers only.

The move: Change the counts, keep the structure. If the autopsy was “down 50% without base,” invent a new pair with a base present and a second pair with base missing. Unaided. Then check.

Talk-to-yourself while you practice

Use the locked opening on every try-it. After one attempt, try a parallel jacket: same structure, new numbers (another tile, another sticky, another invented toy). Near transfer is the honest hope — sample labels on news **and** work **and** leaflets in the same week. Far transfer to an unseen domain without knowledge is not promised.⁹³

One incorrect example to diagnose (blocked practice)

Slide: “Customer satisfaction up 40%.”

Speaker: “So customers are much happier company-wide.”

Chart: bars from 72 to 74 on an axis that starts at 70. No sample size.

No time window on the slide.

Diagnosis. Relative “40%” does not match $72 \rightarrow 74$ (that rise is roughly a 2.8% relative change and a 2-point absolute change — and even those need units and a clear metric). Axis truncation inflates the leap. Population claim (“company-wide”) has no sample or census named. Fluent slide $\neq$ check.

Repair. Write actual values; redraw from a fair baseline; label sample or census; refuse the overreach sentence; add the time window if known.

Mixed practice (when the move is warm)

When you can name the wrong twin above, mix in one sitting:

1. Sample sticky from a survey sentence.

⁹³Near-transfer design continuity with Willingham-style transfer caution: vary jackets, keep structure; far transfer without knowledge is not promised. Word-problem transfer demos are not workplace numeracy RCTs.

2. Base-missing percent from a dashboard.
3. Truncated bar redraw.
4. Absolute/relative pair with invented percents labelled invented.
5. One interloper from chapter 2 (stacked discount) so you do not only rehearse stats vocabulary.

Blocked practice builds the new move. Mixed practice builds *when* to use it.

A short spacing week (not a 180-day plan)

- Monday micro: sample vs population sticky.
- Tuesday micro: dashboard % base check.
- Wednesday desk: Curcio/Friel three questions.
- Friday micro: truncation glance.
- Optional Sunday deep: absolute/relative toy + done-enough checklist.

There is no adult commute RCT that proves this exact week. Named sessions with named moves are the honest ambition.⁹⁴

If it isn't clicking

Diagnostic 1 — You keep treating the sample as the population.

What you notice: You write “customers prefer...” after a small survey; you say “our average is...” as if everyone were measured.

Likely meaning: Stats overreach; the sample sticky was skipped.

⁹⁴Murray and colleagues (2025) math meta-analysis: spacing versus massed practice about $g \approx 0.28$ in math contexts — labelled encouragement for spacing across days, not an adult commute RCT of this book's objects. Retrieval practice hedged where confidence intervals crossed zero in that synthesis. Dose honesty: there is no IES practice guide titled “Adult Commute Stats Hour.”

Next move: Force two stickies every time. Sticky A: “In this sample of , **we saw** .” Sticky B: “Someone might claim about the population that ____.” Only promote B when you can say what evidence would be needed. Slow down. No shame. Place by skill: if sample labels still fail, stay here even if you once sat a college stats course.

Diagnostic 2 — Percent claims feel complete without a base.

What you notice: “Down 50%” feels finished; you cannot name the old count.

Likely meaning: Relative framing without schema; chapter 2 base habit not yet automatic on dashboards.

Next move: Return to the percent chapter’s base habit for one micro-session, then add the stats sticky “base missing.” Write absolute and relative only when both values exist. If anxiety spikes, one magnitude win (old value / new value) un-timed, then stop. Short sessions and early wins beat a countdown timer as default “rigor.”⁹⁵

Diagnostic 3 — Charts feel like decoration; you read only the title.

What you notice: You remember “On track” and nothing about axes.

Likely meaning: Title trusted over representation.

Next move: Ban the title for two minutes. Name axes, units, time window, comparison first. Then read the title as rhetoric. If a willing colleague is available, ask them to point at the axis start — privately, not as a public quiz. If a class or tutor is available, bring a composite chart, not a proprietary export, to a public tool or a shared table.

When to go ahead: you can label sample vs population on a new surface, refuse one overreach sentence, and notice a missing base. When to ask a human: a real decision that depends on a chart you do not yet trust — still without pasting proprietary data into a public consumer tool. Correct a wrong reason without crushing the attempt — applied to yourself. “Never correct yourself” is not this book’s rule. Silence in the face of your own wrong reason is not kindness.

⁹⁵Erin A. Maloney and Sian L. Beilock, “Math Anxiety,” *Trends in Cognitive Sciences* 16, no. 8 (2012); Sian L. Beilock and Daniel T. Willingham, *American Educator* (Summer 2014). Anxiety loads working memory; short sessions and early wins. Not a clinical protocol in this book.

Tools, including AI

Keep this short. Point back to the refresh-hour AI box for the full allow/ban list.

Honest tools for this strip. Pencil and two stickies. A calculator to check an absolute and relative pair *after* you estimate. A spreadsheet to redraw a bar from zero *after* you sketch by hand. OpenStax *Introductory Statistics* (or a similar free text) for definitions you want to sit with longer — as a backbone, not a diploma.⁹⁶

You may, after you have tried: ask a tool to explain *to you* what “sample versus population” means using a named lesson title; request isomorphic practice with answers on **your** page; ask for a hint after an attempt; diagnose work you already did (crop to paper — no face); use a labelled calculator or dynamic tool after the try to check or explore.

You may not: paste a workplace dashboard export and ask the tool to finish the critique during the attempt; ask “what’s the answer” mid-try; photo-to-key a chart problem; park an unsupervised chatbot as your only partner; paste proprietary employer data into a public consumer chat; invent sample sizes or axis starts the slide never gave; print a fluency certificate from a chatbot; detector-grade your own unaided write-up.

One mechanism reminder, labelled ON-AGE-adjacent: in a high-school math field experiment (grades 9–11, nearly 1,000 students, Turkey), an unguarded chatbot made practice look better and left students worse on the test with the window closed.⁹⁷ **Not an adult RCT.** The lunch-break rule still holds: the model may prepare you and the next problem; it may not do your problem during the attempt.

Spreadsheet and dashboard software are tools for display. They do not replace axes, units, time window, and comparison named in plain language. Fluency with buttons is not statistical judgment — that was wrong answer 5 for a reason.

⁹⁶OpenStax Introductory Statistics / Statistics titles — free digital OER backbone. Not a diploma by themselves. Fit details in the resources chapter. No opened WWC report crowns OpenStax for adult kitchen self-study.

⁹⁷Hamsa Bastani and colleagues, *Proceedings of the National Academy of Sciences* (2025): high-school math field experiment (grades 9–11, nearly 1,000 students, Turkey) — unguarded chatbot condition improved practice appearance and hurt closed-exam performance relative to control; guarded tutor withheld answers. **Not an adult RCT.** Mechanism reminder only; full AI rules live in *The Refresh Hour*.

What “done enough” looks like

Before you move on from this strip, check yourself by **skill**, not by diploma year and not by birthday:

- ☐ I can label a number as about a **sample** or as a **population claim** (or say unclear and what I would need).
- ☐ I can refuse an **overreach** sentence and say what would support a broader claim on ordinary stakes.
- ☐ I can name **axes, units, time window, and comparison** on a chart.
- ☐ I notice a **missing base** on a percent claim and write “base missing” when needed.
- ☐ When both values are known, I can write **absolute** and **relative** change.
- ☐ I can spot a **truncated** habit and say what changes if the axis starts at zero (or at a fair baseline).
- ☐ I treat association lightly: “together in the data” is not automatically “caused.”
- ☐ I hear **margin of error** (when present) as honesty about uncertainty, not as “the study failed.”
- ☐ I can diagnose the incorrect slide example in this chapter without peeking.
- ☐ I kept at least one **dated line** a colleague who missed the huddle could almost follow.

A “math refresher certificate” is a receipt, not the work — and this book never invents one. Never Emerging Adults Math I. Never Math Refresher I. Optional CLEP and college course names (including Introductory Statistics, if that is your path) live in keeping track and resources if credit is your goal. Sitting a stats exam is not required to finish this chapter. No diploma was required to open it.

Life of the habit tip. Next Tuesday, reopen one dashboard or one news sticky.

Same question: sample or population? Space a truncation glance across the week. Maintenance weeks revisit last month's percent *and* this strip's sticky; rebuild weeks dig into absolute/relative toys or a longer chart autopsy. Stop when the next life demand is met — then keep a light maintenance rhythm so the strip does not re-rust. Growing fluency is more miles on the same road, not a new faculty and not a math birthday.

You try this week. On the go: label one workplace or news number as a sample or a population claim. Deeper sit: redraw a truncated comparison from zero; write both absolute and relative change when a base is known, and “base missing” when it is not.

You do not need to be a statistician. You do need to hear a sample mean offered as a population fact — and to ask the locked question before you nod at the slide. You can do this on the commute and in a deeper sit.

Chapter 8

Keeping track

There is often **no** “Math Refresher I” as a job title, and this book will not invent one. A stranger — a manager, a later hiring panel, a college registrar, a colleague who missed the huddle — already reads work they understand: a dated note, a worked check, a skill named in ordinary language, an optional official score report. Never Emerging Adults Math I. Never Math Refresher I. Never a fake certificate of fluency.

This chapter is the file you keep. The teaching chapters rebuilt meaning and fluency. Here you place yourself by skill, log what you practiced, leave a dated line someone else could almost follow, and — only if credit is your goal — decide honestly whether an optional exam is worth the fee.



Figure 11: Dated notebook with skill-strip checklist — no diploma seal

What you are making

You are making a **skill-strip log**, a **self-placement habit**, and a **dated practice note** (often with one worked example). Stacked over months, that file is the honest adult work product. You are not making a national diploma. You are not making a chatbot badge. You are not making Certified Math Refresher.

Titles a stranger already understands:

- the **skill** (percent and ratio; linear equations; geometry reasons; sample vs population)
- the **job artifact** (invoice check; dashboard read; cut list; APR structure comparison)

- a **college course name** only if the work maps (College Algebra, Intermediate Algebra, Quantitative Reasoning, Introductory Statistics)
- an **optional CLEP** exam name if you sat it and the receiving college cares

Combining objects is allowed. Combining does not mint a franken-title. A Khan streak plus an OpenStax chapter plus a dated dashboard read is still the skill strip — not “Online Math Refresher I.”

Here is an illustration, labelled as such, not a reported workplace:

5 September 2026. Skill strip: percent & ratio + dashboard read. Object: composite KPI tile (blurred names). Old value 40 defects / 200 units; new value 20 / 200. Relative —50%; absolute —10 percentage points (20% → 10% defective). Axis on slide started at 8%; redraw from 0% shrinks the leap. Sample: checked batches, not yet every line. What would change my mind: line-level counts for two more weeks. Files: practice sheet 2026-09-05.

That is readable by a colleague who was not in the room. It is not a badge. Place by skill, not by brand.

The skill-strip log

Keep a simple table or checklist you will actually reopen. Columns that earn their keep:

Date	Skill strip	Object	What I tried	Exit sticky	Next cue
		(stub / shelf / chart / equation...)		/ cautious sentence	

Skill strips this book uses (match the teaching chapters):

1. Number sense, fractions, decimals as numbers

2. Percent and ratio
3. Algebra refresh (linear / quadratic meaning)
4. Geometry with reasons
5. Functions and trigonometry (only if this strip is in your plan)
6. Calculus edges (optional, ready adults only — not for everyone)
7. Statistics for life and work (sample vs population; dashboards)

You do not need every strip. A trade adult may live in fractions, percent, measure, and dashboard reads. A STEM bridge adult may add functions and calculus edges. Placement is by life demand and skill, not by a podcast that said “everyone needs Calc.”

Rebuild week vs maintenance week. A rebuild week opens a rusty or new strip: more solved autopsies, a deeper sit, blocked practice before mixing. A maintenance week revisits last month’s percent or linear check with three micros and one mixed interloper — no heroics. Adults who only rebuild and never maintain will re-rust. Adults who only maintain and never rebuild will not open the next gate. Name both in the log (“R” or “M” in the Next cue column is enough).

Life of the habit, here: the note is how the habit leaves a trace. Same kind of object next Tuesday, new date. “I used to treat ‘down 50%’ as finished; now I write the base or ‘base missing’ ” is growth. It is not a diploma ceremony.

Self-placement by skill (not diploma year)

A college graduate can still fail a multi-step percent-of-percent on a dashboard. A GED-holder can be fluent at shop fractions. **No diploma was required** to open this book, and no diploma year places the next module.⁹⁸

⁹⁸National Mathematics Advisory Panel, *Foundations for Success* (2008), Finding 15: readiness depends on prior opportunity to learn — adult flip: place by skill strip, not “too old,” not diploma year. Access date for sources in these notes: 5 September 2026.

Use short probes — mirrors, not oracles. Vendor placement quizzes (platform diagnostics, commercial adaptive tools, college guided self-placement) place by skill; they are not IES-certified RCTs of this book.⁹⁹

Probe ladder (self-check, not a published test):

- **Whole numbers & place value.** Multi-digit ops; decimal place meaning. If cash and inventory counts still wobble, stay here.
- **Fractions & ratios.** Equivalence; part/whole; unit rates. Recipes, shops, label rates (units only).
- **Percents.** Of, increase/decrease, stacked. Tax, tip, discounts, dashboards.
- **Proportional reasoning.** Solve a proportion; scale a drawing. DIY, maps.
- **Linear algebra edges.** Solve $ax + b = c$; rearrange a formula you actually use.
- **Graphs & data displays.** Axes, trends, truncation; sample vs population.
- **Functions & nonlinear edges.** Slope as rate; family from a table — if needed.
- **Calculus edges (optional).** Average rate in units; accumulation sketch — only if ready.

Rule of thumb: if you cannot yet name a base on a percent, you are not ready to skip to derivative meaning. If fractions still feel opaque, you are still in the gate chapter, whatever your diploma. Patch the hole in front of the next real task. You may be strong at shop fractions and weak at dashboard percents in the same week — that is ordinary, not a character flaw.

⁹⁹Vendor placement tools (platform diagnostics, commercial adaptive systems, college guided self-placement) are mirrors. IES / What Works Clearinghouse does not certify them as RCTs of this book.

Dated practice note / worked example

Keep what a stranger can use.

A dated note. Object named. Skill strip named. What you tried. The date is 5 September 2026 or whatever day it actually was — not “Q3 math.”

One worked check. APR vs interest rate using CFPB definitions (literacy, not product advice); unit-price comparison with matched units; dashboard read with axes/units/time/comparison; dose-rate arithmetic as units only (not medical advice); map/scale or cut-list reason.

Sources/numbers named. If “it” needs the meeting, rewrite.

What not to keep as the proof: a random PDF “certificate of completion” from a site that is not transcript credit; a LinkedIn badge; a percentile; a franken-title. Those can be receipts. They are not the work.¹⁰⁰ Prefer: dated notebook; community-college noncredit completion letter on letterhead if you earned one; official CLEP score report if you chose that path; actual college transcript.

How much is enough? A line after a micro-session is enough for a first week. A stack of dated checks over a quarter is enough for a later reader. A binder labelled “Math Refresher, Level 2” is not more lawful than the line. Frequency the evidence actually supports at the desk grain for *this* book is unknown — there is no adult commute RCT at this exact grain. The honest pitch is often enough that the question is familiar, and dated enough that next Tuesday can find last Tuesday.

If writing itself is the wall, keep the parts anyway: skill, object, because, check, date. A human colleague who already writes clear notes can help with the writing shape. You still own the math question.

What employers and colleges already read

Most workplaces do not have a line item called “math refresher.” They have invoices, dashboards, cut lists, quotes, and handoffs. Put the dated check inside those

¹⁰⁰Fake certificates and invented course titles (Emerging Adults Math I / Math Refresher I) are refused as stranger-readable proof. Prefer dated notes, letterhead completions, official score reports, transcripts.

objects. An auditor's workpaper energy — what did I look at, what did I conclude, what would change my mind — is the grain, applied to quantitative claims. A nurse's chart, an analyst's repo note, a lead's decision log: same idea. The title stays the job you hold.

Colleges read transcripts, placement results, and sometimes CLEP. They do not need a homemade course code. If you completed a local noncredit refresher, keep the letter on letterhead. If you sat CLEP and credit posted, keep the official report. If you only used this book and free practice, say that honestly when someone asks how you prepared — the log is still the proof of *practice*, not a substitute transcript.

PIAAC and other large adult-skills surveys describe a national pond. They are not your pretest cut score and not a grade for your notebook.¹⁰¹

Optional CLEP — honesty, not spine

CLEP is **optional** credit-by-exam. It is a bridge when a specific institution will grant credit. It is not required to refresh math for work, money, health-numeracy representation, or DIY. Sitting CLEP $\neq$ finishing this book.

Opened College Board pages (5 September 2026): College Algebra, Precalculus, and Calculus list at **\$97.00** each, plus a test-center administration fee or a remote-proctor fee.¹⁰² Rough shapes: College Algebra about 60 questions in 90 minutes; Precalculus about 90 minutes in two sections; Calculus about 44 questions in about 90 minutes (limits/differential and integral shares per the listing). Credit-granting scores are **institution-set**. ACE notes on exam pages are notes — colleges may differ. Sitting without checking transfer rules wastes money — say that plainly.

DANTES funding for eligible service members is a neighboring path — confirm locally. AP Calculus / Statistics sittings are high-school logistics with coordinator rules; they are not the adult default year. SAT Subject Tests were discontinued in

¹⁰¹National Center for Education Statistics / IES, PIAAC 2023 highlights: U.S. adult numeracy context (mean 249 vs international 263; Level 1 or below 34%). Survey pond, not this book's placement cut score.

¹⁰²College Board CLEP exam pages for College Algebra, Precalculus, and Calculus, opened 5 September 2026: fee **\$97.00** each plus administration or remote-proctor fee. Confirm live fee pages at publication if you quote remote-proctor add-ons; some educator materials list an approximate remote fee — verify before you budget.

June 2021 — one sentence, then stop.¹⁰³

If your goal is credit, prep from the skill strips you still miss, confirm the receiving college, then sit. If your goal is a cleaner dashboard read next Tuesday, skip the fee and keep the log.

If a later reader asks

A manager who sees “Math Refresher I, A” has been given a slogan. A manager who sees “Aug–Sep 2026 practice log: percent applications; APR vs interest using CFPB definitions; five dashboard reads with axes/units noted” has been given a month they can use.

A registrar who sees an official CLEP score report and a catalog match has a credit question. A registrar who sees a chatbot PDF certificate has a paperweight.

Export inside the job’s real title or a real college course name only when the work maps. Adding “Certified Numeracy Practitioner” to a résumé does not make the year more lawful. It may confuse the next stranger.

You try this week

Open one page in a notebook or a plain file. Date it. Name the skill strip you touched. Paste or rewrite one worked check (unit price, APR structure, dashboard read, or sample-vs-population sticky). Mark rebuild or maintenance. Write one next cue for Tuesday.

If you have three dated lines already, glance back: which strip reappears? That strip may need a maintenance week. Which strip never appears though life keeps asking for it? That strip may need a rebuild week.

That is the record. It is not a diploma. You can stop when the next life demand is met — and you can return without starting from shame.

¹⁰³College Board: SAT Subject Tests discontinued June 2021. AP remains a high-school exam object — not the adult refresher default.

Chapter 9

Resources

This chapter names programs a working adult actually meets. It does not rank them. It does not sell them. Fit is cost shape, time shape, social shape, credential shape, depth, and tone — whether a commute, a desk, or a weekend sit can use the object this week.

Paper and a free text before any paid platform. Kitchen **\$0** practice before a subscription. Confirm a current price on the live page before you buy; this chapter does not invent list prices it did not open.

Whenever a community-college pathways model is named below, copy the evidence label with it: What Works Clearinghouse’s Dana Center Mathematics Pathways review (June 2021, Developmental Education) found **positive effects** on progressing in developmental education (improvement index **+21**; 3 studies; 46,012 students) and progressing in college (**+8**; 2 studies; 11,163 students). Those outcomes are sequence/credit progression for enrolled community-college students with supports — **not** achievement on a kitchen refresher quiz, and **not** a solo commute RCT.¹⁰⁴

No opened WWC report crowns Khan Academy or OpenStax for adult self-study.

¹⁰⁴Institute of Education Sciences, What Works Clearinghouse, Dana Center Mathematics Pathways, Developmental Education intervention report (June 2021, WWC 2021-014): positive effects on progressing in developmental education (improvement index **+21**; 3 studies; **46,012** students) and progressing in college (**+8**; 2 studies; **11,163** students). Reviewed evidence from Texas community-college settings among others as summarised; other protocol outcomes (degree attainment, employment, earnings) not known under that summary table. Co-requisite model revisions after some studies — confirm local 2026 practice. Access date for sources in these notes: 5 September 2026.

Secondary-school WWC math ratings stay school-labelled if named at all.



Figure 12: Three fit cards: \$0 practice, local class, optional credit — no logos

How to read a program

Six questions do the work.

Cost shape. Nothing tonight? Free digital text? Local adult-ed fee? Tuition? Exam fee (\$97 plus proctor for CLEP)? Subscription?

Time shape. Asynchronous twenty minutes? Scheduled evening class? One sitting after weeks of prep?

Social shape. Alone? Instructor? Cohort?

Credential shape. None? Local completion letter? Transcript credit? Optional CLEP score report?

Depth shape. Arithmetic and percent refresh? College algebra bridge? Contest olympiad?

Tone shape. Patient remediation? Lecture-heavy? Contest puzzle culture?

What a stranger can still read is the skill you rebuilt or the college course you actually took. Combining two honest objects does not mint Emerging Adults Math I. Never a fake certificate as the year's proof.

Kitchen \$0 first

Pencil, paper, your own bills, shelf tags, blurred dashboard numbers rewritten onto *your* sheet, CFPB pages for interest-versus-APR literacy (not product advice), library print of a free textbook PDF.¹⁰⁵

Fit. Tonight, one adult, low load, low cost, offline-friendly.

Title. The skill strip or the job artifact. Never Math Refresher I.

This is not “failing the specialist.” It is the object this book already taught. Wait. One question. One look. One dated line.

Khan Academy — \$0 practice backbone

Fit card. Cost: \$0. Mode: asynchronous videos and mastery-style problem sets across arithmetic through early college mathematics and statistics. Credential: none. Strength: low-friction drills and gap-finding between shifts. Weakness: the loop can feel school-coded; workplace examples may be thin. Evidence: no

¹⁰⁵Consumer Financial Protection Bureau, Ask CFPB: interest rate versus APR — compare APR to APR; literacy, not lending advice.

WWC adult self-study report opened for this stream.¹⁰⁶ Placement quizzes on the platform are **vendor tools** — mirrors, not oracles.

Adult use. Diagnose a hole (fractions, percent, linear equations). Drill it. Move on. Pair with a dated error log so the energy bar is not the only memory.

Not. A diploma. An employer ID. CLEP. A licence to skip unaided attempts.

OpenStax — \$0 text backbone

Fit card. Cost: \$0 digital (low-cost print editions exist per OpenStax higher-ed copy). Mode: peer-reviewed open textbooks with exercises. Credential: none unless a college adopts the book inside a credit course. Strength: stable scope and sequence mirroring college courses — good when you want a book that stays put. Weakness: alone, less adaptive; requires self-pacing. Assignable homework layers, when offered, are a **paid** LMS add-on — confirm live pricing; this chapter does not invent a 2026 student fee.¹⁰⁷

Commonly listed math titles adults meet: Prealgebra, Elementary Algebra, Intermediate Algebra, College Algebra, Algebra and Trigonometry, Precalculus, Calculus Volumes 1–3, Introductory Statistics / Statistics, Introductory Business Statistics (catalogue consensus; live pages can be script-heavy — open the subject page you need).

Adult use. Read a section. Work problems away from the screen. Keep a dated error log. Pair with Khan for interactive reps when that helps.

Not. A diploma by itself. “WWC-approved OpenStax” — no such opened report.

¹⁰⁶Khan Academy and OpenStax (Rice University nonprofit OER): catalogue consensus for free practice and free digital textbooks. No opened WWC adult self-study crowning report for either in this research stream. OpenStax Assignable is a paid layer when used — confirm live price.

¹⁰⁷Khan Academy and OpenStax (Rice University nonprofit OER): catalogue consensus for free practice and free digital textbooks. No opened WWC adult self-study crowning report for either in this research stream. OpenStax Assignable is a paid layer when used — confirm live price.

Pairing shape

Khan for interactive reps; OpenStax for longer problem sets and definitions that stay put; kitchen objects (stub, shelf, chart) for transfer. Neither free backbone replaces a community-college transcript if you need credit. Neither replaces the talk-to-yourself wait before the key.

Community college — noncredit, developmental, pathways

Noncredit refresher / adult education — fit card. Cost: local tuition or adult-ed fee (varies; not invented nationally here). Mode: scheduled or flexible. Credential: sometimes a local completion letter on letterhead. Strength: human instructor, peers, a door into credit pathways. Weakness: calendar, transport, and the stigma some adults feel walking into a “remedial” room — stigma is social weather, not a measure of your skill.

Developmental / co-requisite credit path — fit card. Cost: tuition plus time. Credential: transcript progress when successful. Strength: for pathways models like Dana Center Mathematics Pathways, WWC reports positive progression effects (+21 / +8) in reviewed community-college studies as labelled above.¹⁰⁸ Weakness: bureaucracy; co-requisite load is real work. Co-requisite designs evolved after some reviewed studies — do not describe your 2026 local catalog from older papers alone. Ask the college what it runs now.

OCTAE College and Career Readiness Standards for Adult Education are a program framework for adult basic education providers — not a consumer syllabus you must finish as a PDF.¹⁰⁹

¹⁰⁸Institute of Education Sciences, What Works Clearinghouse, Dana Center Mathematics Pathways, Developmental Education intervention report (June 2021, WWC 2021-014): positive effects on progressing in developmental education (improvement index +21; 3 studies; 46,012 students) and progressing in college (+8; 2 studies; 11,163 students). Reviewed evidence from Texas community-college settings among others as summarised; other protocol outcomes (degree attainment, employment, earnings) not known under that summary table. Co-requisite model revisions after some studies — confirm local 2026 practice. Access date for sources in these notes: 5 September 2026.

¹⁰⁹U.S. Department of Education / OCTAE, College and Career Readiness Standards for Adult

Tie evidence only when the college actually runs a pathways model. Do not assume every noncredit evening class is DCMP. Do not treat DCMP progression outcomes as proof that your commute app works.

CLEP — optional credit bridge

Fit card. Cost: **\$97** per exam plus test-center or remote-proctor fee (College Board pages opened 5 September 2026).¹¹⁰ Mode: one sitting after prep. Credential: possible transcript credit if the receiving college accepts the score and maps it. Strength: fast bridge for adults who already know the material. Weakness: no credit if the institution says no; sitting anxiety; prep time still required.

College Algebra, Precalculus, and Calculus are the math exams adults most often mean. Check transfer **before** you pay. DANTES funding for eligible service members — confirm locally. Sitting CLEP $\neq$ finishing this book. CLEP $\neq$ a fake certificate.

AP Calculus / Statistics remain high-school exam objects with coordinator logistics — not the adult default year. Mention once if you already teach or sit near a school; then return to adult fit.

Print spines and paid platforms (labelled, not crowned)

Saxon (print tradition). Fit for adults who already know the incremental, problem-heavy style from their own schooling or from teaching teens. Offline strength. WWC Saxon Algebra I (school intervention report) found **no discernible effects** on algebra under that review — **school-labelled**, not an adult

Education — ABE program framework, not a consumer syllabus mandate for kitchen self-study.

¹¹⁰College Board CLEP College Algebra, Precalculus, and Calculus pages, opened 5 September 2026: **\$97.00** each plus administration or remote-proctor fee. Institution sets credit. Confirm live fees before budgeting.

kitchen endorsement.¹¹¹ Do not sell Saxon as WWC-proven for solo refreshers. 2026 kit prices omitted unless you open a specific product page.

Commercial adaptive collegeware (ALEKS and cousins). Fit when a class requires them. Strength: placement plus practice inside an institutional course. Weakness: can train click patterns — check whether you can still do the paper disclosure or the dashboard read without the software. Feature cards and prices **vendor**; not opened as RCTs of this book.

Video curriculum vendors familiar from school/homeschool catalogs (names vary). Age packaging is often parent-and-teen. Adult fit is incidental: use a clear video algebra spine if it helps, after checking age labels. Do not default over Khan/OpenStax without a reason (offline preference, pedagogy you already trust). Prices omitted unless opened.

Employer L&D “business math” or Excel modules. Fit: sits on real workbooks; internal completion at best. Weakness: often thin on conceptual percent sense; may teach buttons not meaning. Still a vendor/HR object.

AoPS — not the adult default

Art of Problem Solving Online recommendations (opened) point to texts and classes for students in **grades 5–12**, Beast Academy for ages **6–13**, and a contest ladder (AMC / AIME / WOOT).¹¹² Deep and excellent for that pond. Wrong default for an adult refreshing percent, APR structure, unit price, and dashboard reading.

Optional only if you explicitly want contest-depth problem solving and accept the high-school/olympiad grain. “Are you ready?” diagnostics on that site are vendor tools for that curriculum — not this book’s placement.

¹¹¹WWC secondary mathematics intervention reports (Saxon Algebra I — no discernible effects; Cognitive Tutor / UCSMP / Core-Plus with mixed or potentially positive/negative ratings depending on product) are **school-labelled**. PDFs available in research primaries from sibling streams. Not adult kitchen proof.

¹¹²Art of Problem Solving Online recommendations page, opened 5 September 2026: grades **5–12**; Beast Academy ages **6–13**; contest ladder. Not the adult refresher default.

Secondary WWC and other school objects — one pointer

If you remember Cognitive Tutor, UCSMP, Core-Plus, or similar names from school-era debates: those WWC ratings are **school** intervention reports (mixed, potentially positive, or potentially negative depending on product and outcome).¹¹³ One pointer for writers and curious adults. Do not paste them as adult kitchen proof.

Education Endowment Foundation secondary maths findings are age-labelled UK school objects if named at all. Do not mash Philosophy for Children trial results into a math resource list.¹¹⁴ UK adult-numeracy pilots exist abroad; they are not this book's U.S. WWC lock.

Dual enrollment and credit courses (neighbor, not reprint)

Some adults sit credit math through dual enrollment, evening credit, or online college sections with placement gates. State and college rules vary. That path is a neighbor to CLEP: transcript credit when you pass the course the college actually offers. It is not a reprint of high-school dual-enrollment or athletic-eligibility machinery from *Math for Emerging Adults*. One pointer to that book if you also teach a teen — then return to your adult load.¹¹⁵

Placement tests and guided self-placement at the college are institutional tools. Use them as mirrors. They do not license skipping fraction fluency because a schedule wants you in College Algebra next Monday.

¹¹³WWC secondary mathematics intervention reports (Saxon Algebra I — no discernible effects; Cognitive Tutor / UCSMP / Core-Plus with mixed or potentially positive/negative ratings depending on product) are **school-labelled**. PDFs available in research primaries from sibling streams. Not adult kitchen proof.

¹¹⁴Do not mash Education Endowment Foundation Philosophy for Children effectiveness results (Key Stage 2 / Year 6 school trial) into adult math resource claims. EEF secondary maths findings stay age-labelled if used at all.

¹¹⁵*Math for Emerging Adults* covers high-school transcript and dual-enrollment grain for teens. One pointer for adults who also teach or guide a teen — then return to adult self-placement and adult fit. This book is not a reprint of that lesson bank.

What not to chase

- Fake PDF certificates and invented course titles (**Emerging Adults Math I, Math Refresher I**, “Certified Math Refresher”).
- “WWC-proven Khan” or “WWC-approved OpenStax” for kitchen self-study — no such opened report.
- AoPS or AP as the default adult year.
- Productized financial tips or medical advice dressed as numeracy resources.
- Invented claims that most employers require CLEP or a numeracy badge.
- Treating this book as a replacement for developmental math policy at a college — it can prepare you to walk in less afraid; it does not rewrite the catalog.

If you want proof, prefer the dated notebook, a letterhead completion, an official score report, or a real transcript.

Choosing without ranking — a five-minute decision

1. What skill strip is in front of you this week?
2. Do you need **credit**, or do you need **usable fluency** for a stub, shelf, chart, or cut list?
3. If fluency only: start kitchen \$0 or Khan/OpenStax.
4. If credit: open the receiving college’s CLEP and placement pages *before* paying an exam fee or buying a platform.
5. If a local class fits your calendar and stigma budget, walk through that door — and keep the skill-strip log anyway.

Fit is personal. A night-shift adult and a remote analyst can both be right with different rows of the table above.

Fit table (use; do not rank)

Tonight's need	Object	Load	Note
Nothing spent	Kitchen objects + dated note	Low	Not a fake certificate.
Free drills	Khan Academy	Low–moderate	\$0; not a diploma.
Free text backbone	OpenStax math titles	Moderate	\$0 digital; self-pace.
Human instructor / local door	CC noncredit or adult ed	Moderate–high	Local fees; letter sometimes.
Transcript progress	Developmental / co-requisite / pathways	High	DCMP +21/+8 labelled CC developmental ed — not commute RCT. ¹¹⁶
Optional credit-by-exam	CLEP math exams	Prep + sitting	\$97 + fees; check college first. ¹¹⁷
Familiar print sets	Saxon etc.	Moderate	School WWC $\neq$ kitchen proof. ¹¹⁸
Contest depth	AoPS	High / HS grain	Not adult default. ¹¹⁹
Required by a class	Adaptive collegeware	Gated	Buttons $\neq$ meaning.

¹¹⁶Institute of Education Sciences, What Works Clearinghouse, Dana Center Mathematics Pathways, Developmental Education intervention report (June 2021, WWC 2021-014): positive effects on progressing in developmental education (improvement index **+21**; 3 studies; **46,012** students) and progressing in college (**+8**; 2 studies; **11,163** students). Reviewed evidence from Texas community-college settings among others as summarised; other protocol outcomes (degree attainment, employment, earnings) not known under that summary table. Co-requisite model revisions after some studies — confirm local 2026 practice. Access date for sources in these notes: 5 September 2026.

¹¹⁷College Board CLEP College Algebra, Precalculus, and Calculus pages, opened 5 September 2026: **\$97.00** each plus administration or remote-proctor fee. Institution sets credit. Confirm live fees before budgeting.

¹¹⁸WWC secondary mathematics intervention reports (Saxon Algebra I — no discernible effects; Cognitive Tutor / UCSMP / Core-Plus with mixed or potentially positive/negative ratings depending on product) are **school-labelled**. PDFs available in research primaries from sibling streams. Not adult kitchen proof.

¹¹⁹Art of Problem Solving Online recommendations page, opened 5 September 2026: grades **5–12**; Beast Academy ages **6–13**; contest ladder. Not the adult refresher default.

Tonight's need	Object	Load	Note
Employer module	L&D business math / Excel	Low–moderate	Internal receipt at best.

You try this week

Name *why* tonight's object fits — usually kitchen \$0 or Khan/OpenStax before paid. Open one free section or one drill set that matches the skill strip in front of you. Do one unaided parallel item on a stub, shelf tag, or dashboard tile. Date the line.

If credit is on the table, open *one* receiving-college page and write the CLEP or placement rule in your notebook before you spend \$97. If credit is not on the table, do not let an exam brochure become this week's homework.

If a branded pathways program is named at your college, remember the WWC +21 / +8 progression evidence is for **community-college developmental education**, not a solo commute proof — then learn the math on *your* objects anyway. The file is still the proof. Fit first. Rank never. Start free when free will do. Return to the skill strip when a catalog page starts to feel like shopping.

A Note on Sources

Studies named in the chapters are listed in Notes, in one series at the back. That is where the full citations live, so the teaching pages can stay a teaching voice.

Some items the research behind this book did not open stay out of the teaching voice. I did not invent a coefficient, a statute, or a product feature to fill a hole. If a study is in the chapter, it is in the notes. If we could not open it, it is not used as a finding here.

Program hours, exam fees, and product descriptions in the resources chapter are taken from the companies' or agencies' own pages. Access date 5 September 2026.