Math for Young Minds

Michael Gannotti

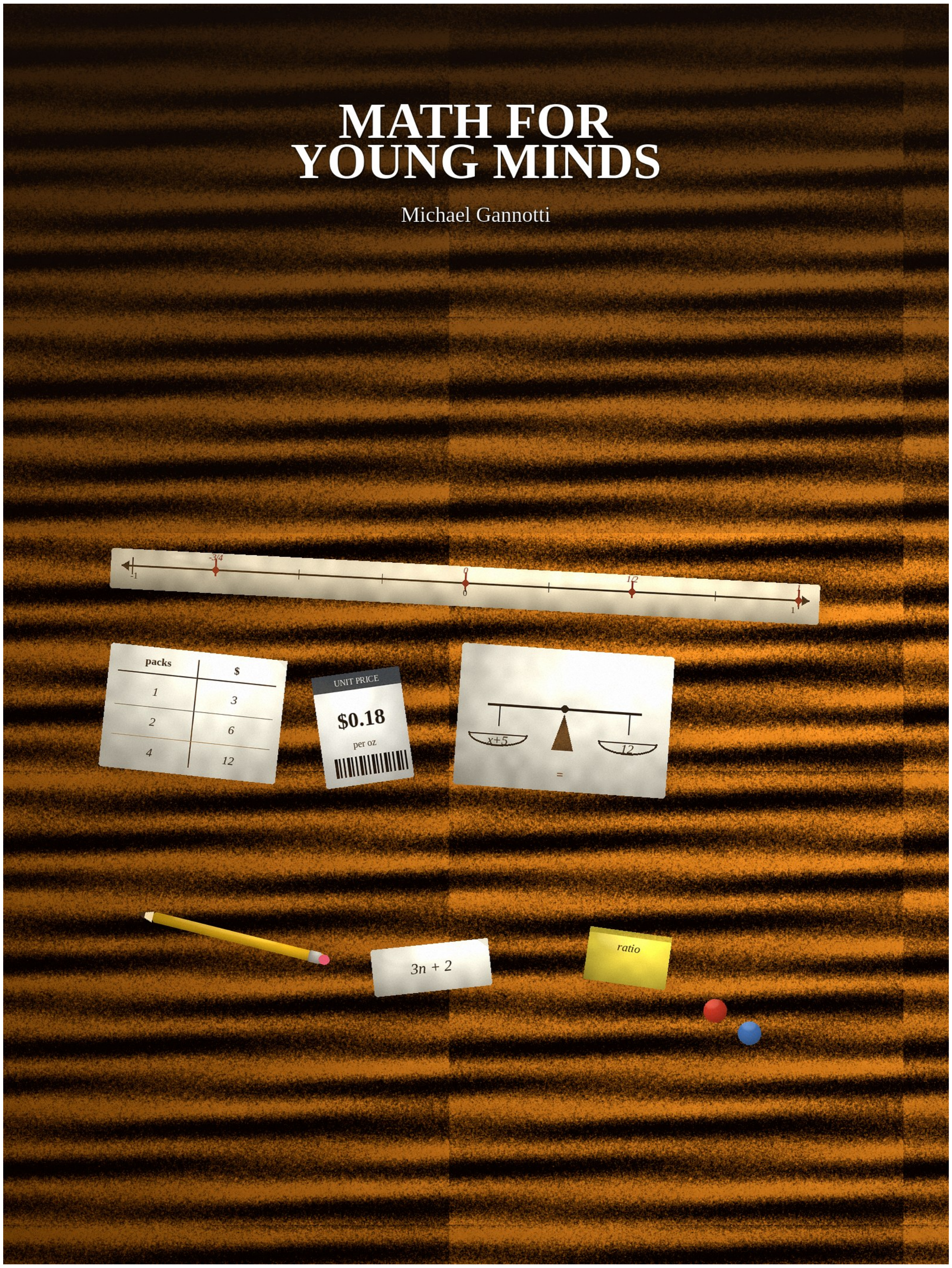

Figure 1: Cover

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This illustrated edition is the teaching-manual of 4 September 2026. Endnotes run in one series. Chapter-opening images are original still-lifes, not portraits of persons or children. Documents still constrain the facts.

Contents

Welcome

This book exists because the math hour at your table is the whole program. There is no math department down the hall and no specialist waiting after lunch. There is you, an eleven- to fourteen-year-old, and today's idea — a ratio table or a number line that runs both ways, a question worth waiting for, an equation they write and check. That is enough, if you know what to do with the hour.

I wrote this for a capable, busy, willing parent of an eleven- to fourteen-year-old. You may be teaching two ages at once, eleven-to-twelve and thirteen-to-fourteen, at one table. You may be fitting a math sit between a job, a younger sibling, and a store run. You may love number, or you may remember middle-grades math as a fog of worksheets and a birthday that somehow meant Algebra I. Many adults feel rusty when they sit down to teach. That feeling is common. It is not a verdict. We will move on from it. This book will make you fluent enough in today's idea to hear a wrong turn, and to ask a good question — without taking the pencil.

You do not need to be a mathematician. You do need to hear cross-multiply with no ratio table as a missing-meaning move, not a cute slip. You need to hear "of means multiply," = as "write the answer," pizza offered as the number $-3/4$, letter-moving without structure, a formula card offered as geometry, and a birthday offered as Algebra I readiness. You need a session shape you can run on a Tuesday. You need a few sentences that actually help. That is the job. A path through ordinary middle-grades challenges — fractions, decimals, and percents as numbers (including negatives), ratio and proportion with meaning, signed numbers, expressions, equations by same-to-both-sides, early functions and linear thinking, geometry with reasons, data and chance with story-problem schemas — is possible at a kitchen table. The path is not a personality trait and it is not a percentile. It is a small set of moves, practiced on mathematics the student already somewhat knows, with you in the chair.

About 3.4 percent of U.S. students ages 5–17 were homeschooled in 2022–23 — roughly 1.765 million children.[1] In the middle grades, ages that map to grades 6–8, the cell is about 3.0 percent.[2] That figure is context, not a ranking. You are one of those tables. Many readers of this book will not be in that pond. They will be after-school, weekend, or kitchen-table parents of enrolled adolescents. The moves still fit. Homeschooling is legal in all fifty states and the District of Columbia. The paperwork is not one load: Texas asks almost nothing of the state agency; New York asks for a written plan and regular reports.[3] There is no national homeschool diploma and no national "Young Minds Math I" credit.[4] None of Texas, North Carolina, Pennsylvania, Virginia, or New York requires a critical-thinking course. Math is the usual required or assumed middle-grades object.[5] In the last federal

[1]Ellena Sempeles and Jiashan Cui, *Parent and Family Involvement in Education: 2023*, NCES 2024-113 (Washington, DC: National Center for Education Statistics, September 2024), Table A-6: 3.4 percent homeschooled, approximately 1,765,000 students, ages 5–17 with a K–12 grade equivalent, 2022–23; grades 6–8 cell 3.0 percent. https://nces.ed.gov/pubs2024/2024113.pdf. Access date for URLs in these notes: 4 September 2026. Latest federal count this book uses; not a 2026 national total. A neighboring 5.2 percent received instruction at home (homeschooled or full-time virtual) and is a different bucket.

[2]Ellena Sempeles and Jiashan Cui, *Parent and Family Involvement in Education: 2023*, NCES 2024-113 (Washington, DC: National Center for Education Statistics, September 2024), Table A-6: 3.4 percent homeschooled, approximately 1,765,000 students, ages 5–17 with a K–12 grade equivalent, 2022–23; grades 6–8 cell 3.0 percent. https://nces.ed.gov/pubs2024/2024113.pdf. Access date for URLs in these notes: 4 September 2026. Latest federal count this book uses; not a 2026 national total. A neighboring 5.2 percent received instruction at home (homeschooled or full-time virtual) and is a different bucket.

[3]Texas Education Agency, "Home Schooling," https://tea.texas.gov/families-and-students/finding-school-your-child/home-schooling. New York State Education Department, "Home Instruction Questions and Answers," https://www.nysed.gov/nonpublic-schools/home-instruction-questions-and-answers. Homeschooling is legal in all fifty states and D.C.; these two pages illustrate a light-process and a heavier-process pattern, not a national template. This book is not legal advice.

[4]Texas Education Agency, "Home Schooling": the State of Texas does not award a diploma to students that are home schooled. New York State Education Department, "Home Instruction Questions and Answers": a high school diploma may only be awarded to a student enrolled in a registered secondary school. North Carolina Division of Non-Public Education, "Home School Records Retention & Diplomas FAQs," https://www.doa.nc.gov/divisions/non-public-education/home-schools/faqs/records-retention-diplomas: the State does not issue a diploma or a transcript; the home-school chief administrator does. "Young Minds Math I" is not a credit in any opened statute.

[5]Texas Education Agency, "Alternative Schooling," restating *Leeper*: bona fide instruction using a written curriculum that includes math. North Carolina Division of Non-Public Education: annual nationally standardized test including mathematics. Pennsylvania 24 P.S. § 13-1327.1: el-

tables that listed subjects taught at home, the middle math rows were arithmetic at 66 percent and Algebra I at 41 percent for grades 6–8 that year.[6] The hour in front of you is the work. Placement into Mathematics, Pre-Algebra, or Algebra I is by skill, not by birthday.

This book is not a reprint. *Math for Little Thinkers* is for ages 5–10. One pointer, if you have a younger sibling at the table. Then we teach at eleven to fourteen. *Mathematics for Homeschooling* covers grades 1–12. It compressed middle grades into one chapter and then ran through algebra, geometry, statistics, and calculus. One pointer, if you need the later years later. Then we teach here. This book is not a sequel. *Critical Thinking for Young Minds* is the same house, a different subject — reasons, testimony, changing a mind. One pointer, then we teach this age's math.

Two refusals belong on the first day. Birthday is not Algebra I readiness. Fractions, decimals, percents, and negatives still finish in this band; the National Mathematics Advisory Panel named them Critical Foundations for a reason.[7] Cross-multiply-first and keyword lists are not proportional reasoning. Ratio meaning comes before the shortcut. A table, a double number line, or a strip diagram comes before the chant.

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ementary and secondary subject lists include arithmetic or mathematics. Virginia Department of Education, "Home Instruction": the parent writes a list of subjects. New York 8 NYCRR 100.10: required subjects by grade band include arithmetic or mathematics. None of these five opened maps names a critical-thinking course. Math is the usual required or assumed middle object. The parent reads their own statute.

[6]Jiashan Cui and Rachel Hanson, *Homeschooling in the United States: Results from the 2012 and 2016 Parent and Family Involvement Survey*, NCES 2020-001 (Washington, DC: National Center for Education Statistics, 2019), Table 9: among grades 6–8 homeschoolers that year, arithmetic taught 66 percent; Algebra I taught 41 percent. Quote the middle math rows. Do not reuse the elementary arithmetic 83/86 cells as this band. The 2023 First Look did not republish the subject-taught tables. https://nces.ed.gov/pubs2020/2020001.pdf.

[7]National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Critical Foundations of Algebra: proficiency with whole numbers, fractions (including decimals and percents), and certain aspects of geometry and measurement. Finding 10: conceptual understanding, computational fluency, and problem-solving skills belong together. Finding 15: claims that children of particular ages cannot learn certain content because they are "too young," "not in the appropriate stage," or "not ready" have consistently been shown to be wrong if prerequisite knowledge is present. https://files.eric.ed.gov/fulltext/ED500695.pdf.

What a good math hour looks like

You sit down already knowing today's idea. An object is on the table — a ratio table on scrap paper, a number line in both directions, signed chips of two colors, a drawn balance, a unit-price tag, a map scale. The student warms up on facts or placements they can already get right. You model one problem out loud, short. Then they try, and you wait. You ask one good question — not "did you get it?" but "what stays in the same ratio?" or "what did we do to both sides?" Three seconds feels long. It is the work. Practice mixes the new move with last week's. The hour ends with one or two items they do alone. You stop talking sooner than feels polite. The student talks and builds. You hold the key. That is the hour. The next long piece of this front matter, *The Math Hour*, will teach it in full. Later chapters will not reinvent it.

Figure 2: A kitchen-table math hour with a ratio table

Students this age already do middle-grades mathematics. They do not become algebra-ready because a candle was lit, and they do not become proportion-fluent because they can chant cross-multiply. A student who cross-multiplies with no table is applying a prior procedure, not displaying an absence of math. Pizza can introduce a share. The number, when we are done, lives on a line — including negatives.

What you will actually get

Each teaching chapter does eight jobs, always in the same order, so you are never hunting for the move.

You will learn why this week's idea is worth the struggle. You will understand it yourself, with the wrong answers you should be able to hear. You will get a session you can run this week: exact wording, named try-its in and out of the home, and a talk box. Your student will get a short section of their own. If it isn't clicking, three diagnostics and a next move. Tools, including AI, stay optional and adult-side. And you will get a plain checklist for "done enough," so you can place by skill rather than by birthday.

The eight teaching chapters follow the work: fractions, decimals, and percents as numbers; ratio and proportion; integers and signed numbers; expressions; equations; early functions and linear thinking; geometry with reasons; data, chance, and story problems. Records and resources come last. Algebra I edges at age fourteen are labelled only.

This week you can learn the session shape well enough to hear cross-multiply with no table. Today the student can build a ratio table, place a rational on a line, or write what was done to both sides.

What this book will not do

This book will not hand you 180 days of worksheets. A year of photocopies is not a teaching method, and I will not pretend it is. Coverage is a map. Depth is the hour.

It will not sell you a curriculum. Later, a short resources chapter names common programs by fit — parent load, style, how they place a student — so you can choose. Combining does not turn a brand into a course name. The transcript still says Mathematics, Pre-Algebra, or Algebra I — only if the year's work was that course.

It will not treat a birthday as Algebra I readiness, a cross-multiply chant as ratio meaning, or a keyword list as story problems. It will not franchise bar models as a miracle.

It will not promise a score. There is no guaranteed percentile, no math diploma, and no certificate of fluency inside these pages. What this book can promise is a path: the ideas in order, at the skill the student actually has, until they can do the next one unaided.

It is not "Young Minds Math I." One math-hour AI box lives in the next long piece. Later chapters point back.

It is not a culture-war pamphlet. Safe math objects only: a ratio table, a double number line, signed chips, an equation balance, an angle walk, a unit-price tag, a recipe scale, a map scale, a sports box score as a ratio. A price tag is a number. Live fights stay off this table.

And this is not a book that lectures your student about you. The student is a person, not a percentile. When this book speaks to them, it speaks with respect.

The promise

If you only remember one sentence, remember this: a path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

The work is to take this student through mathematics they can actually use — where a rational lives on the line, what stays in the same ratio, what a signed number means, what an expression is saying, what was done to both sides — without skipping the gate because they are "ready for algebra," and without parking them in a workbook that is too easy because a catalog printed a grade on the cover. You will sometimes slow down. You will sometimes skip ahead. Both are teaching. Birthday is not placement. Algebra I is by skill, and the edges are labelled only.

Three things have to hold. You understand today's idea well enough to hear a wrong turn. The student attempts first, with a representation, then the written equation. Any helper — including an AI tool — stays on your side of the table: a supplement you host and constrain, not a partner during the attempt, not a photo-to-key, and not a secret friend.

You can do this. You do not have to know next year's idea today. You have to know this week's idea well enough to sit still while they struggle, then ask one good question. Start here. Read *How to Use This Book This Week*, then the one-page list of five things, then *The Math Hour*. After that, open the chapter that matches the skill in front of you. You will know more after one chapter than you know this morning. Your student will have something to try today. We can do this.

How to Use This Book This Week

Start at the skill in front of you, not on page one because a catalog, a birthday, or a well-meant relative said so.

This book is a handbook you open to this week's idea, not a novel you read cover to cover. The eight teaching chapters follow the work, not twelve thin grade labels. Records and resources come last. Age bands here are 11–12 and 13–14. They tell you the grain of the same ideas. They do not tell you where *this* student sits.

This book does not replace the program already on the shelf. If a math curriculum, a kitchen ratio table, a library placement pretest, or a store hour you already use is working, keep it. Use this book to hear a wrong turn, to run the hour, and to name the work a stranger can read. Combine honestly. The title is still Mathematics, Pre-Algebra, or Algebra I — only if the year's work was that course. Never a brand. Never Young Minds Math I.

Two ages at one table

You may be teaching an eleven-to-twelve-year-old and a thirteen-to-fourteen-year-old in the same sit. That is ordinary. The session shape in *The Math Hour* still holds; the object on the table and the grain of the question change.

The younger student talks and builds. A ratio table with one missing value, a rational placed on a line (including a negative), a one-step equation by same-to-both-sides, a story type named in ordinary words — that is enough. The check is near: the table, the line, the balance. The older student can hold a little more: multi-step percent, proportional relationships toward linear, more equation strategy choice, similarity with reasons, richer data and chance talk. You may run a thirty-minute fraction-line sit with one student and a forty-minute proportion sit with the other. You do not need two personalities. You need two first problems.

Pick the chapter by skill, not by birthday

Open the chapter you think is right. Skip to **What "done enough" looks like** at the end of the *previous* chapter, or to **If it isn't clicking** in the one you opened. If the student can already do those checks unaided, you are too early. If the checks from two chapters back are still failing, drop back.

If they still treat 3/4 as pizza pieces without magnitude — if they can chant a fraction and still fail to place it on a line, including as a negative — they are still in the fraction-line work of the first teaching chapter, whatever their age. Students this age already do mathematics. What they can do depends on what they have already been invited to try, not on a birthday.[8]

If they cannot yet build a ratio table for a missing-value problem, they are still in the meaning stage of ratio, whatever the birthday. A fourteen-year-old missing fraction magnitude does eleven-to-twelve fraction-line work without apology. Place by skill, not birthday.

If they can already place rationals on a line, build a ratio table, and solve a one-step equation by same-to-both-sides, and they are still in a "grade 7 math workbook" only because the cover says so, skip ahead. A publisher's grade label is a scope, not a legal grade. Common Core State Standards for Mathematics is a map of typical U.S. public-school placement, not a homeschool statute.[9] Use this book's checklists, then teach. Algebra I edges at age fourteen are labelled only. This book does not dump the course.

[8] National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Finding 15: claims that children of particular ages cannot learn certain content because they are "too young," "not in the appropriate stage," or "not ready" have consistently been shown to be wrong if prerequisite knowledge is present. Access date for URLs in these notes: 4 September 2026. https://files.eric.ed.gov/fulltext/ED500695.pdf.

[9] Common Core State Standards for Mathematics (Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010): "These Standards do not dictate curriculum or teaching methods"; what students can learn at a grade "depends upon what they have learned before." A map of typical U.S. public-school placement, not a homeschool statute. http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

How you use the parent half

Most of each teaching chapter is for you. Read it *before* the lesson, not over the student's shoulder.

Why this matters tells you what this idea unlocks. **For the parent: understand it yourself** gives one everyday picture, one precise picture, and three to five wrong answers you should be able to hear. Sit with those.

How to teach it this week assumes the session shape from *The Math Hour*. It will not rebuild the hour. It will give you this week's named try-its — at least one in the home and one outside it — the wording for *this* idea, and a talk box: an exact opening question, three to five follow-ups, how to wait, and what a stuck silence usually means. Age-band moves for 11–12 and 13–14 live inside that heading. Later chapters supply this week's questions. They do not reinvent the shape.

Practice that actually builds learning names the try-its with time, materials, safety, the fun, and the skill. The kitchen, the walk, the store, and the sports sideline *are* the practice. They still have to land on the number line, the ratio table, or the written equation. **Tools, including AI** is optional, short, and for the adult. The rules live once in *The Math Hour*. **What "done enough" looks like** is how you leave: unaided work, not a perfect Tuesday.

You do not have to read the whole chapter tonight. You do have to read the parent half of *this week's* idea before you sit down with the student.

"Lesson plans" in this book means this week's session shape. "Assessment" means a short skill check, an exit ticket, a done-enough checklist, and optional publisher placement. Neither is a 180-day dump. Neither is a percentile battery.

The five-minute parent warm-up

Five minutes. Student not yet in the chair. Phone face down.

1. Read today's idea until you can say it in one sentence.
2. Look at one representation yourself — the ratio table, the number line, the balance. Ask, out loud, what would count as a check. Stay off any tool until you have looked.
3. Glance at the "wrong answers you should be able to hear." Name the one you would have given at fourteen.

4. Write one sentence you will actually say. Not a speech. Example: "What stays in the same ratio?" Or: "What did we do to both sides?"
5. Close the book to the student page. You are ready.

If you are learning the idea *while* they are stuck, you will talk too much. Prepare first. Then sit still.

How the student uses "For the student"

Every teaching chapter includes a short section written to the student, not about them. One or two pages. Warmer. Direct. What the idea is, a tiny worked example, two tries, an "explain it back" prompt, and one challenge. Hand it over after your short model, not instead of it. You stay in the room.

The student page is not something to send off with an unsupervised chatbot. The attempt is still theirs. You still hold the key.

In the home and outside it

Every teaching chapter names try-its at the table and try-its on a walk, at a store, on a sports sideline, or with a map. Kitchen, money, and making motivate. They do not replace the number line, the ratio table, or the written equation. No live controversy in the examples. A price tag is a number. Sports stats are ratios and percents.

When to skip ahead

Skip ahead when this chapter's "done enough" checklist is already true *unaided*. Slow down when the same wrong turn repeats after a clear look. **If it isn't clicking** will give you three likely causes and a next move. Hearing a wrong answer, then asking a better question, then naming a better move if needed, is teaching.

For this week: pick the chapter by skill. Do the five-minute warm-up. Run the hour as *The Math Hour* describes it. Let the student page be theirs. Stop talking sooner than you want to.

If You Only Remember Five Things

Keep this page. The chapters will add wording, representations, and this week's questions. They will not replace these.

1. Birthday is not Algebra I readiness. Fractions — including decimals, percents, and negatives — still finish here. A candle does not make someone algebra-ready. Proficiency with fractions as numbers on a line is a Critical Foundation, not a topic you abandon because the catalog printed "pre-algebra" on the cover. Place by skill. A fourteen-year-old missing fraction magnitude does eleven-to-twelve fraction-line work without apology.[10]

2. Understanding, fluency, and problem solving belong together. Ratio meaning comes before cross-multiply. Build a ratio table, a double number line, or a strip diagram first. The equal sign means "the same as" — still live in this band, including expressions and equations on both sides. A program that advertises only one strand is selling a fragment.[11]

[10] National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Critical Foundations of Algebra: proficiency with fractions (including decimals, percents, and negative fractions) is foundational for algebra. Finding 15: claims that children of particular ages cannot learn certain content because they are "too young," "not in the appropriate stage," or "not ready" have consistently been shown to be wrong if prerequisite knowledge is present. Access date for URLs in these notes: 4 September 2026. https://files.eric.ed.gov/fulltext/ED500695.pdf.

[11] National Mathematics Advisory Panel, *Foundations for Success*, Finding 10: conceptual understanding, computational fluency, and problem-solving skills belong together. Robert Siegler et al., *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, NCEE 2010-4039 (Washington, DC: Institute of Education Sciences, September 2010), Recommendation 4, minimal evidence: develop strategies for solving problems involving ratios, rates, and proportions before exposing students to cross-multiplication as a procedure. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/fractions_pg_093010.pdf. Eric J. Knuth, Ana C. Stephens, Nicole M. McNeil, and Martha W. Alibali, "Does Understanding the Equal Sign Mat-

3. The student talks and builds on a representation. You ask one good question and wait. Three seconds is a convention, not a sacrament.[12] After they stop, wait again. If the silence is hard, look at the table, the line, or the balance, not at the student. Struggle before rescue: ask, wait, hint, then model. You do not grab the pencil. You do not finish the item. A helper, including an AI tool, stays on your side of the table. It does not sit in the chair during the attempt.

4. Word problems are schemas, not keywords. Ratio, percent, compare — never "of means multiply." Geometry needs reasons, not only formula cards. Equations: same to both sides. Hearing a wrong answer, then asking a better question, then naming a better move if needed, is teaching. Silence in the face of a keyword error is not kindness.[13]

5. A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not. If a stranger asks what you taught, the title is Mathematics, Pre-Algebra, or Algebra I — only if the year's work was that course. Never a brand. Never Young Minds Math I.[14] Everyday stakes. No live controversy on the table.

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ter? Evidence from Solving Equations," *Journal for Research in Mathematics Education* 37, no. 4 (2006): 297–312: relational ("the same as") as best definition was 32 percent of sixth graders, 43 percent of seventh, and 31 percent of eighth — still live in this band.

[12] Robert J. Stahl, "Using 'Think-Time' and 'Wait-Time' Skillfully in the Classroom," ERIC Digest ED370885 (1994), https://files.eric.ed.gov/fulltext/ED370885.pdf: think-time; about three seconds as a convention; classroom-origin. Mary Budd Rowe, "Wait-Time and Rewards as Instructional Variables," NARST paper, ERIC ED061103 (1972), https://files.eric.ed.gov/fulltext/ED061103.pdf, is elementary science class, not a homeschool trial and not a math randomized trial.

[13] John Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8*, NCEE 2012-4055 (Washington, DC: Institute of Education Sciences, 2012), Recommendations 2–3, strong evidence: monitor and reflect; visual representations. Lynn S. Fuchs et al., *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*, WWC 2021006 (Washington, DC: Institute of Education Sciences, 2021), Recommendation 5: teach word-problem types / schemas, not keyword-to-operation matching — labelled intervention guide with middle-edge overlap, not a kitchen RCT. Geometry-with-reasons and same-to-both-sides equation moves are developed in later chapters; this page only names the refusal of keyword theater.

[14] Ellena Sempeles and Jiashan Cui, *Parent and Family Involvement in Education: 2023*, NCES 2024-113 (Washington, DC: National Center for Education Statistics, September 2024): no national homeschool diploma. Transcript titles in this book are Mathematics, Pre-Algebra, or Algebra I by content — never a brand, never "Young Minds Math I." Jiashan Cui and Rachel Hanson, *Homeschooling in the United States*, NCES 2020-001 (2019), Table 9 grades 6–8: arithmetic 66 percent; Algebra I 41 percent that year. https://nces.ed.gov/pubs2024/2024113.pdf; https://nces.ed.gov/pubs2020/2020001.pdf.

Kitchen, money, and making motivate. They do not replace the number line, the ratio table, or the written equation. Algebra I is by skill, not birthday; age-fourteen edges are labelled only.

The student attempts first. You hold the key. If this week needs a compass, this is it. Five things. Then sit down and teach.

The Math Hour

The hour has a shape. Learn it once. Later chapters will give you today's idea, today's try-it, and today's questions. They will not rebuild this hour. When a chapter says "run the session," it means this.

You do not need a school bell. You need a warm-up, a short model, a real attempt, one good question then wait, mixed practice, and an unaided exit. An eleven-to-twelve-year-old may finish nearer thirty minutes; a thirteen-to-fourteen-year-old may need fifty. The shape does not change.

Sit down having already done the five-minute parent warm-up from *How to Use This Book This Week*. You know today's idea well enough to hear cross-multiply with no ratio table as a missing-meaning move. The student talks and builds. You hold the key. Thirty to fifty minutes, most weekdays, plus one named try-it outside the home, is the right ambition.[15] There is no national table of homeschool math minutes.

Life of the habit, as a session rhythm, lives here: revisit yesterday; mix last week; stop.

1. Warm-up

Three to five minutes. Known facts, or a quick number-line or integer warm-up. Already-right material. This is retrieval, not a test of character.

Say: "We're going to start with things you already know."

[15] Session length in this chapter is pedagogical design for the kitchen table, not a national math-minutes study (none was found). North Carolina DNPE recommends (not law) five hours across all subjects; Pennsylvania statute names 180 days or whole-program hours. None of these is a national math block. Access date for URLs in these notes: 4 September 2026.

Ages 11–12: place a few rationals on a short line, or name equivalent fractions. "Where does 3/4 live? Where does 0.5 live?" Ages 13–14: a quick integer net-change, or a one-step same-to-both-sides check. "What is $-3 + 8$? Take your time."

If they already get these right untimed, you may use a brief timer — one to five minutes, not the lesson.[16] Say: "We'll time this only because you already get these right. Ready?" If they do not already get them right, skip the timer. Fluency is accurate first, then sometimes timed. You are building retrieval, not an identity. A timed algebra kit is not fluency.

Keep this short. The warm-up is not the lesson.

2. Short model

Five to eight minutes. One worked example, think-aloud, representation visible — ratio table, double number line, signed chips, or balance — then the notation. Then you stop. Show the move. Then fade.[17]

Say: "I'm going to show this one short. Then you'll try." "Watch. Both sides stay the same. That's why I can subtract 4 from both sides."

On the second pass, leave a hole. When the idea is new, show one incorrect example — cross-multiply with no table, "of" glued to multiply, = treated as "write the answer," pizza offered as the number $-3/4$ — and ask what went wrong. If you are still talking at minute nine, close the model.

[16] Lynn S. Fuchs et al., *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*, WWC 2021006 (Washington, DC: Institute of Education Sciences, 2021), Recommendation 6, strong evidence: timed activities last 1 to 5 minutes; they are not the entire focus; add them once students have been working on a concept over many lessons. Population includes grades that overlap this band's lower edge for intervention, not a parent-hour trial. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/WWC2021006-Math-PG.pdf. National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Finding 11: automatic recall frees working memory for more complex problem solving.

[17] National Mathematics Advisory Panel, *Foundations for Success*, Finding 27: explicit instruction (clear models, extensive practice, think-alouds, extensive feedback) has consistently positive effects for students with mathematical difficulties; this does not mean all instruction should be explicit. Finding 23: high-quality research does not support instruction that is entirely student-centered or entirely teacher-directed. https://files.eric.ed.gov/fulltext/ED500695.pdf.

3. Student attempt

This is the center of the hour. Ten to eighteen minutes. One to four items of *today's type*. Representation first, then the written equation, as needed. Hand them "For the student" or the first try-it. Then you talk less than you want to.

Say: "This one is yours. I'll be quiet." Then be quiet. If they stall, use this order: ask, wait, hint, then model. Not the reverse. Ask: "What stays in the same ratio?" Wait. Count a slow three. The silence is the work. If you fill it, you took the problem back.

Hint, one hint: "You already know we can fill another column that multiplies both quantities by the same factor. Try that on the table." Then, if they are still stuck after a real try: "I'm going to show you this one step. Then you take it from here." Keep your hands off their pencil. The student talks and builds. You hold the key.

4. One good question, then wait

One to three minutes of clock time that feels longer. The talk around it can run using the talk-box shape below. Not "did you get it?"

An authentic question is one for which you have not already written the answer. "What is the vocabulary word?" is recitation. Say: "What stays in the same ratio?" Or: "What did we do to both sides?" Or: "Where does this live on the line?"

Then wait. A slow three after you ask. A slow three again after they stop. Robert Stahl called that silence *think-time*; that label is classroom-origin, not a homeschool trial. About three seconds is a convention, not a sacrament.[18] Mary Budd Rowe found the same pause in elementary science class. That was science class. This book uses the pause so already-present reasoning can be heard.[19] If they are

[18] Robert J. Stahl, "Using 'Think-Time' and 'Wait-Time' Skillfully in the Classroom," ERIC Digest ED370885 (1994), https://files.eric.ed.gov/fulltext/ED370885.pdf: eight categories of classroom silence; about three seconds as a convention. Classroom-origin. Not a finding that wait-time teaches mathematics, and not a homeschool randomized trial.

[19] Mary Budd Rowe, "Wait-Time and Rewards as Instructional Variables: Their Influence on Language, Logic, and Fate Control," NARST paper, ERIC ED061103 (1972), https://files.eric.ed.gov/fulltext/ED061103.pdf: elementary science classes; teachers typically waited about one second after a question; extending to three to five seconds changed discourse. Rowe is science class. This book does not cite Rowe 1972 as a homeschool randomized trial or as a math trial.

mid-reason, do not cut them off. If the silence is hard, look at the table, the line, or the balance, not at the student. One question. Maybe a follow-up. Then stop.

5. Mixed practice

Eight to twelve minutes. Yesterday and last week mixed with today. Not forty of the new item.[20]

Say: “Two from today. One from last week. Then we stop.”

Kitchen, money, and making can sit inside this mix. They do not replace the line, the ratio table, or the equation.

6. Exit ticket

Three to five minutes. Two to four items, one of them yesterday’s skill. Book closed. No hints. No chatbot. Say: “These few are yours. I won’t help. That’s the point.” Done-enough is “right, or wrong-with-a-reason we can use tomorrow,” not a percentage.

After you ask, wait. If you fill the silence, you took the problem back. Praise a clear column on a ratio table, a mark on the line, a reason for a similar triangle, or a mind changed after a check — not speed. Hands in your lap. If a wrong answer keeps coming, hear it, ask a better question, then name a better move if needed. Hearing a wrong answer and leaving it uncorrected is not kindness.

If the hour goes sideways

Use this as a debug, not a verdict.

- Cross-multiplies with no meaning → ratio table or double number line first; ask what stays in the same ratio.
- Keyword-grabs “of” or “altogether” → name the schema (ratio / percent / compare); draw the quantities.

[20] Fuchs et al., Recommendation 1: mix previously and newly learned material. Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8*, NCEE 2012-4055 (Washington, DC: Institute of Education Sciences, 2012), Recommendations 2–3, strong evidence: visual representations; teach problem types / schemas, not keyword matching.

- Treats "=" as "write the answer" → solved balanced equations; same-to-both-sides out loud.
- Fast and wrong on facts → untimed accuracy; brief known-fact warm-ups; delay the clock.
- Waits for rescue → count three; sit on your hands.
- Stuck after a real try → one think-aloud, then a parallel item.
- Geometry as formula only → "Why these sides?" sketch before plug-in.
- Session is only worksheets → put one item on a table or line; mix yesterday.
- Parent feels rusty and reaches for the pencil → put the representation back; let them build and say.[21]

Revisit yesterday. Mix last week. Stop. That is the habit. It is not a diploma.

The talk-box shape, once

Later chapters will fill this week's opening question. They will not reprint this shape. When a chapter prints a talk box, it means this.

Opening question. Authentic. About where a number lives on the line, what stays in the same ratio, what a signed number means, what an expression is saying, what was done to both sides, what the input and output are, what must be true and how you know, or what kind of story this is. No answer you have already written. Not "did you have fun" and not "what is the vocabulary word." Locked openings live in the teaching chapters.

Three to five follow-ups. Pick; you do not need all five every Tuesday. Meaning: What does that number mean? Representation: Show me on a ratio table, double number line, strip, number line, or balance. Type or magnitude: Is this a ratio story, or a percent story? Is this more than one half? Check: Does that answer make sense? Mind-change: You may change your mind. Try it another way.

How to wait. After you ask, a slow three. After they stop, wait again. If they are mid-reason, do not cut them off. Look at the table, the line, or the balance, not at the student, if the silence is hard.

[21] Erin A. Maloney, Gerardo Ramirez, Elizabeth A. Gunderson, Susan C. Levine, and Sian L. Beilock, "Intergenerational Effects of Parents' Math Anxiety on Children's Math Achievement and Anxiety," *Psychological Science* 26, no. 9 (2015): 1480–88. School homework-help study, not a homeschool trial. The portable caution: let the student build and say.

What a stuck silence usually means. The question was vague. Wait-time after the question was zero. They are hunting a keyword. They are guessing what you want. They reached for cross-multiply with no table. They think "=" means "the answer comes next." They think "of" means multiply. The numbers or the signed structure are too new. Pose, wait, and point back at the representation. You do not stack five questions or switch to a live controversy.

The in-home try-it shape, once

Later chapters will fill named types. They will not reprint this spine. When a chapter says "run the in-home try-it," it means this.

Time. A short sit, then stop. Ten minutes with a recipe scale. Fifteen with a balance and coins. Pedagogical design, not a national minutes table.

Materials. Household objects: a recipe already in the drawer, an allowance ledger, a weather app for lows through zero, phone-plan numbers, coins or weights for a balance, a board-game score pad. You do not need a kit.

Safety. Ordinary kitchen. Ordinary food hygiene. Tone: curious, not courtroom. Let the student move the work.

The fun. Scaling a recipe. Finding a better buy in the pantry. Watching temperature through zero. Naming what a phone plan charges flat plus per unit.

The skill. Ratio table. Percent bar. Integers with meaning. Expression before solving. Same-to-both-sides. Reading a table of scores as data.

Types you will meet, not a lesson bank: recipe scale (½×, 1½×, 3×); double a recipe with mixed numbers; allowance or budget percent; hot/cold weather through zero; phone-plan expression; balance-scale equation with coins or weights; board-game or card scores as data. A recipe scale uses measuring cups already in the drawer. It is not a nutrition sermon.

The out-of-home try-it shape, once

"Outside" means the walk, the store, the sports sideline, the map, the ramp or stairs. Later chapters will fill named types. They will not reprint this spine.

Types you will meet: store unit price / better buy; walk angles on fences or similar shadows; sports sideline stats as ratios and percents; map scale on a hike or transit map; ramp or stairs rise-over-run. A walk is ratio, angle, and measure you were already going to meet.

Safety, once. Roads, water, aisle courtesy. No shoplifting a "test." No lecture to a stranger's adolescent. No public quiz of a cashier. No climbing for a "similarity photo." A price tag is a number. It is not a sermon about a food system. Sports stats are ratios. They are not identity fights.

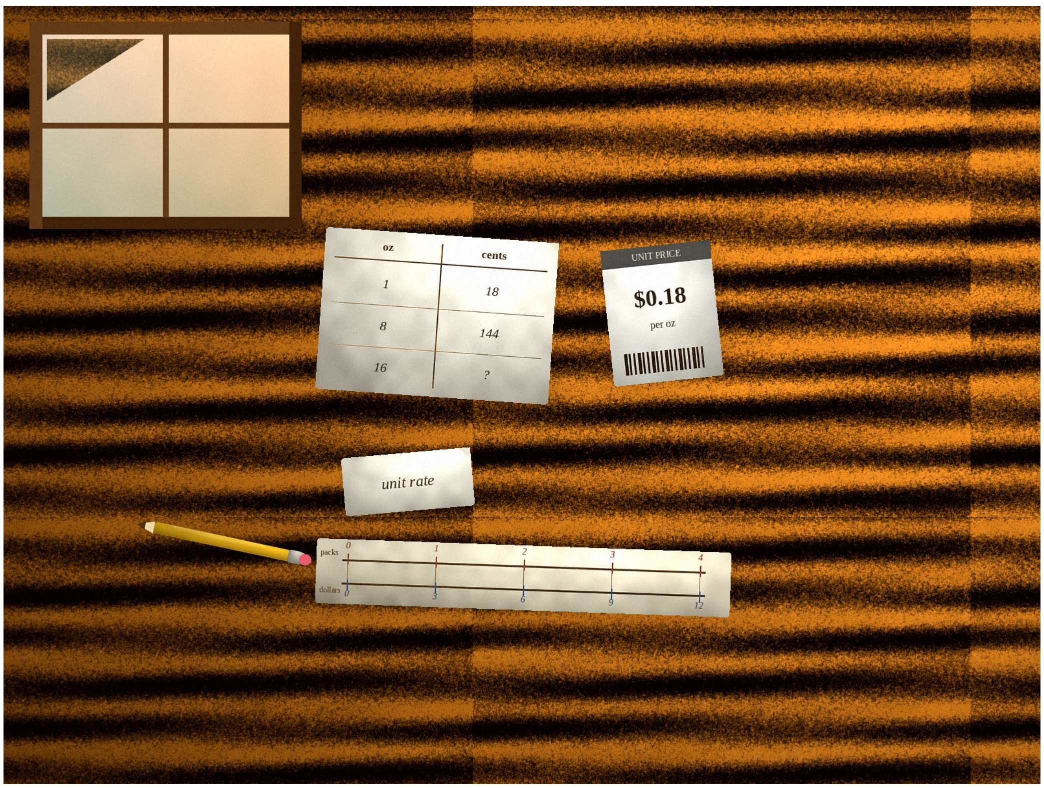

Figure 3: A generic unit-price tag beside a ratio table

This week, said plainly

One focus skill — unit rate on a double number line, same-to-both-sides on a two-step equation, similar triangles with corresponding side ratios. Four or five math hours of the session shape, numbers adjusted to the student. Mixed review of last week's skill inside every hour. One in-home try-it and one out-of-home try-it that use the same type. Friday: look at the week's exit tickets; pick one diagnostic wrong answer; decide whether next week repeats, narrows, or moves on. That is the honest answer to "lesson plans," not 180 days written in advance.

A short skill check is three objects, not a battery: an exit ticket of two to four items, one of them not today's new skill; one diagnostic wrong answer the student explains — classify the error (cross-multiply-first, keyword grab, operational =, additive thinking on a ratio, formula without reason, sign error); a done-enough checklist by skill, not by grade. Optional fourth: a publisher placement test after a gap — not a yearly identity, not a percentile.

The AI rules, once

Later chapters will point here in one sentence. They will not reprint this box. The student talks to the parent. The parent may use a tool. The student does not sit alone with an open chat as the only partner. The math is on the table: ratio table, double number line, signed chips, equation balance, angle walk, unit price. The student attempts first. The parent holds the answer key.

Ages 11–12 (COPPA under-13): parent holds the account. Scripts and parent–student talk. Not live open chat as the student's partner. You may generate a four-line SCRIPT for after the attempt, and you may ask a tool to explain a named page to you. No student-facing open chat as partner. No student account.

Ages 13–14 (COPPA off; still parent in the room): parent co-holds the account. Turning 13 is not a licence. The student may sit next to you while you use a tool. They may hear a hint after they have tried. They may not have their own unsupervised companion account as the only partner. They may not be asked to "prompt the tutor until it agrees." Unaided first. SCRIPT after the try.

You may, on your account, after the student has tried:

- Explain this idea *to you* from a named lesson (title, page, or today's object: ratio table, double number line, signed chips, equation balance, angle walk, unit price). You still have to understand it well enough to hear a wrong turn.
- Extra isomorphic practice with the answer key held by you.

Same structure, new numbers. The student never sees the key. Do not generate "practice" by photographing tonight's assigned worksheet.

- A labelled SCRIPT after the try — a short spoken sequence for *you* to say, not a paragraph for the student to copy as their work. Example: "Fill another column. Multiply both quantities by the same factor. Check that the ratio stayed the same."
- A hint after an attempt. A hint is a question or a named idea, not the finished number. "What did we do to both sides?" is a hint. "The answer is 12" is not.
- Diagnose work the student already produced. You hear the diagnosis and decide. Crop to the paper. Do not upload the student's face.

You may not:

- Paste the worksheet and ask the tool to complete it.
- Ask "what's the answer" or "just give me the steps" during the student's attempt, or let the student ask that of a window as the only partner.
- Photo-to-key. Point a camera at the page (Photomath, Mathway, Symbolab, or any scan-and-solve) so a solution pops up for the exact problem. That loop is a solver. Photo-to-answer is the ban.
- Park an unsupervised chatbot as the student's only partner during the attempt — not at 11, not at 12, not at 14. No "math friend" account.
- Write the student's work. The model does not fill "the unit rate is ___ because ___."
- Invent facts (the unit price on the shelf, the angle on the page, which chip is positive, what the ratio table already shows). Look.
- Grade the student's work from a detector score.
- Print a certificate of fluency.
- Hand the student an agent (Hermes Agent, Grok Bot, or any cloud-computer worker) as a math partner.

In a high-school math trial with nearly 1,000 Turkish students in grades 9–11, an unguarded chatbot (GPT Base) raised assisted practice about 48 percent relative to control, then cut unaided exam grades about 17 percent; a guarded tutor that withheld the answer (GPT Tutor) raised practice about 127 percent and left the unaided exam about the same as control (Bastani et al., 2025).[22] This is a useful study, not a promise that every home will see the same result. That is not an 11–14 RCT — closer than 5–10, still not this band's trial. The kitchen-table rule it supports: the model may prepare the adult and the next problem; it may not do the student's problem.

Federal practice guides and the National Mathematics Advisory Panel name visual representations, solved problems, structure, schemas, fractions as numbers, and same-to-both-sides reasoning. They do not name a chatbot as the math teacher.[23]

Detectors mislabelled more than half of some human essays as machine-written (Liang et al., 2023: average false-positive rate 61.22 percent on human TOEFL essays).[24] Do not police a twelve-year-old's "the unit rate is 3 per 4 because every column multiplies by the same factor" with a score. Language models invent sources (Walters and Wilder, 2023: 55 percent of GPT-3.5 citations and 18 percent of GPT-4 citations fabricated in that study).[25] If you asked a tool for a paper, open the paper. IES and NMAP are public PDFs.

Names, in ordinary language. Hermes 4 is a model. Hermes Agent is an agent. Grok is a chat assistant. Grok Bot is a cloud-computer

[22] Hamsa Bastani, Osbert Bastani, Alp Sungu, Haosen Ge, Özge Kabakcı, and Rei Mariman, "Generative AI without guardrails can harm learning: Evidence from high school mathematics," *Proceedings of the National Academy of Sciences* 122, no. 26 (2025): e2422633122. Field experiment, nearly 1,000 Turkish high-school math students, grades 9–11: GPT Base +48% practice / -17% unaided; GPT Tutor +127% practice / unaided $\approx$ control. This is a high-school math trial, not an 11–14 RCT.

[23] Search of IES fractions, problem-solving, and algebra practice guides and National Mathematics Advisory Panel, *Foundations for Success* (2008), for a recommendation to sit a student with a generative chatbot: it is not there. They name visual representations, solved problems, structure, schemas, fractions as numbers, and same-to-both-sides reasoning.

[24] Weixin Liang, Mert Yuksekgonul, Yining Mao, Eric Wu, and James Zou, "GPT detectors are biased against non-native English writers," *Patterns* 4, no. 7 (2023): 100779: average false-positive rate on human TOEFL essays 61.22 percent.

[25] William H. Walters and Esther Isabelle Wilder, "Fabrication and errors in the bibliographic citations generated by ChatGPT," *Scientific Reports* 13 (2023): 14045: 55 percent of GPT-3.5 citations and 18 percent of GPT-4 citations were fabricated in that study.

agent. Game artist makes pictures. None of these is an 11–14 math class. There is no Grok Bot education SKU. Agents are not chatbots.

COPPA covers children under 13. Ages 11–12 are under 13. The 22 April 2026 compliance date has passed.[26] Keep accounts, logs, and keys on the parent side.

Not a secret friend. A tool that talks in the first person is still a tool. It does not get a bedroom, a private channel, or a promise to keep secrets from the parent.

You can teach this entire book with no AI. The hour still has the same shape.

A Tuesday, said plainly

Here is an illustration, not a reported family. A parent has spent five minutes looking at a ratio table and a double number line. The student warms up on placing 3/4 and 0.5. The parent shows a missing-value ratio on the table, think-aloud, then puts the pencil down. The student tries. The parent asks, "What stays in the same ratio?" and waits. The student reaches for cross-multiply. The parent hears it, asks them to fill another column first, and they rebuild. Exit: "These two are yours. I won't help." On a thirteen-to-fourteen day the same shape holds, with a two-step equation and "What did we do to both sides?"

You can run that hour. You need today's idea, a sentence you refuse to finish, and the willingness to stop talking. Later chapters will say: run the session as in *The Math Hour*.

[26]Children's Online Privacy Protection Act, 15 U.S.C. §§ 6501 *et seq.*; 16 C.F.R. Part 312. Federal Register 90 FR 16918 (22 April 2025): amended Rule effective 23 June 2025; general compliance date 22 April 2026. Bright line: children under 13. Keep accounts on the parent side. Photomath "a solution will pop up for your exact problem" sits next to "designed for learning" — the loop is the ban, not a trial. Khanmigo "never gives you the answer" is a design goal on a product page, not an 11–14 trial.

Chapter 1 — Fractions, decimals, and percents as numbers

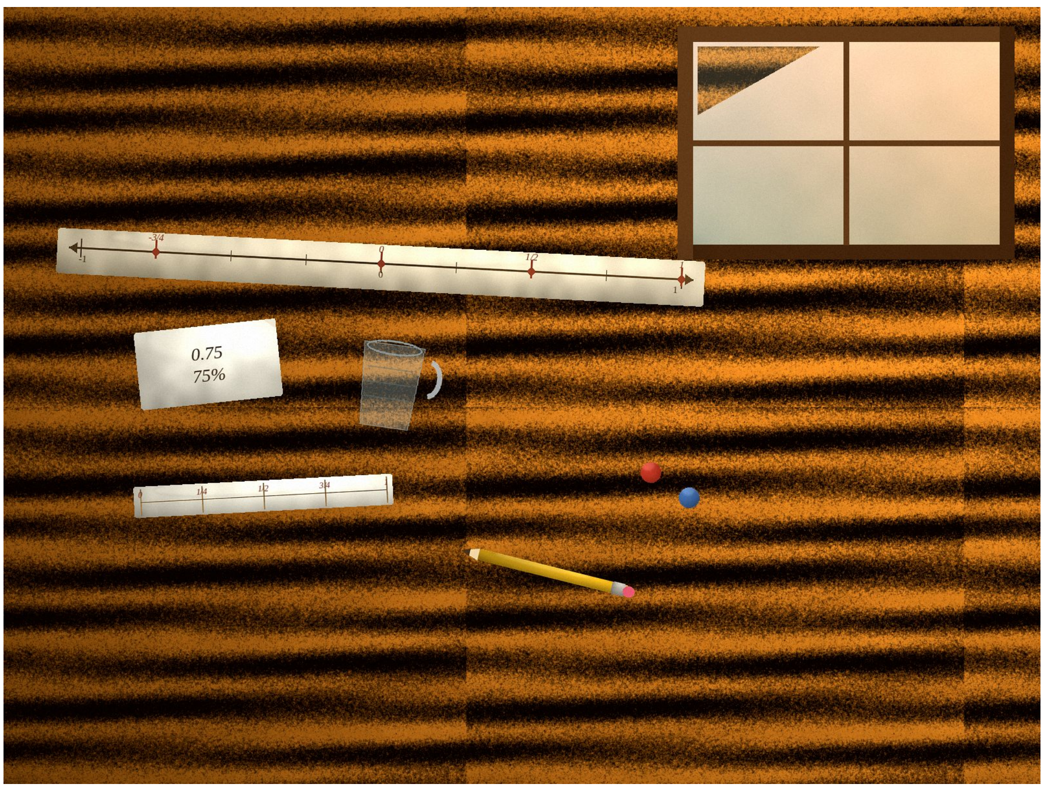

Figure 4: A kitchen-table still-life from a high three-quarter view: a number line both ways on register tape with —3/4, 0, 1/2, and 1 marked; a scrap showing 0.75 and 75%; a measuring cup in soft focus. No pizza as the only object. No people. No logos.

Why this matters

Where does this live on the line?

That is not a vocabulary quiz. It is not a request for a pizza drawing as the finished number. It is the question that turns $-3/4$, 0.75, and 75% from three school topics into one location — the same magnitude, three names. A half of a recipe is a fair share. A half on a line that runs past zero and past one is a number. Both belong in the week. The share is still a useful introduction. The line is the destination of this chapter.

If you used *Math for Little Thinkers* with a younger child, you already met fractions as numbers on a 0–1 line. This book does not reprint that lesson bank. It keeps the same house rule and raises the stakes: by ages 11–14, fractions, decimals, percents, *and negative fractions* still finish as a Critical Foundation for algebra. They are not "elementary leftovers" you abandon because a prealgebra book arrived in the mail.

The National Mathematics Advisory Panel (NMAP) was blunt. A major goal for K–8 mathematics should be proficiency with fractions — including decimals, percents, and negative fractions — because that proficiency is foundational for algebra and, at the time of the Panel's report, seemed severely underdeveloped. By the end of Grade 6, the Panel's Table 2 wanted multiplication and division of fractions and decimals *and* all operations with positive and negative integers. By the end of Grade 7, all operations with positive and negative fractions, and problems involving percent, ratio, and rate extended toward proportionality. That calendar sits inside this book's age band. Conceptual understanding, computational fluency, and problem solving still belong together. A program that advertises only one strand is selling a fragment.[27]

A third reason sits next to those two, labelled so it does not become a kitchen

[27] National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Critical Foundations of Algebra / Finding 4: proficiency with fractions — including decimals, percents, and negative fractions — is foundational for algebra and seemed severely underdeveloped. Table 2 Grade 6–7 benchmarks: by end of Grade 6, multiplication and division of fractions and decimals and operations with positive and negative integers; by end of Grade 7, all operations with positive and negative fractions, and problems involving percent, ratio, and rate toward proportionality. Finding 10: conceptual understanding, computational fluency, and problem-solving skills belong together. Access date for URLs in these notes: 4 September 2026. https://files.eric.ed.gov/fulltext/ED500695.pdf.

promise. Siegler and colleagues found that age-10 fraction knowledge uniquely predicted high-school algebra and overall mathematics years later, after other elementary skills were controlled. In the UK Birth Cohort Study, a one-standard-deviation increase in age-10 fractions was associated with +0.15 standard deviations of later algebra; in the US PSID-CDS sample, +0.17. This is a useful study, not a promise that every home will see the same result. It is a prediction finding — continuity from the younger book's spine — not a classroom experiment proving a teaching method for your Tuesday. It is a reason to keep locating, comparing, and operating on fractions, decimals, and percents as *numbers*, including negatives. It is not a reason to dump invert-and-multiply onto day one, or to treat a birthday as Algebra I readiness.[28]

What this idea unlocks is almost everything later that looks like "part of," "off," or "below zero." Equivalence as the same point. Percent as a ratio per hundred that still lives on a line or a percent bar. Money and shopping as motivation that still lands on written math. If a pizza drawing still stands in for −3/4, later pages arrive as three unrelated vocab lists, and 0.75 never meets 75%. Hearing pizza offered as the finished number −3/4, then putting −3/4, 0.75, and 75% on one line, is worth the struggle.

You do not need to be a mathematician. You do need to hear a pizza forever diet, or three disconnected school topics, as an unfinished number system — and to ask where this lives on the line.

Money and shopping motivate percent. A clearance tag is a real object. The written percent-as-ratio and the number-line or percent bar still have to appear the same week. Kitchen measuring cups motivate fraction operations. The number still has to be written and, often, placed. Motivation without representation is a pleasant hour that does not finish the Critical Foundation.

This week you can learn to hear that miss. Today the student can place −1/2, 2/3, 0.75, and 50% on one line and say which are the same.

[28]Robert S. Siegler, Greg J. Duncan, Pamela E. Davis-Kean, Kathryn Duckworth, Amy Claessens, Mimi Engel, Maria Ines Susperreguy, and Meichu Chen, "Early Predictors of High School Mathematics Achievement," *Psychological Science* 23, no. 7 (2012): 691–697. UK Birth Cohort Study N = 3,677: +0.15 SD later algebra per 1 SD age-10 fractions after controls; US PSID-CDS N = 599: +0.17 SD. Prediction finding / continuity from *Math for Little Thinkers*, not a kitchen teaching-method RCT.

For the parent: understand it yourself

Many adults feel rusty on why −3/4, 0.75, and 75% name the same magnitude, and why a negative fraction is still a location, not a punishment. That is ordinary. Five minutes of this section, then the warm-up at the end, is enough for tomorrow. The student still places.

Everyday picture. A strip of paper or register tape. Mark 0 in the middle for a line that runs both ways, or mark 0 near the left and extend left for negatives. Mark 1 to the right. −3/4 lives three fourth-lengths left of 0. 0.75 lives three fourth-lengths right of 0 — seventy-five hundredths is three fourths. 75% is seventy-five per hundred, which is the same three fourths of a whole. Pouring three quarter-cups, reading a receipt that says 75% off, and writing −0.75 for a change that went down are kitchen and money cousins. They motivate. The line is still the number.

Precise picture. A fraction is a number that expands the number system past whole numbers — including past zero into negatives. The Institute of Education Sciences fractions guide, Recommendation 2, rated *moderate*, wants number lines as a central tool so students treat fractions as numbers, not only as pizza pieces. Recommendation 1's part–whole sharing is rated *minimal*; the panel still wanted the share as an introduction. Recommendation 3 wants procedures that make sense — also *moderate*. Pizza is Rec 1. The line is Rec 2. Procedures that you can justify land after meaning, not instead of it.[29]

Decimals are a place-value way of writing the same magnitudes: tenths, hundredths, thousandths. Percents are a per-hundred way of writing ratios that often sit next to money: 25% of $40 is a quarter of forty. Unifying the three names on one line (and on a percent bar when helpful) is Grade 6–7 map work. Dividing fractions with meaning — how many thirds fit into two wholes? — belongs here before invert-and-multiply is chanted as magic. Multi-step percent (tax then discount, percent change) belongs more fully at 13–14 and will lean on Chapter 2's ratio meaning. Wait until percent-as-location works, then hand the heavier stories to the ratio table.

[29] Robert Siegler et al., *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, NCEE 2010-4039 (Washington, DC: Institute of Education Sciences, September 2010). Recommendation 1, minimal: part–whole sharing as introduction. Recommendation 2, moderate: fractions as numbers; number lines as a central tool. Recommendation 3, moderate: procedures that build from conceptual understanding. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/fractions_pg_093010.pdf.

Negative fractions are not a side quest. $-1/2$ is the opposite of 1/2 on the line. -0.75 and -75% are the same story with different names. The Panel's Critical Foundation named them on purpose. Chapter 3 will deepen integers and signed meaning. This chapter already places them so the student does not treat "fractions" as only the positive pizza half of the world.

One more precise picture: percent bar beside the line

A percent bar is a strip marked 0 to 100 (or 0% to 100%). 75% is three-quarters of the way along that bar. Drawing the bar next to a 0–1 number line helps the student see that 75% of *one whole* matches 0.75 and 3/4. When the whole is $40, the bar still shows 25% as a quarter of the bar; then they scale: a quarter of forty. The bar does not replace the line. It is another picture of the same per-hundred idea. Strip diagrams return in Chapter 2 for ratio. Here they serve percent-as-location.

Procedures that make sense

When you multiply 0.75×8, you can think "three-fourths of eight." When you divide $2 \div 1/3$, you can ask how many thirds fit into two. Invert-and-multiply can be shown later as a compressed consequence after the meaning story works — not as day-one magic. The IES guide's Recommendation 3 wants procedures that make sense. Sense first. Speed second. A student who can chant invert-and-multiply but cannot place 2/3 has the chant without the Critical Foundation.

Wrong answers you should be able to hear

1. *A pizza drawing offered as the finished number $-3/4$, with no line in the week.* Part–whole happened for a positive picture. The signed number never arrived. Say: "That is three of four parts of a pizza. Where does $-3/4$ live on our line?" Mark left of zero. The pizza can stay in the room. It cannot be the destination.

2. *Treats 0.75 and 75% as unrelated school topics.* Three vocab lists, no shared magnitude. Say: "Show me both on the same line. Are they the same place as 3/4?" If they mark three different points, the names never met. Unify before you compute.

3. *Places 1/2 to the right of 2/3.* Magnitude is wrong. The denominator-as-size habit ("3 is bigger, so 2/3 is smaller") or a tick-counting error. Fold

or partition equal lengths. Ask which is farther from 0. Estimate before computing.

4. *Computes first, estimates never.* A long decimal multiplication appears before "Is the answer near 1 or near 0?" The check is missing. Ask: "About where should this live?" Then compute. Then look back at the line.

5. "*Fractions are done — this is algebra.*" Birthday or book title treated as a skill gate. The Critical Foundation is unfinished. A fourteen-year-old who cannot place −3/4 still needs this chapter's line work without apology. An eleven-year-old who unifies −3/4, 0.75, and 75% is ready to keep going. Place by skill.[30]

A sixth you will also hear: a store percent tag enjoyed as "twenty percent off," never written as a ratio or placed on a bar. The aisle motivated. The number never arrived. Look at the tag, then write the percent-as-ratio at home the same week.

Five-minute parent warm-up

Do this before the lesson, with a strip and a pencil, no student in the room.

Minute 1. Mark −1, 0, and 1. Place −3/4. Say out loud: "Negative three-fourths lives here. It is a number."

Minute 2. Place 0.75 and write 75% above the same point as 3/4. Notice they share a location with each other on the positive side — and −0.75 would mirror −3/4.

Minute 3. Imagine a pizza drawing offered as −3/4. Decide, without performing, what a scold would do. Decide what "Where does this live on the line?" would do instead.

Minute 4. Compare 1/2 and 2/3 by placement only. Say which is larger and why, looking at distance from 0.

Minute 5. Write the sentence you will actually say: "Where does this live on the line?" Put the pencil down. That sentence is the lesson.

[30] National Mathematics Advisory Panel, *Foundations for Success*, Finding 15: age/"too young"/"not ready" claims are wrong when prerequisite knowledge is present. Place by skill, not birthday. Algebra I edges stay labelled; this chapter does not become a full algebra course because a candle was lit.

If you can do those five minutes, you are ready to sit down. The student places. You hear.

How to teach it this week

A good math hour this week has a shape. The math-hour chapter in the front of this book is the full version. Here is the shape scaled to fractions, decimals, and percents as numbers.

Warm-up (3–5 minutes, unaided). Place three familiar benchmarks on a line: 0, 1/2, 1 — then, once negatives are in the week, −1 and −1/2. Known locations. Sometimes a brief timed retrieval of already-accurate fraction–decimal pairs, only after they are accurate untimed.

Short model (5–8 minutes). One worked placement: locate −3/4 and 0.75; think aloud that 0.75 is three fourths, same magnitude as 75% of one whole on the positive side; show −3/4 left of zero. You talk for a few minutes. Then you stop.

Student attempt (10–18 minutes). One to four placements of *today's* type: positives and at least one negative; fraction, decimal, and percent names mixed. Adult does not hover-solve. Struggle before rescue: ask, wait, hint, then model on a *different* line.

One good question, then wait. "Where does this live on the line?" Not "what is a percent?" Not "shade three of four." After you ask, a slow three. Stahl's think-time is a classroom-origin convention, not a sacrament.[31] Use the pause. Look at the line, not at the student's face, if the silence is hard.

Mixed practice (8–12 minutes). Yesterday's integer or whole-number location next to today's percent. A recipe half next to 0.5. Mixing is choosing: is this a location, an operation, or a money story that still needs a written check?

Exit ticket (3–5 minutes, unaided). Two placements plus one "which is larger?" Done-enough is a point you can see, or a wrong point with a reason you can use tomorrow — not a percentage score.

[31] Robert J. Stahl, "Using 'Think-Time' and 'Wait-Time' Skillfully in the Classroom," ERIC Digest ED370885 (1994), https://files.eric.ed.gov/fulltext/ED370885.pdf: about three seconds as a classroom-origin convention. Not a mathematics RCT and not a homeschool trial. Mary Budd Rowe, "Wait-Time and Rewards as Instructional Variables," NARST, ERIC ED061103 (1972), is elementary science class.

That shape is a practice you impose on whatever line, cup, or aisle is already in the week. It is not a 180-day fractions calendar.

Exact wording you can say

On location:

“Where does this live on the line?”

“Between which two wholes?”

“Show me −3/4 and 0.75 — same place? Same magnitude?”

“How do you know 75% is three-fourths?”

On estimating:

“About where should the answer live — near 0, near 1/2, or past 1? Then compute.”

On a pizza that showed up:

“That is three of four slices. Is that the number, or a picture of a part? How could we check on the line — including left of zero if the number is negative?”

On three names:

“Fraction, decimal, percent — three names. One magnitude. Show me.”

When you are about to take over:

“Your mark. I’ll wait.”

Then wait.

Age-band moves: 11–12 / 13–14

11–12. Unify positive fraction, decimal, and percent on a 0–1 line and extend through negatives for simple cases (−1/2, −0.5, −50%). Place and compare. Divide fractions with a meaning story (how many groups?) before invert-and-multiply as a chant. Recipe scales with halves and thirds that still get written as numbers. Percent of a quantity with a percent bar or line — “what is 25% of…?” — before multi-step tax-and-discount theater. Money motivates; the written ratio still appears.

13–14. Multi-step percent with meaning (percent of, percent change, discount then tax) once percent-as-location and Chapter 2’s ratio table are working — hand heavy proportion work to Chapter 2 rather than dumping it here. Operate with positive and negative fractions as numbers. Estimate before computing as a habit. A fourteen-year-old missing magnitude still does 11–12 line work without apology. Algebra I edges stay labelled; this chapter does not become a full algebra course because a candle was lit.

Parent learns this week. Hear pizza offered as the number −3/4, and ask for the line.

Student tries today. Place −1/2, 2/3, 0.75, and 50% on one line and say which are the same.

First try-it for the student

A strip with −1, 0, and 1 marked:

place −1/2, 2/3, 0.75, and 50%.

Say: "Where does each live on the line? Which are the same magnitude?"

Wait. If they put 0.75 far from 3/4, the decimal never met the fraction. If they refuse to place −1/2 left of zero, negatives are still a slogan. If they shade a pizza and never mark, they stayed in part–whole. Ask the location again. Model on a *different* strip if needed — you mark −3/4 and 0.75, you say "same magnitude, different sides of zero for the negative" — then hand them a fresh strip.

Later the same week, the diagnostic item:

a pizza drawing offered as −3/4, then a blank line through zero.

Hear a refusal to leave the pizza. Hear a mark at +3/4. Hear a mark at the right place. After the student has attempted, you may show a worked *incorrect* example (labelled as an illustration) that left the pizza as the finished work for a negative fraction and ask what that person thought the number *was*. Generation first. Then the named miss.

How to fade help. First sitting: you mark together, they echo "−1/2 lives here." Second: they mark, you wait, you hint ("left of zero for negatives"). Third: they place fraction, decimal, and percent and say which match. Fourth: a cousin — store percent tag written at home — with the first strip closed.

When to stop talking. When you hear yourself explaining 75% while their strip is still blank. When the sit has become a lecture titled *What a Percent Is*. One good question. Stop while they still have a mark left in them.

Practice that actually builds learning

Blocked, for a new move. The day you introduce negatives on the fraction line, give a short set that is only placements through zero: −1/2, −3/4, 1/4, 0.5. The day you unify percent, only percent-as-location next to fraction and decimal names. Mixing too early makes the student hunt for a pizza picture or a keyword.

Mixed, for when to use it. After two or three blocked sits, mix: place a number, compare two, rewrite 0.2 as a percent, scale a recipe half and write the number. Mixing is the real test. A student who can only place when you say "use the line" has not yet owned the move.

Talk box

Opening question (locked): "Where does this live on the line?"

Follow-ups (3–5):

1. Between which two wholes?
2. Show me −3/4 and 0.75 — same place? Same magnitude as 75% of one?
3. How do you know 75% is three-fourths?
4. Estimate before computing — about where should this live?
5. You may change your mind. Try it another way on the percent bar.

How to wait. After you ask, a slow three. After they stop talking, wait again. If they are mid-reason, do not cut them off. Look at the line, not at the student, if the silence is hard. Do not stack five questions. Do not fill the silence with the right mark.

What stuck silence usually means. The question was vague; wait-time 1 was zero; they are stuck in pizza forever; they treat fraction, decimal, and percent as three vocab lists; they fear negatives; they are guessing what you want. Your job is to pose, wait, and point back at the line — not to grab the pencil, and not to turn the table into a fight.

Named try-its

1. Number-line still-life (in-home)

- **Time:** 10–15 minutes.

- **Materials:** Register tape or scrap paper marked −1 to 1 (and a second strip 0–1 if helpful).
- **Safety:** Ordinary table; ordinary tone — curious, not courtroom.
- **The fun:** Placing −3/4, 0.5, 75%, and watching three names land as locations.
- **The skill:** Magnitude; equivalence; negatives appear. Write the numbers. The tape is the representation; a snack picture is not the destination.

2. Recipe fraction ops (in-home)

- **Time:** 10–20 minutes.
- **Materials:** Measuring cups already in the drawer; a simple recipe you were going to cook anyway.
- **Safety:** Ordinary food hygiene. No nutrition sermon. A half-cup is a number, not a moral.
- **The fun:** Scaling to 1½× or ½× and tasting the result.
- **The skill:** Fraction multiplication or division with meaning; still write the number and, when useful, place it. Kitchen motivates. The line or the written equation still appears the same week.

3. Percent as location + money (in-home)

- **Time:** 10–15 minutes.
- **Materials:** Allowance ledger, play receipts, or round-number prices on scrap paper.
- **Safety:** Ordinary; play money is fine.
- **The fun:** "What is 25% of…?" and watching a percent bar fill.
- **The skill:** Percent as ratio per 100; percent bar or number line; estimate then compute. Land on the written check: 25/100 of the amount.

4. Store unit / percent tag (out-of-home)

- **Time:** 10–15 minutes of a trip you were already taking.
- **Materials:** Shelf tags; a small notebook or phone note for numbers (not a photo of a stranger's face).
- **Safety:** Aisle courtesy; no quiz of the cashier; no shoplifting a "test"; no lecture to a stranger's adolescent. A price tag is a number. It is not a sermon about a food system.
- **The fun:** "Is 20% off the same as paying 80%?" Choose one tag. Compare mentally.

- **The skill:** Percent with meaning; land on the written percent-as-ratio or percent bar at home the same day. The aisle motivates. The notebook finishes the math.

Incorrect example to diagnose (labelled illustration). A student is asked where $-3/4$ lives. They shade three of four pizza slices, smile, and stop. They have offered part–whole as the finished signed number. Ask: "Where does $-3/4$ live on the line?" If they mark +3/4, they heard the fraction and missed the sign. If they refuse the line, pizza is still the destination. Correct the move without crushing the attempt: the pizza named three fourths; the sign needs a direction from zero.

More on unifying the three names

Spend one sitting where the only job is translation. Write a small table with three columns: fraction, decimal, percent. Fill six rows together the first time: $1/2 \leftrightarrow 0.5 \leftrightarrow 50\%$; $1/4 \leftrightarrow 0.25 \leftrightarrow 25\%$; $3/4 \leftrightarrow 0.75 \leftrightarrow 75\%$; $1/5 \leftrightarrow 0.2 \leftrightarrow 20\%$; $1 \leftrightarrow 1.0 \leftrightarrow 100\%$; $-1/2 \leftrightarrow -0.5 \leftrightarrow -50\%$. Then erase the middle column and ask the student to rebuild it. Then erase the percent column. The point is not speed. The point is that the names share a magnitude on the line.

When money enters, keep the same table nearby. "25% of $40" is easier after 25% already lives next to 1/4. Compute 0.25×40 only after the estimate ("a quarter of forty is ten") has been said out loud. If the student jumps to a calculator and lands on 10 without the estimate, ask them to place 25% on the percent bar first next time.

Continuity without reprinting

Math for Little Thinkers owned ages 5–10 and the early number-line fraction work. *Mathematics for Homeschooling* compressed middle grades into a wider 1–12 map. This chapter owns 11–14 unification, including negatives. One pointer each, then teach. You do not need to reopen the younger book's lesson banks. You do need the same refusal: pizza is introduction; the line is destination — now with decimals, percents, and signs in the same week's breath.

A week's shape without a 180-day dump

Monday: warm-up placements; model $-3/4$ with 0.75; student places four numbers including one negative; talk box once; exit two placements.

Tuesday: blocked percent-as-location with the three-name table; mix one recipe half written as 1/2 and 0.5.

Wednesday: in-home number-line still-life try-it; incorrect pizza-for-negative illustration after their attempt.

Thursday: money percent bar; estimate then compute; mixed review of Tuesday's rows.

Friday: out-of-home store tag numbers collected (or remembered from a trip earlier in the week); written percent-as-ratio at the table; look at exit tickets; pick **one** diagnostic wrong answer; decide whether next week repeats location, adds ops, or moves toward Chapter 2's ratio table for harder percent stories.

That is a this-week plan. It is not a district pacing guide. Adjust numbers to the student. Stop while the hour is still honest.

Fluency note

Accuracy first. Brief timed retrieval only on pairs already known cold — for example, flashing "3/4" and expecting "0.75" after weeks of correct untimed work. Timing a student who still places 1/2 right of 2/3 trains panic, not magnitude. The Panel's automaticity finding frees working memory for later algebra; it does not turn this chapter into a stopwatch kit.

For the student

You already know that numbers live on a line. Whole numbers do. Fractions do too — and so do decimals and percents. They are different names for amounts. Sometimes the amount is less than zero. That is still a number. It lives to the left of zero on the line we draw.

Pizza can help you *start*. It is not where this idea ends. $-3/4$ is not a sad pizza. It is a location three-fourths of a unit left of zero. 0.75 is three-fourths of a unit right of zero. 75% of one whole is the same size as 0.75 and 3/4. Three names. One magnitude (with a sign when the story needs one).

Tiny worked example. Mark -1, 0, and 1. $-1/2$ is halfway from 0 toward -1. 0.5 is halfway from 0 toward 1. 50% of one is the same place as 0.5. So $-1/2$ and 0.5 are opposites. 50% matches 0.5, not $-1/2$.

Try 1. On a line from −1 to 1, place −3/4, 1/2, 0.75, and 25%. Which two are the same magnitude on the positive side? Which is farthest left?

Try 2. A shirt costs \$40. A tag says 25% off. About how much do you pay — near \$10, near \$30, or near \$40? Estimate first. Then write 25% as a fraction or decimal and check.

Explain it back. In your own words: why can −3/4, 0.75, and 75% belong in the same conversation? Use the word *line* or *place*.

Challenge. Place 2/3 and 0.6. Are they the same? How do you know without a calculator? You may change your mind when you look again.

You are allowed to wait. You are allowed to mark, erase, and mark again. Showing it on the line *is* the work.

If someone tells you fractions are "done" because you are older now, you can still open a line. Age is not a placement test. Skill is. A clear mark at −3/4 is real math. A shaded pizza that never becomes a mark is only a start.

If it isn't clicking

Diagnostic 1 — Pizza forever (or pizza for a negative). They shade slices and stop. Next move: same week, insist on a line through zero. Keep the pizza in the room if it helps start. Require a mark left or right of zero. One think-aloud on a parallel strip, then their strip again.

Diagnostic 2 — Three vocab lists. 0.75 and 75% live on different planets. Next move: blocked practice that only asks for the *same point* under three names. No long computation until the names meet. Percent bar beside the number line for one sitting.

Diagnostic 3 — Magnitude or sign errors. 1/2 marked right of 2/3, or −1/2 marked right of zero. Next move: estimate first ("which is closer to 1?"). Count equal jumps, not tick marks as if 0 were a jump. For signs, walk a taped line on the floor: stand at 0, step left for negative.

When to slow down. If placements are guesses and explanations are empty, stay on location and equivalence. Delay multi-step percent. Delay invert-and-multiply as a chant.

When to go ahead. If they unify three names, place negatives, and estimate before computing, mix in recipe ops and a store tag write-up. Begin handing proportion-heavy percent stories to Chapter 2's ratio table.

When to get a human tutor. If weeks of calm line work still leave magnitude scrambled, and you cannot hear the miss even after the parent warm-up, a short run with a person who will use a number line (not only a worksheet packet) is reasonable. Shame is not a tool. Correcting a wrong mark without crushing the attempt *is* teaching.

Composite illustration (not a reported family). An adult hears "we're in prealgebra, so we skipped fractions." The student's exit ticket places 1/2 right of 2/3 and offers a pizza for −1/4. The next move is not a scolding about curricula. It is painter's tape on the table, four placements, and the locked question. Two weeks later the same student matches 0.75 with 75% and marks −1/2 left of zero. That is placement by skill. Birthday did not fix it. The line did.

Tools, including AI

Optional helpers for you, the adult. Keep this short. The full AI rules live in The Math Hour at the front of this book — point back there.

Restated lightly for this week: the student attempts unaided first on the line or percent bar. You hold the answer key. At 11–12, you hold the account. At 13–14, you still stay in the room; turning thirteen is not a licence for an unsupervised chat partner. No photo-to-key. No pasting tonight's worksheet into a window and asking it to finish. After a real try, you may ask a tool to explain *this idea to you*, to build isomorphic practice with the key on your side, or to offer a short SCRIPT for you to say. The model may prepare you. It may not do their placement.

A number-line app or a blank printable strip is a fine $0-adjacent helper. A scan-and-solve loop for the exact homework item is not. Kitchen cups motivate. They do not replace the line.

What "done enough" looks like

Before you move on, check skill — not birthday.

- ☐ Places positive fractions, decimals, and percents on one line and names matches (e.g., 3/4, 0.75, 75%).
- ☐ Places at least simple negative fractions or decimals (e.g., −1/2, −0.75) left of zero and says what the sign means as direction.
- ☐ Estimates location or size before computing, then checks the answer against the estimate.
- ☐ Hears a pizza-only answer for a signed or percent number and can restart on the line without a fight.
- ☐ Writes one money or recipe result as a number (fraction, decimal, or percent) the same week the kitchen or aisle happened.
- ☐ Exit tickets show right answers *or* wrong answers with a reason you can teach from tomorrow.

A "grade 7 math workbook" is a publisher's scope, not a legal grade and not a transcript line. If the year's work is general middle-grades math, the stranger-readable title is **Mathematics**. If it is honestly prealgebra content, **Pre-Algebra**. **Algebra I** only if the year's work was that course — by skill, not because someone turned fourteen.

A fourteen-year-old who still cannot place −3/4 stays in this chapter's line work without apology. An eleven-year-old who can unify the three names and place negatives keeps going. Revisit mixed placements next week inside later chapters. Done enough is a path through the Critical Foundation — not a percentile, and not a diploma.

Carry forward: keep a few mixed placements inside Chapter 2 and Chapter 3 warm-ups so the line does not vanish when ratio tables and integer chips arrive. The Panel's continuity finding is why. The kitchen promise is not. If next Tuesday's exit ticket asks for a better-buy, still glance once at whether 25% and 1/4 still match in their head. That glance is life of the habit as session rhythm — not a ninth teaching chapter.

Chapter 2 — Ratio and proportion

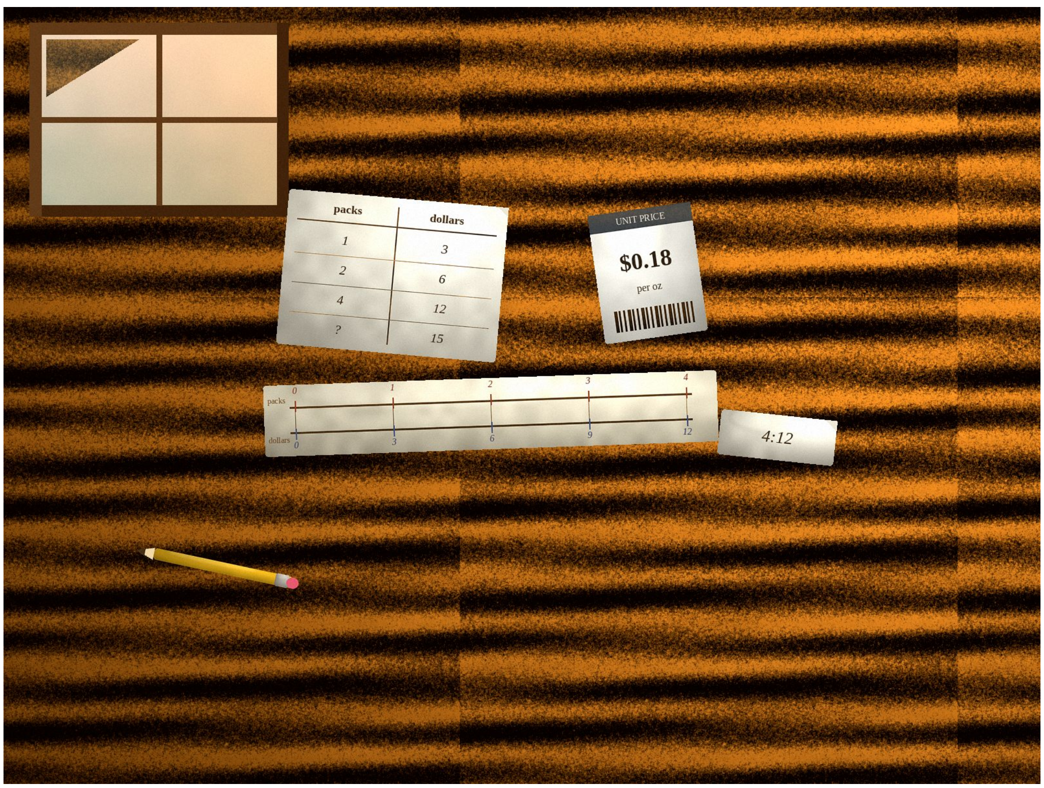

Figure 5: A kitchen-table still-life: scrap-paper ratio table with packs and dollars in two columns, a double number line on register tape beside it, a generic unit-price tag with no brand logo. No people.

Why this matters

What stays in the same ratio?

That is not a request for the word *proportion* recited from a glossary. It is not a cue to circle "of" and multiply. It is the question that turns "3 packs for $7.50" into a relationship you can scale — on a ratio table, on a double number line, or on a strip — before anyone reaches for cross-multiply as a first move.

Proportional reasoning is the signature cognitive work of ages 11–14. Kitchen doubling and store unit-price comparisons motivate it. They must land on the table and, when ready, on a written equation. Motivation without a representation is a pleasant aisle that does not finish the idea.

The Institute of Education Sciences fractions practice guide, Recommendation 4, asks teachers to develop conceptual understanding of strategies for ratio, rate, and proportion problems *before* exposing students to cross-multiplication as a procedure. The evidence tier is *minimal*; the panel still recommended the sequence. Build proportional relations. Use visual representations — the guide's own figures include ratio tables. Discuss alternative strategies. The 2012 problem-solving guide's Recommendation 3, rated *strong* for grades 4–8, wants visual representations that show relationships among quantities: tables, strip diagrams, percent bars. Those visuals are teaching moves. They are not a franchise effectiveness claim. When bar models appear in this chapter, they appear as strips that show quantities — not as proof that a named curriculum "works."[32]

Lamon's 1993 study of sixth-graders *before* instruction found that relative thinking

[32] Robert Siegler et al., *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, NCEE 2010-4039 (Washington, DC: Institute of Education Sciences, September 2010), Recommendation 4, minimal evidence: develop conceptual understanding of strategies for solving problems involving ratios, rates, and proportions before exposing students to cross-multiplication as a procedure; use visual representations including ratio tables. John Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8*, NCEE 2012-4055 (Washington, DC: Institute of Education Sciences, 2012), Recommendation 3, strong evidence (grades 4–8): visual representations that depict relationships among quantities. Access date for URLs in these notes: 4 September 2026. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/fractions_pg_093010.pdf; https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/MPS_PG_043012.pdf. What Works Clearinghouse, *Singapore Math Intervention Report* (December 2015), Primary Mathematics protocol, K–8: no studies of Singapore Math that fall within the scope of that protocol meet WWC group design standards; the WWC is unable to draw conclusions about effectiveness. Bar models / strip diagrams in this chapter are representations, not a franchise finding.

and unitizing related to sophistication; part–part–whole situations did not force proportional reasoning; stretcher/shrinker problems were hardest because students missed the multiplicative structure. You do not need the full classroom protocol book open on the table. You do need to hear additive thinking ("add 3 down the column") on a multiplicative story, and to ask what stayed the same.[33]

What this idea unlocks is unit rate, better-buy, scaling recipes, map scale, percent as a ratio story, and the bridge toward constant rate and linear thinking in later chapters. What it refuses is keyword lists — "of means multiply," "per means divide" — that train look-up instead of structure. Word problems in this band are schemas: equivalent ratios / missing value; unit rate / better buy; percent of / percent change; multiplicative compare versus additive compare. Ask what kind of story it is. Never circle the keyword as the operation.[34]

You do not need to be a mathematician. You do need to hear cross-multiply with no table as a missing-meaning move, not a cute shortcut, and to ask what stays in the same ratio.

Kitchen doubling is not the whole of proportional reasoning, and neither is a single better-buy trip. The week needs both the lived context and the written representation. If the pancakes get eaten and the table never appears, the Critical Foundation work of ratio and rate did not finish. If the table appears with no context all month, students may scale columns without knowing what the numbers meant in the world. Aim for both in the same week.

This week you can learn to hear that miss. Today the student can build three rows of a ratio table and find a missing cell without being told to cross-multiply.

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[33] Susan J. Lamon, "Ratio and Proportion: Connecting Content and Children's Thinking," *Journal for Research in Mathematics Education* 24, no. 1 (1993): 41–61. N = 24 sixth-graders pre-instruction: relative thinking and unitizing related to sophistication; part–part–whole situations did not force proportional reasoning; stretcher/shrinker problems were hardest. Abstract/DOI opened for this book; page-level classroom protocols beyond the abstract remain unverified and are not claimed as opened HTML.

[34] Woodward et al., *Improving Mathematical Problem Solving*, Recommendations 2–3. Fuchs et al., *Assisting Students Struggling with Mathematics*, WWC 2021006 (2021), Recommendation 5: word-problem types / schemas, not keyword matching — intervention guide with middle-edge overlap, not a kitchen RCT. Russell Gersten et al., *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (IES/WWC, 2009): word-problem structures and visual representations in the middle-school-inclusive guide.

For the parent: understand it yourself

Many adults feel rusty on why cross-multiply works, and clearer on the chant than on the meaning. That is ordinary. Five minutes here, then the warm-up, is enough for tomorrow. The student still builds the table.

Everyday picture. Three snack packs cost $6. Six packs should cost $12 if the price per pack stays the same. On scrap paper, two columns: packs | dollars. Rows: 3 | 6; 6 | 12; 1 | 2. The unit rate is $2 per pack. A double number line shows packs on one line and dollars on the other, marks lined up. Scaling the recipe to 1½× is the same idea with flour and eggs. The kitchen motivates. The columns are still the math.

Precise picture. A *ratio* compares two quantities. A *rate* is a ratio with different units (miles per hour, dollars per ounce). A *unit rate* is the amount for one of something. A *proportion* says two ratios are equal. A ratio table lists equivalent ratios by scaling (×2, ×10, ÷2) or by finding the unit rate and building out. A double number line keeps the two quantities aligned so the same scale factor is visible. A strip diagram (or percent bar) shows parts of a whole or parts compared to parts when that picture fits.

Cross-multiplication — if $a/b = c/d$, then $ad = bc$ — is a compressed consequence of equivalent fractions. Teach it *after* the student can explain the proportion on a table or double number line, and teach *why* from equal fractions. "Just cross-multiply, it's faster" as week one skips the representation the guides asked for first.[35]

Additive thinking is the common miss: the packs go 3, 6, 9 so the dollars go 6, 9, 12 (+3 each time) instead of doubling. Multiplicative structure means the same scale factor on both quantities. Stretcher/shrinker stories ("this photo is enlarged so every length is 3/2 as long") are harder pre-instruction; stay with missing-value and unit-rate tables until scaling both directions feels natural.

Money picture, still precise. Unit price on a shelf tag is already a rate someone else computed — often. Your student's job is to verify or compare, not to trust the tag blindly and not to quiz the cashier about the algorithm. Write the two rates. Compare. Go home. The aisle is not a courtroom.

[35] Siegler et al., NCEE 2010-4039, Recommendation 4 and Example 5: why cross-multiplication works is shown *after* proportional meaning, not as week-one magic.

Sports picture. Shots made to shots taken is a ratio; a shooting percent is that ratio per hundred. Box scores motivate. They still want a table or a clear fraction/percent link from Chapter 1. Sports stats are ratios and percents — not identity fights. Keep the example on the numbers.

Constant of proportionality language (y = kx) can appear lightly when the table is fluent — especially at 13–14 — without dumping a full linear-functions unit. Chapter 6 will own that bridge. This chapter owns meaning before cross-multiply.

Wrong answers you should be able to hear

1. *Cross-multiplies immediately with no table or double number line.* Procedure before meaning. Say: “Show me the rows first. What stays in the same ratio?” Fill three rows. Then, later in the week or later in the term, show how cross-multiply compresses the same equality.

2. *Adds 3 down a column when the structure is* $\times$. Additive thinking on a multiplicative story. Point at the table: “Did packs double or increase by three? What must dollars do?”

3. *Circles “of” and multiplies.* Keyword habit. Ask: “What kind of story is this — missing value, unit rate, percent, or compare?” Draw the quantities. Never treat the circled word as the operation.

4. *Treats every fraction word problem as a proportion.* Part–part–whole or simple fraction-of stories get forced into cross-multiply. Lamon’s reminder: not every fraction situation is proportional reasoning. Name the schema. If it is “half of twelve apples,” that may be fraction-of, not a missing-value proportion.

5. *Cannot say what stayed the same.* They produced a number and cannot name the invariant. Ask: “What stays in the same ratio?” Require one sentence: “Dollars per pack stayed $2” or “Both columns $\times$3.”

Five-minute parent warm-up

Minute 1. Draw packs | dollars. Write 2 | 5. Scale $\times$3 to 6 | 15. Say: “Same ratio.”

Minute 2. Find the unit rate: 1 pack costs $2.50. Build a row for 4 packs.

Minute 3. Sketch a double number line with the same numbers. Notice the alignment.

Minute 4. Imagine a student who immediately writes 2/5 = 6/x and cross-multiplies. Decide what “What stays in the same ratio? Show me on a table” does instead of a scold.

Minute 5. Write the locked question on a sticky: “What stays in the same ratio?” Put the pencil down.

How to teach it this week

Warm-up (3–5 min). Scale a known ratio $\times 2$ and $\times 10$. Already-right material. “3 for 6 — what about 6? What about 30?”

Short model (5–8 min). One missing-value on a ratio table; think aloud scaling or unit rate; refuse cross-multiply in the model. Then the notation if useful: 3/6 = 9/x as “same ratio,” still filled from the table first.

Student attempt (10–18 min). Two missing-value items; student chooses ratio table or double number line. You wait.

One good question. “What stays in the same ratio?” Wait a slow three. Look at the table.

Mixed practice (8–12 min). Fraction location from Chapter 1 next to today’s missing value. A percent-as-location item next to a unit-rate item.

Exit ticket (3–5 min). One better-buy without specifying which visual. Done-enough: right, or wrong with a reason (additive thinking named, keyword named, etc.).

Exact wording

“What stays in the same ratio?”

“What scaled — both quantities by the same factor?”

“Show me on a table / double number line.”

“Is this additive or multiplicative?”

“Does the answer’s size make sense?”

“You may change your mind.”

When they reach for cross-multiply first: "Table first today. We'll connect the shortcut when the rows are clear."

Age-band moves: 11–12 / 13–14

11–12. Write and interpret a ratio; build a ratio table; find a unit rate; solve missing-value by scaling *without* cross-multiply-first. Recipe scale on a table (½×, 1½×, 3×). Store better-buy on a double number line. Percent of with a percent bar once Chapter 1's percent-as-location works. Refuse keyword lists every week.

13–14. Identify proportional versus not; constant of proportionality in table language; multi-step percent with meaning; map or model scale as ratio; stretch toward y = kx language without dumping Chapter 6. Cross-multiply may appear *after* meaning, with why from equal fractions. A fourteen-year-old who still adds down the column stays on tables without apology.

Parent learns this week. Hear cross-multiply with no table, and ask for the rows.

Student tries today. Build three rows of a ratio table and find a missing cell without being told to cross-multiply.

First try-it. Scrap paper: "4 markers cost $6. What do 10 markers cost if the rate stays the same?" Require a table with at least three rows (include the unit rate or a ×/÷ path). No cross-multiply instruction. Hear additive +2.50 errors. Hear a correct build. Fade: first sitting you start the columns; later they choose table or double number line alone.

How to fade help. Co-build columns → they scale one row → they choose the representation → they explain what stayed the same in a sentence.

When to stop talking. When you are narrating every scale factor while their pencil is idle. One good question. Stop.

Practice that actually builds learning

Blocked. Day of unit rate: only better-buy and "per one" rows. Day of missing value: only scale-factor tables. Day of percent bar: only percent-of with play numbers.

Mixed. After blocked fluency, mix missing value, better-buy, and a Chapter 1 placement. Mixing trains choosing the representation.

Practice that actually builds learning — retrieval note

After the blocked days, ask for a cold unit rate on Wednesday with no model first. Retrieval of a known strategy is part of fluency. If they blank, that is information: the table is not yet theirs. Return to a short model, then parallel try. Do not interpret a blank as laziness; interpret it as “needs another representation pass.”

Keep a running list on the fridge for a week: three ratios from real life (recipe, price, map, sports). Each evening, add one row to a table for one of them. Tiny habit. Real ratio.

Talk box

Opening (locked): “What stays in the same ratio?”

Follow-ups:

1. What scaled?
2. Show me on a table / double number line / strip.
3. Is this additive or multiplicative?
4. Does the answer’s size make sense?
5. You may change your mind. Try another representation.

How to wait. Slow three after the ask; wait again after they stop; look at the table if silence is hard; do not fill with the missing cell.

Stuck silence usually means. Keyword hunt; cross-multiply reflex; additive thinking; wait-time 1 was zero; question vague; numbers too new. Point at the representation. Do not turn the table into a fight.

Named try-its

1. Recipe scale on a ratio table (in-home)

- **Time:** 15–20 minutes.
- **Materials:** Scrap paper columns; a recipe you will actually cook or a play recipe with round numbers.
- **Safety:** Ordinary kitchen hygiene; no nutrition sermon.
- **The fun:** 1½× pancakes or cookies; tasting the scaled batch.
- **The skill:** Scaling; unit rate; refuse cross-multiply-first. Both ingredient columns scale by the same factor. Write the table. Cooking motivates. The

table is the math.

2. Percent bar / discount (in-home)

- **Time:** 10–15 minutes.
- **Materials:** Paper strip marked 0–100%; play price for a household item.
- **Safety:** Ordinary.
- **The fun:** A "sale" with play numbers — 20% off $50.
- **The skill:** Percent of / (at 13–14) percent change. Shade the bar; write the ratio; compute. Hand messy multi-step percent to a week when ratio meaning is already steady.

3. Store better-buy (out-of-home)

- **Time:** 10–15 minutes of a trip.
- **Materials:** Two shelf tags for comparable items; notebook for numbers.
- **Safety:** Aisle courtesy; no cashier quiz; no shoplifting a test; no public lecture. A price tag is a number.
- **The fun:** Which is cheaper per ounce (or per count)?
- **The skill:** Unit rate; double number line or table; schema not keyword. Finish the written comparison at home if the aisle is rushed.

4. Map scale / model scale (out-of-home)

- **Time:** 10–20 minutes.
- **Materials:** Trail map, transit map, or model-kit scale; ruler optional.
- **Safety:** Ordinary outdoor/road sense; no climbing stunts for a "scale photo."
- **The fun:** "How far really?" from map distance to real distance.
- **The skill:** Scale as ratio (lean 13–14). Ratio table: map cm | real km. Same factor both ways.

Incorrect example to diagnose (illustration). Problem: 5 notebooks for $12.50; cost for 8? Student writes 5/12.50 = 8/x and cross-multiplies, gets a number, cannot say what stayed the same. Ask for three table rows and the unit rate sentence. If they instead write $12.50 + $3 for each extra notebook (additive), name the multiplicative miss and rebuild $\times$ and $\div$ rows.

More on refusing keywords

Gersten and colleagues' recommendation on word-problem structures, and later anti-keyword findings in the intervention literature, travel into this chapter as a practical rule: name the story type, draw quantities, then compute. "Of means multiply" fails on "3/4 of the students who play soccer also…" style complexity and on compare stories. When the student circles a word, smile, and ask the locked question anyway. Chapter 8 will deepen schema sorting. This chapter installs the refusal inside ratio work.

When cross-multiply earns a seat

After several weeks of tables and double number lines, show one proportion the student already solved by scaling. Write a/b = c/d. Show that multiplying both sides by bd (or using equivalent fractions) yields ad = bc. Name it as compression, not magic. Then put the table back for the next problem. Speed is allowed after meaning. Speed as week one is the residue this book declines.

A week's shape

Monday: $\times 2/\times 10$ warm-up; model missing value on a table; attempt two items; talk box; exit.

Tuesday: unit-rate blocked set; better-buy with play tags at home.

Wednesday: recipe table try-it.

Thursday: percent bar; mix one Chapter 1 placement.

Friday: store better-buy numbers; written finish; one diagnostic wrong answer for next week's plan.

Ratio table, double number line, and strip — same family

Think of these three as one family with different strengths. The ratio table is easiest to start: columns, rows, scale factors you can write. The double number line makes the alignment visible — helpful when students lose track of which number belongs to which quantity. The strip diagram (including the percent bar) shines when parts of a whole or part-to-part comparisons need a single bar picture. You do not need all three every day. You do need at least one visual before the cross-multiply chant. The 2012 problem-solving guide's strong recommendation for visuals in grades 4–8 is on-age for this book. Use it. Do not turn the family into a collage of twelve disconnected pictures in one hour.

When Singapore-family materials use bar models, you may use the bar as a strip. You may not treat a federal review of Singapore Math as proof that a franchise works at your table. Representations are allowed. Crowns are not.

Unit rate as a habit

Train the sentence: "For one ____, there are ____." One ounce costs…. One mile takes…. One batch needs…. Once the unit rate is solid, missing values get easier because students can go down to one and back out — or scale in friendlier jumps ($\times 2$, $\times 10$) when the numbers invite it. Better-buy is unit rate with a comparison. Map scale is unit rate with different units. Percent of a quantity is a per-hundred rate applied to a whole. Same family. Different costumes.

Multiplicative compare vs additive compare

"Three times as many" is multiplicative. "Three more" is additive. Students mix them. A short sort once a week helps: write three sentences, ask which kind, then decide whether a ratio table is the right tool. Chapter 8 will deepen story-type sorting. Plant the distinction here so ratio work does not swallow every word problem.

Linking back to Chapter 1

Keep placing a fraction or percent on a line during warm-ups. Ratio tables that use 1/2 and 0.5 as scale factors need Chapter 1's unification. If 25% still floats free of 1/4, fix that before a multi-step discount week. Continuity is a session rhythm, not a birthday.

Parent load and video

If ratio tables make you anxious, it is allowed to let a clear video or a purchased program give the first explanation while you sit beside the student and insist on the table on paper. The student still attempts unaided after the model. Your job is still to hear cross-multiply-first and additive thinking. You do not need to invent every worked example from scratch on a rusty morning.

Worked parent moves you can steal

Move A — missing value. "8 markers for $10. Cost for 6?" Columns: markers | dollars. Rows: 8|$10; 4|$5; 2|$2.50; 6|$7.50. Narrate $\div 2$, $\div 2$, $\times 3$ — or go to 1 then $\times 6$.

Move B — better buy. "15 oz for $2.40 vs 20 oz for $3.00." Unit rates: $0.16/oz vs $0.15/oz. Double number line optional. Sentence: "B is cheaper per ounce."

Move C — recipe 1½$\times$. Eggs 2 $\to$ 3; flour 1½ cups $\to$ 2¼ cups. Table with ingredients. Same scale factor 3/2 on every row.

Move D — after meaning, why cross-multiply. Student already has 2/5 = 6/15 from a table. Show $2\times 15 = 5\times 6$. Name equality of products as a consequence. Next problem: table first again.

Incorrect patterns, labelled illustrations

Illustration 1: Student sees "3 out of 4 students prefer tea" and sets up a proportion to find how many prefer tea in a class of 28 — sometimes correct schema, sometimes they cross-multiply without knowing what the 3 and 4 meant. Ask them to label parts.

Illustration 2: "A rope 12 m long is cut into pieces each 3/4 as long as the previous" treated as simple $\times 3/4$ once — stretcher/shrinker complexity. If it is too hard, park it and return after basic scaling is fluent.

Illustration 3: Keyword "altogether" circled; student adds when the story was a rate. Name the schema miss.

Practice volume without a worksheet dump

Four or five math hours this week beat forty identical cross-multiply items. Each hour: warm-up, short model, attempt, one question, mix, exit. One in-home try-it. One out-of-home try-it. Friday: one diagnostic wrong answer. That is enough grain for a real week.

How to teach — fuller session notes

On Monday's model, write the columns headers *before* any numbers: "What are we comparing?" Force the labels. Unlabelled columns are how dollars get multiplied by packs by accident.

On the student attempt, if they stall, offer a hint that is a question: “What would one pack cost?” or “What if we double both?” Do not offer the finished cell. Struggle before rescue: ask, wait, hint, then model a *parallel* item with different numbers while they watch, then hand back their item.

On mixed practice, include one deliberate non-proportion: “Maya had 3 apples and bought 4 more. How many now?” If they build a ratio table, smile and ask whether anything was supposed to stay in the same ratio. Sorting matters.

On the exit ticket, one item should not announce “use a ratio table.” See whether they reach for it. If they only use the table when the worksheet says “complete the table,” the representation is still a compliance move, not a tool.

Out-of-home without theater

A better-buy try-it fails as teaching when it becomes a public performance. Whisper the question to your student. Write the two prices. Leave the store. Compute at the car or at home. Courtesy is part of the try-it’s safety line. The skill still completes on paper.

Map scale works on a walk you were taking anyway. Measure a map segment with a finger joint if you lack a ruler; rough scale still teaches ratio. Precision can improve later. A simple walk with a map you already own is enough for one proportion — no wilderness franchise required.

Connecting percent change (13–14)

Percent change = amount of change / original, as a percent. A percent bar from 100% down to 80% shows 20% off. A table can show original | new for several items at the same discount rate. If the student still cannot place 20% from Chapter 1, repair location first. Multi-step “discount then tax” waits until each step has meaning — and until they can say what stayed the same at each stage.

Language that helps

Prefer: “same ratio,” “scale factor,” “for each one,” “both quantities $\times 3$.” Prefer less: “cross multiply and hope,” “of means multiply,” “just use the butterfly method" as a first explanation. When informal speech appears ("the dollars go with the packs"), translate once into labelled columns, then let them talk human again.

Done-enough edge cases

If the student builds beautiful tables but cannot connect them to a store tag, add one out-of-home finish this week. If they ace better-buy but freeze on "3:5" notation, teach colon and fraction notation as the same ratio — three names again, Chapter 1 style. If they are ready for constant of proportionality language, write "dollars = 2.5 $\times$ packs" under a fluent table and stop — Chapter 6 will deepen.

For the student

You already know fractions live on a line from Chapter 1. Ratios use those numbers as partners. 3/4 can be three-fourths of a whole, or a ratio 3 to 4, depending on the story. That is why "what kind of story" matters. The digits look the same. The structure may not.

A ratio says how two amounts go together — packs with dollars, cups of mix with cups of water, centimeters on a map with kilometers outside. When the relationship stays the same as amounts grow or shrink, you have a proportion. Your job is to see what stayed the same.

A ratio table is two columns that keep the same relationship. If 2 packs cost \$5, then 4 packs cost \$10 (both $\times$2). One pack costs \$2.50 (divide by 2). Ten packs cost \$25 ($\times$5 from the unit rate, or other paths). A double number line shows the same idea with marks that line up.

Cross-multiplying can wait until you can explain the table. Circling the word "of" is not a strategy. Asking what kind of story you have — and what stayed in the same ratio — is.

Tiny worked example. 3 oranges for \$2. What for 12 oranges? Table: 3|\$2; 6|\$4; 12|\$8. Both columns $\times$4 from the first row. Unit rate: 1 orange is \$2/3. Same answer either path.

Try 1. 5 tickets for \$20. Build three rows. Find the cost for 8 tickets. Say what stayed the same.

Try 2. Brand A: 12 oz for $3. Brand B: 20 oz for $4.60. Which is cheaper per ounce? Show a table or double number line for each.

Explain it back. What is the difference between adding down a column and multiplying both quantities by the same factor?

Try 3 (bonus). A map scale says 1 cm : 2 km. A trail measures 7.5 cm on the map. How far is the trail? Show the ratio table.

Try 4 (bonus). Is "12 students own dogs; 3 more own cats than dogs" a ratio-scaling story? Why or why not?

Challenge. A photo is enlarged so every length is 3/2 as long. A line that was 10 cm becomes…? Why is "add 5 cm" the wrong kind of story?

You may change your mind when the table looks back at you. Showing the rows *is* the work.

If a worksheet tells you to cross-multiply on line one, you can still draw the table in the margin first. Showing what stayed the same is not extra credit. It is the point.

If it isn't clicking

Diagnostic 1 — Cross-multiply reflex. Next move: ban the shortcut for a week of tables only. Connect why later.

Diagnostic 2 — Additive thinking. Next move: color the scale factor; require them to say "$\times 3$ on both" out loud before writing.

Diagnostic 3 — Keyword hunt. Next move: sort three short stories (missing value / unit rate / not a proportion). Draw first; compute second.

Slow down if they cannot build three equivalent rows. **Go ahead** into percent change and map scale when unit rate is fluent. **Tutor** if weeks of calm tables still leave only chanted cross-multiply with no explanation — seek someone who will teach representations, not only tricks.

Also check wait-time. If you asked the locked question and answered it yourself in the same breath, the silence was never given a chance. Sit on your hands. Count a slow three. Look at the columns.

Also check number size. Ugly decimals can mask a clean strategy. Use friendlier numbers for one day, then return to the ugly tag from the store once the path is clear.

Composite illustration. A student nails every cross-multiply worksheet and fails a better-buy with odd numbers because the unit rate was never built. The repair is not more cross-multiply drills. It is two shelf tags and a double number line.

Why the struggle is worth it

Without ratio meaning, later linear equations become letter soup, similarity becomes "they look alike," and percent change becomes a calculator sequence nobody can explain. With ratio meaning, Chapter 6's constant rate feels familiar, Chapter 7's similar figures have side ratios waiting on a table, and Chapter 8's story schemas have a home. The struggle this week — building rows when a shortcut beckons — is the foundation those chapters stand on.

A parent who finishes this chapter able to hear cross-multiply-first and ask for the rows has done the adult half of the work. A student who can find a missing cell and say what stayed the same has done theirs. That is enough for a week. Tomorrow you can cook the pancakes. Bring the table to the counter if you want — flour column, egg column, same scale factor, same ratio.

Tools, including AI

Short box. Full rules live in The Math Hour. Unaided first on the table or double number line; parent holds the key; 11–12 parent holds the account; 13–14 still in the room; no photo-to-key; no paste worksheet. After a try, you may ask a tool to explain ratio tables *to you*, or to generate isomorphic practice with the key on your side. The model does not fill "the unit rate is ___ because ___" as the student's work.

Scrap paper is enough. A blank printable double number line helps. A scan-and-solve of the exact homework item does not.

If you use a purchased program this week, keep the ratio table on the table anyway. The program's page is not a substitute for the student building three rows by hand. Fit, not rank: choose materials that leave room for the representation.

Throw nothing out that already works — add the talk box and the named try-its beside it.

What "done enough" looks like

- ☐ Builds a ratio table with at least three equivalent rows and names what stayed the same.
- ☐ Finds a unit rate and uses it for a missing value or better-buy.
- ☐ Uses a double number line or strip/percent bar when it fits, not only when told.
- ☐ Does *not* default to cross-multiply before meaning (shortcut only after explanation).
- ☐ Resists keyword circling; can name missing value vs unit rate vs "not a proportion."
- ☐ Exit tickets usable tomorrow — right, or wrong with a named miss (additive, keyword, empty chant).

Place by skill. Title the year **Mathematics** or **Pre-Algebra** by content, never by brand. Revisit mixed ratio stories next Tuesday inside later chapters. Done enough is proportional meaning — not a percentile, not a diploma, and not a crown for any named curriculum.[36]

What "done enough" also includes is mercy: ugly numbers can wait one more day if the structure finally clicked with friendly ones. Carry into Chapter 3 and beyond: warm up with one unit-rate row so the representation family does not vanish when signed numbers arrive. If Chapter 6's constant rate shows up later, it should feel like a reunion with this chapter's table — not a brand-new planet named Algebra.

[36] What Works Clearinghouse, *Singapore Math Intervention Report* (December 2015): no studies meet WWC group design standards — copied whenever a named curriculum might be crowned. Fit, not rank. Transcript titles remain Mathematics / Pre-Algebra / Algebra I by content.

Chapter 3 — Integers and signed numbers

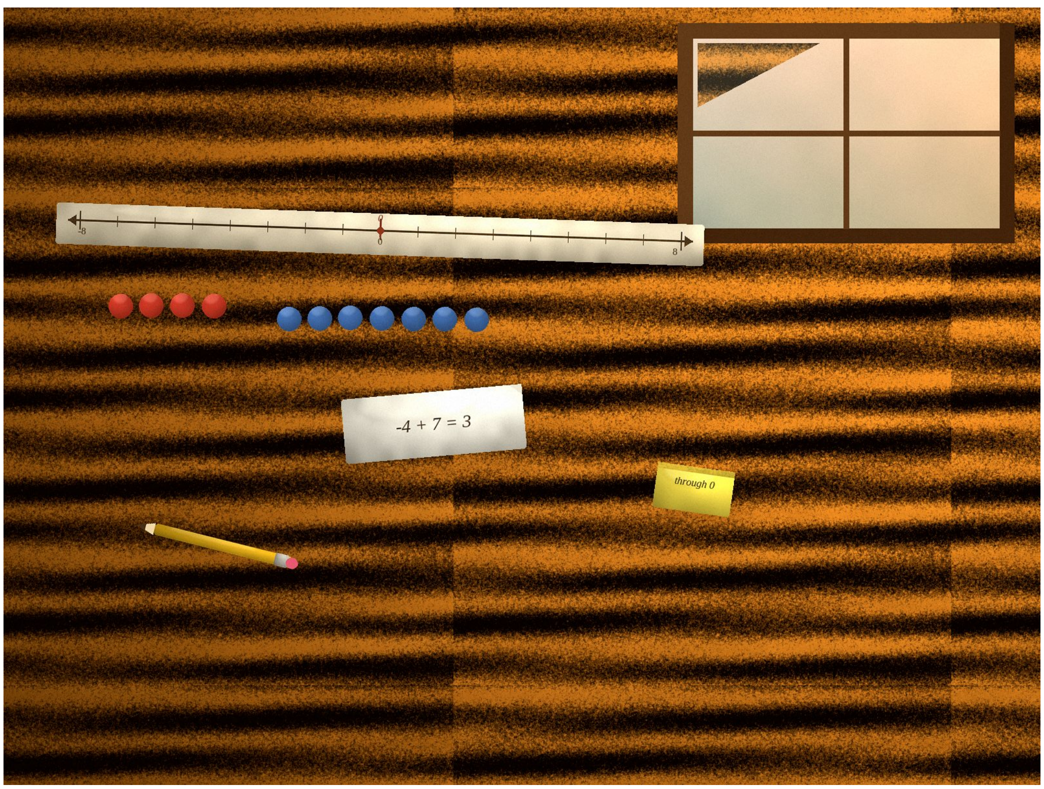

Figure 6: A kitchen-table still-life: painter's tape number line through zero on the table, two colors of chips in small piles, a scrap with −4 + 7 = 3. No people. No logos.

Why this matters

What does this signed number mean here?

If Chapter 1 asked where a number lives, this chapter asks what the sign is doing in this situation — direction, opposite, or net change — so the location keeps its meaning when we operate. That is not a request for the slogan "two negatives make a positive." It is not a request for absolute value as "drop the sign." It is the question that turns -4 from a mark on a worksheet into a direction, an opposite, or a net change — on a line, with chips, in a weather story, on a staircase — before anyone chants a rule with no picture.

Integers and the full rational system are Grade 6–7 Critical Foundation work, not optional enrichment you skip because a prealgebra cover looked exciting. The National Mathematics Advisory Panel's Table 2 wanted proficiency with positive and negative integers by the end of Grade 6, and all operations with positive and negative fractions by the end of Grade 7. That calendar sits inside ages 11–14. Chapter 1 already placed negative fractions on a line. This chapter deepens meaning: opposite, neutralization, net change. Temperature, elevation, and debt are useful *contexts*. They are not the only meaning. The number line in both directions remains central.[37]

Signed numbers are not a side quest. Linear equations, slope, and coordinate work need them. A student who can chant keep-change-change but cannot show $-3 + 5$ as a directed move will hit a wall when variables pick up signs. Meaning first. Rules that match the representation second. Slogans with no picture — never as week one.

There is no separate federal "integers practice guide" that crowns a kitchen method. The claim in this chapter sits under the Panel's benchmarks and the same number-line continuity you used for fractions. This is a useful continuity, not a promise that every home will see the same result on the same Tuesday. Then teach.[38]

[37] National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Critical Foundations / Table 2: proficiency with positive and negative integers by the end of Grade 6; all operations with positive and negative fractions by the end of Grade 7. Access date for URLs in these notes: 4 September 2026. https://files.eric.ed.gov/fulltext/ED500695.pdf.

[38] Robert Siegler et al., *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, NCEE 2010-4039 (Washington, DC: Institute of Education Sciences, September 2010), Recommendation 2, moderate: number lines as a central representational tool for fractions as num-

What this idea unlocks is honest work with net change, absolute value as distance, four quadrants on a plane, and signed fraction operations that Chapter 1 began. What it refuses is "two negatives make a positive" as a song without chips or a line. You do not need to be a mathematician. You do need to hear a chant without a representation as unfinished meaning — and to ask what this signed number means *here*.

Kitchen, weather, and stairs motivate. They do not replace the number line or the chips. A week of only temperature talk with no tape still leaves the Critical Foundation thin. A week of only tape with no story still leaves students asking what negatives are *for*. Aim for both in the same week — representation on the table, context in the try-it.

This week you can learn to hear that miss. Today the student can show −4 + 7 on a line and with chips, then write the sentence.

For the parent: understand it yourself

Many adults remember a rhyme for multiplying negatives and feel less clear on why −3 + 5 lands at 2. That is ordinary. Five minutes here, then the warm-up, is enough. The student still walks the line.

Everyday picture. Painter's tape on the floor or table: … −3, −2, −1, 0, 1, 2, 3 …. Stand at −4. Adding 7 means seven steps in the positive direction — land at 3. Subtracting can mean moving the other way, or adding the opposite. Two-color chips: one color +1, the other −1. A + chip and a − chip cancel (neutralize) to zero. Five − chips and two + chips leave three − chips: −3. Weather: 4° to −2° is a net change of −6. Elevation: two floors below lobby is −2 if lobby is 0. Debt: owing $5 can be modelled as −5 in a ledger. Contexts motivate. The line and the chips are still the math.

Precise picture. An *integer* is a whole number or its opposite: … −2, −1, 0, 1, 2 …. A *rational* number includes fractions and decimals, positive and negative; Chapter 1 placed them; this chapter operates with meaning. The *opposite* of a is −a; the opposite of −a is a. *Absolute value* |a| is distance from 0 on the line — always nonnegative as a distance. |−4| = 4 because −4 is four units from 0, not because we "dropped the sign" as magic.

bers — continuity used here for signed integers and signed rationals. There is no separate federal "integers practice guide" crowning a kitchen method.

Addition of signed numbers can be taught as directed moves on the line or as combining chips with cancellation. Subtraction as adding the opposite connects to later algebra. Multiplication and division rules should wait until addition/subtraction have a representation — and when the "two negatives" moment arrives for multiplication, it needs a story (pattern in a table; chip groups; opposite of an opposite), not only a chant.

Coordinate plane: four quadrants. (−2, 3) is left 2, up 3. Emerging at 11–12; more fluent at 13–14. Do not skip negatives while chasing worksheet titles that say prealgebra.

Wrong answers you should be able to hear

1. *Chants "two negatives make a positive" with no line or chips.* Slogan without representation. Say: "What does this signed number mean here? Show −3 + 5 on the line." Save multiplication slogans until addition has a picture — and then require a story.

2. *Absolute value as "drop the sign."* They write |−4| = 4 with no distance language, then later write |−4| = −4 under stress, or treat absolute value as optional decoration. Ask: "How far from zero?" Walk it.

3. *Debt (or temperature) as the only meaning.* Every negative must be money owed. Context froze into a prison. Show elevation, time zones light-touch, game scores below par, or simply "left of zero" with no story. Keep the line as the home meaning.

4. *Sign error when combining.* −4 + 7 becomes −11, or 4 − 7 becomes 3. Directed-move miss or chip-count miss. Rebuild on the tape. Require the number sentence after the move.

5. *Refuses to plot in four quadrants.* (−2, 3) placed in QI, or axes ignored. Practice ordered pairs with signs after line work is steady. Emerging vs fluent depends on the band — but refusal is information.

Five-minute parent warm-up

Minute 1. Tape a small line −5 to 5. Walk −4 + 7 with a finger. Say the landing out loud: 3.

Minute 2. Chips: five dark, two light; cancel pairs; name −3.

Minute 3. Write |−4| and say "distance 4 from zero."

Minute 4. Imagine the chant with no picture. Decide what the locked question does instead.

Minute 5. Sticky note: “What does this signed number mean here?”

How to teach it this week

Warm-up (3–5). Opposites on a line: name the opposite of 6, of −2, of 1/2. Already-right material.

Short model (5–8). −3 + 5 as directed moves; then the same sum with chips cancel; then write −3 + 5 = 2. Stop talking.

Student attempt (10–18). Three integer sums/differences with a representation *required* — line or chips — before the sentence.

One good question. “What does this signed number mean here?” Wait a slow three. Look at the line.

Mixed practice. Place a negative fraction from Chapter 1. One unit-rate row from Chapter 2 if it fits the hour.

Exit ticket. One net-change story + number sentence. Done-enough: right, or wrong with a reason (wrong direction, dropped sign, chant only).

Exact wording

“What does this signed number mean here?”

“Where is it on the line?”

“What is its opposite?”

“Show net change — start, move, land.”

“Does ‘two negatives’ have a story here, or only a chant?”

“You may change your mind.”

When keep-change-change appears as magic: “Show me on the line first today.”

Age-band moves: 11–12 / 13–14

11–12. Negatives and absolute value; four quadrants emerging; add/subtract integers with line and chips; place negative fractions (continuity with Chapter 1); weather/ledger net change; stairs language.

13–14. All rational operations with contexts and properties; fluent quadrant plotting; multiplication/division of signed numbers *with* representation or pattern tables, not slogan-only; connect toward equations in Chapter 5. A fourteen-year-old who still chants without a line stays on directed moves without apology.

Parent learns this week. Hear a chant without a line, and ask for a directed move.

Student tries today. Show −4 + 7 on a line and with chips, then write the sentence.

First try-it. Tape line: start at −4, add 7, land, write −4 + 7 = 3. Then chips for the same sum. Compare. Fade help across the week: co-walk → they walk you watch → they choose line or chips → sentence only after representation.

When to stop talking. When you are performing keep-change-change while their tape is blank. One question. Stop.

Practice that actually builds learning

Blocked. Day of directed addition only. Day of chips cancel only. Day of absolute value as distance only. Day of net-change stories only.

Mixed. After blocked sits, mix: place −2/3, compute −5 + 9 with a required sketch, |−8|, a weather change.

Talk box

Opening (locked): “What does this signed number mean here?”

Follow-ups:

1. Where is it on the line?
2. What is its opposite?
3. Show net change (start → move → land).
4. Does “two negatives” have a story here, or only a chant?
5. You may change your mind. Try chips if you used the line (or the reverse).

How to wait. Slow three; wait again after they stop; look at the line or chips; do not fill with the landing point.

Stuck silence usually means. Slogan without representation; absolute value as drop-the-sign; wait-time 1 was zero; fear of negatives; numbers too new. Point at the representation.

Named try-its

1. Sidewalk / tape number line through zero (in-home)

- **Time:** 10–15 minutes.
- **Materials:** Painter's tape; marker.
- **Safety:** Ordinary floor space; no hallway Olympics that knock someone over.
- **The fun:** Walk $-3 + 5$; feel the direction change.
- **The skill:** Directed moves; absolute value as distance. Write the sentence after the walk.

2. Weather through zero / allowance ledger (in-home)

- **Time:** 10 minutes.
- **Materials:** Weather app lows or a simple ledger on scrap paper.
- **Safety:** Ordinary; play ledger amounts are fine.
- **The fun:** "Net change" from morning to night, or allowance in/out.
- **The skill:** Meaning before rules. Context motivates; number sentence finishes.

3. Signed chips cancel (in-home)

- **Time:** 10–15 minutes.
- **Materials:** Two colors of chips, tiles, or paper squares.
- **Safety:** Ordinary small pieces — keep away from toddlers if needed.
- **The fun:** Cancel pairs until one color remains.
- **The skill:** Opposite; neutralization; then write the integer. Chips $\rightarrow$ symbols the same day.

4. Elevation / stairs as signed change (out-of-home)

- **Time:** During a walk or errand you were taking.
- **Materials:** None required; optional notebook.
- **Safety:** Ordinary stair sense; no climbing stunts; hold rails as needed.

- **The fun:** Floors above/below lobby or street level as +/−.
- **The skill:** Integers in the wild; later at the table, plot (−2, 3) if ready (11–12 emerging; 13–14 fluent).

Incorrect example to diagnose (illustration). Student faces $(-3) \times (-4)$. Chants "two negatives make a positive," writes 12, cannot explain. Ask for a pattern table: $(-3)\times1 = -3$; $(-3)\times2 = -6$; $(-3)\times0 = 0$; $(-3)\times(-1) = ?$ Or ask for "opposite of an opposite" language with a chip story. If they only have the chant, the representation was skipped.

Deeper teaching notes

Subtraction as adding the opposite. $5 - (-2)$ confuses everyone who only has "minus means go left." Teach: subtracting a number is adding its opposite. $5 - (-2) = 5 + 2$. Show on the line: taking away a leftward move is a rightward move. Require the rewrite once; then the move.

Order of operations with signs. -3^2 versus $(-3)^2$ is a later precision point — parentheses matter. Touch lightly at 13–14 when exponents appear. Keep this chapter's spine on meaning and directed moves, not a full order-of-operations unit.

Properties. Commutative and associative properties still hold for integers; noticing them connects to NMAP Finding 11's properties bridge toward algebra. Mention when a student rearranges $-2 + 5$ as $5 + (-2)$ to make the walk easier.[39]

Continuity with fractions. Warm-up: place $-3/4$. Operate: $-1/2 + 1/4$ on the line. Signed rationals are the Panel's Grade 7 finish line — meaning first, fluency next.

A week's shape

Monday: opposites warm-up; model $-3+5$ two ways; attempt three sums; talk box; exit net-change.

Tuesday: chips blocked; absolute value as distance.

Wednesday: tape try-it; mix negative fraction placement.

[39] National Mathematics Advisory Panel, *Foundations for Success*, Finding 11: automatic recall of facts and fluency with standard algorithms frees working memory; the Panel also emphasizes properties of operations as a bridge toward algebra. Used here for noticing commutative/associative moves with integers, not as a claim that every home will see identical gains.

Thursday: weather/ledger; subtraction as add opposite.

Friday: stairs language collected outside; plot a point at the table; one diagnostic wrong answer.

Multiplication without the empty chant (when ready)

Build a table for $3 \times n$ with n going 3, 2, 1, 0, −1, −2. Products: 9, 6, 3, 0, −3, −6 — pattern decreases by 3. Then $(-3) \times n$: −9, −6, −3, 0, 3, 6 — pattern increases by 3. The "two negatives" moment becomes a pattern landing, not a song. This is a useful teaching move, not a claimed RCT. Use it when addition/subtraction are steady.

Extended how-to: directed moves in slow motion

Start every new family of problems with a narrated walk. Finger on the tape. "I begin at negative four. Adding seven means seven steps toward the positives. One… two… three… four… five… six… seven. I land on three." Then the student narrates their own walk on a parallel problem. If they skip counting out loud, reinstate the count for two days. Silent guesses recreate the old rhyme habit.

When both addends are negative $(-4) + (-3)$, the walk goes left from −4 three more steps to −7. Students who only memorized "different signs subtract" freeze here. The walk does not freeze. Same for $(-4) + 3$: start −4, three right, land −1.

Chips in slow motion

Build $-4 + 7$ as four minus chips and seven plus chips on the table. Pair them into zero pairs. Count what remains. Photograph nothing that includes a face; if you want a record, photograph only the chips and the written sentence. Move from chips to a sketch of plus/minus marks, then to numerals only. Fading the concrete is the point — not staying in chips forever, and not skipping them.

Absolute value workshop (one sitting)

Items: $|5|$, $|-5|$, $|0|$, $|-12|$, $|3 - 8|$ after they compute inside (optional at 13–14). After each, ask "how far from zero?" If they say "drop the sign" as their only sentence, require a second sentence with the word *distance* or *units from zero*. $|3 - 8| = |-5| = 5$ is a nice bridge toward expressions.

Net change stories bank (safe topics)

- Temperature morning to evening
- Allowance earned and spent
- Game score relative to par (if the family knows golf) or points behind/ahead
- Elevator floors relative to lobby
- Yardage gain/loss in a sport box score as integers

Avoid live controversy. Avoid nutrition-as-moral-war. A ledger is a ledger.

Subtraction clinic

Give four items that all rewrite as adding the opposite: 1. $6 - 9$ 2. $6 - (-2)$ 3. $-4 - 3$ 4. $-4 - (-5)$

Require the rewrite line before the answer. Then the walk or chips. Then the answer. This clinic undoes more damage than a poster of rules.

Quadrant mini-lesson (11–12 emerging / 13–14 fluent)

Draw axes. Plot (2, 3), (−2, 3), (−2, −3), (2, −3). Name quadrants I–IV. Ask which coordinate controls left/right and which controls up/down. Stop. Distance formula and slope formula theater wait for Chapters 6–7 edges.

Common parent trap

Finishing their walk because the silence feels long. Sit on your hands. Stahl's think-time is a classroom-origin convention — about three seconds after the question, and again after they pause. It is not a math RCT. It is still the right kitchen move.[40]

Why refuse the slogan as week one

The slogan sometimes produces correct products and still leaves students unable to evaluate $-x$ when x is -3, or to explain why a slope is negative, or to combine like terms with minus signs in Chapter 4. Representation now is cheaper than repair

[40] Robert J. Stahl, "Using 'Think-Time' and 'Wait-Time' Skillfully in the Classroom," ERIC Digest ED370885 (1994): about three seconds; classroom-origin. Mary Budd Rowe, NARST/ERIC ED061103 (1972), is science class. Not a mathematics RCT.

later. When the pattern table finally supports “negative times negative,” the slogan can become a summary — the way a formula becomes a summary after geometry reasons in Chapter 7.

Mixed practice recipes

Set A (blocked walks): six additions, representation required. Set B (blocked chips): six combination problems. Set C (mixed): two walks, one absolute value, one Chapter 1 placement, one net-change word story. Set D (exit style): one story + sentence; one pure computation with a required sketch in the margin.

Volume: prefer two thoughtful sets over a 40-problem drill packet that trains chanting.

Connecting to rationals

Once integer walks are steady, replace $-4 + 7$ with $-1/2 + 3/4$. Equal jumps of fourths on the line. Same meaning, finer grain. That is the Panel’s Grade 7 finish — positive and negative fractions with operations — arriving as continuity, not as a surprise quiz.

For the parent — one more everyday picture: the elevator

Lobby = 0. Parking one level down = -1. Offices up to 3. Riding from -1 to 3 is a net change of +4. Riding from 2 to -1 is a net change of -3. Say the start, the end, and the change as three numbers. Many students can name start and end and still botch the change. The change is the signed number that answers “what does this mean here?” in motion language.

Precise picture — properties without a lecture

$a + 0 = a$. $a + (-a) = 0$. These are not trivia. They are why zero pairs work and why later equations can add the same quantity to both sides. When chips cancel to empty, say “we made zero.” When a walk returns to start by taking the opposite steps, say “opposites sum to zero.” Short. Human. Enough.

Wrong-answer drill for the adult ear (illustrations)

Illustration A: Student computes $(-2) - (-5)$ as -7. They treated both minuses as "more negative." Repair: rewrite as $(-2) + 5$; walk.

Illustration B: Student says $|-3| = -3$ because "the absolute value bars failed." Repair: distance language; compare to $|3|$.

Illustration C: Student places $(-3, -2)$ in quadrant II. Repair: x controls left/right first.

Illustration D: Student refuses negatives in a recipe scale ("you can't have negative cups"). Agree that cups of flour aren't negative — then show a temperature or elevation where negatives earn their keep. Context fit matters.

How to teach — fading script

Sitting 1: You walk; they echo the landing. Sitting 2: They walk; you point only if direction reverses wrongly. Sitting 3: They choose line or chips; you ask the locked question once. Sitting 4: Sentence first, then prove with a sketch. Sitting 5: Mix in a negative fraction sum.

Practice — out-of-home stairs protocol

1. Agree on zero (lobby, sidewalk, driveway).
2. Note start floor/level.
3. Note end after the errand segment.
4. At home: number line sketch; net change sentence.
5. Optional: if you also moved east/west on a grid-like campus map, plot a rough ordered pair — only if it stays fun.

Safety: hold rails; no racing; no blocking doors for math.

Student page companion ideas

Invite them to invent one signed story that is *not* debt. Collect three family-safe stories on the fridge. Rotate them into warm-ups. Ownership helps the meaning stick.

Tools companion

A simple spreadsheet can show a pattern table for multiplication of signed numbers — parent builds it, student predicts the next cell before revealing. That is isomorphic practice with the key on the parent side. It is not photo-to-key.

For the student

A signed number tells you *how much* and *which way* — or *which side of zero*. −4 is four units left of zero on the line we draw. Its opposite is +4. Absolute value is how far from zero: $|-4| = 4$ because the distance is four, not because someone erased a minus sign for fun.

You can add on the line by walking. Start at −4. Add 7: walk seven steps right. Land at 3. Write $-4 + 7 = 3$. You can also use two colors of chips. Plus chips and minus chips cancel in pairs. What is left is the answer.

Temperature, stairs, and money ledgers are stories that use signed numbers. The stories help. The line is still home base. If someone only teaches you a chant for two negatives, ask for a picture. You deserve a reason.

Tiny worked example. $-3 + 5$. Line: start −3, five right, land 2. Chips: three minus, five plus → cancel three pairs → two plus left → 2. Sentence: $-3 + 5 = 2$.

Try 1. Show $-4 + 7$ on a line and with chips. Write the sentence.

Try 2. Morning −2°, afternoon 5°. What was the net change? Write a number sentence.

Explain it back. What does absolute value mean as distance? Why is "drop the sign" an incomplete explanation?

Challenge. Compute $5 - (-3)$ by rewriting as adding the opposite. Show both on the line.

You may change your mind when the chips disagree with your chant. Looking *is* the work.

If it isn't clicking

Diagnostic 1 — Slogan only. Ban the chant for a week. Line and chips only. Reintroduce patterns later for multiplication.

Diagnostic 2 — Absolute value as drop-sign. Walk distances. Ask "how far?" before "what do we write?"

Diagnostic 3 — Sign errors / quadrant refusal. Slow down to single moves. Color code left/right. Plot only after line addition is steady.

Slow down if directed moves are guesses. **Go ahead** into signed rationals and quadrant fluency when net change stories are solid. **Tutor** if weeks of calm tape work still leave only rhymes — find a person who will use representations.

Composite illustration. A student aces a "rules for integers" matching worksheet and cannot show $-2 + (-5)$ on a line. The repair is tape, not another rhyme poster.

Also check wait-time. If you fear negatives from your own school memory, do the five-minute warm-up alone first. Maloney-style caution from early-grade school research is not a middle-grades trial — but a calm adult next to the tape still helps. Let the student move the chips.

If it still feels thin after two weeks

Narrow the focus to addition only for five straight math hours. No multiplication slogans in the room. No worksheet titled "all integer operations." Mastery of directed addition unlocks subtraction-as-opposite and, later, patterns for multiplication. Going wide too soon recreates the rhyme poster.

If the student is already fluent with walks and chips, do not bore them with another week of $-3 + 5$. Move to signed rationals, quadrant plotting, and evaluating simple expressions at negative inputs. Placement by skill cuts both ways.

Life of the habit as session rhythm

Revisit one signed warm-up inside next week's Chapter 4 work. Mix last week's net-change story into Friday's exit. Stop when the hour is honest. That rhythm lives in the math hour and in done-enough checklists — not in a ninth teaching chapter.

Friday diagnostic protocol

Look at the week's exit tickets. Pick **one** wrong answer. Classify it: slogan-only, drop-sign absolute value, direction error, context freeze (debt only), quadrant mix-up. Decide next week: repeat walks, narrow to subtraction clinic, or advance to signed rationals. One diagnostic beats a vague feeling that "integers are hard."

Write the classification in the dated notebook. Tomorrow's you will thank today's you.

What not to overclaim this week

Chips and tape are teaching moves, not a claimed effectiveness rating. Meaning before full rational fluency. Negatives stay in the week even when a prealgebra worksheet title beckons. Keep-change-change is later informal speech, not the identity of algebra. Teach the walk. Teach the cancel. Teach the sentence. That is the week.

Tools, including AI

Point back to The Math Hour for full rules. Unaided first on the line or chips; parent holds the key; 11–12 parent holds the account; 13–14 still in the room; no photo-to-key; no paste worksheet. After a try, a tool may explain signed addition *to you*, or build isomorphic practice with answers on your side. It may not walk their tape for them.

Painter's tape and two colors of scrap paper are enough. A number-line applet is fine. A scan-and-solve of tonight's integer worksheet is not.

If a purchased program is already on the shelf, keep using it — and keep requiring a sketch in the margin for every signed computation this week. Fit, not rank. The talk box still opens with the locked question. The exit ticket still asks for a net-change sentence. Program pages do not replace those moves.

What "done enough" looks like

- ☐ Shows integer addition/subtraction on a line *or* with chips, then writes the sentence.

- ☐ Explains absolute value as distance from zero (not only "drop the sign").
- ☐ Uses at least two contexts (weather, elevation, ledger, game) without freezing negatives as only debt.
- ☐ Places negative fractions on the same line (Chapter 1 continuity).
- ☐ Does not lead with "two negatives make a positive" without a representation or pattern story.
- ☐ Exit tickets usable tomorrow — including naming direction errors.

Place by skill. **Mathematics / Pre-Algebra** by content. Carry signed warm-ups into Chapter 4's expression evaluation (try $n = -2$). Done enough is meaning — not a percentile, not a diploma.

If multiplication of signed numbers is still shaky but addition walks are strong, you may still begin Chapter 4's expressions using only addition structures ($n + -3$, $5 + n$) and return to signed multiplication in parallel. Skill placement is a door, not a wall. A dated notebook showing walks, chips sketches, and corrected sign errors is better evidence than a matching worksheet with rhymes.

Revisit next Tuesday: one net-change exit item mixed into whatever chapter you are in. That is the habit. That is enough.

More practice detail for parents

When the student writes $-8 + 3 = -11$, they often added absolute values and kept a minus. Ask them to walk it. The walk corrects what a lecture on "different signs, subtract" sometimes fails to correct. Rules summarized *after* ten correct walks stick better than rules before any walk.

When the student subtracts incorrectly — $3 - 8 = 5$ — they may have computed $8 - 3$ and ignored order. Rewrite as $3 + (-8)$. Walk from 3 eight steps left. Land at -5. The rewrite is the teaching move; the slogan "keep change change" can be informal speech *later* for "add the opposite," not week-one magic.

Zero pairs are gold. Making zero on purpose (add $+2$ and -2 to an expression of chips without changing the total) prepares the ground for algebra moves that add the same to both sides — Chapter 5's cousin. Mention lightly: "We added a zero pair; the total stayed the same."

For ordered pairs, play battleship-style with no brand game required: call $(-1, 2)$ and mark a dot. Swap roles. Keep it short. Four quadrants become geography, not a fear.

Out-of-home without theater

Stairs try-its fail when they become public quizzes of strangers in an elevator. Whisper. Count floors. Write later. A trail elevation marker, if present, is a number — read it, note the change from the last marker, go home, number sentence. Courtesy and safety beat a perfect data set.

Linking chapters

Chapter 1: negative fractions on the line — keep placing. Chapter 2: ratios can be negative in rate-of-change talk later; do not force it early. Chapter 4: evaluate 2n + 1 for n = −3. Chapter 5: same-to-both-sides with negative constants. Chapter 6: slope can be negative — steepness with direction.

Honest limit

We are not claiming a named integers RCT for your kitchen. We are teaching with representations the broader guidance already endorsed for number lines and for Critical Foundations.[41] Limit line once: this is useful practice design, not a promise of identical gains. Then walk the tape again tomorrow.

Student challenge extension

If Try 1–2 were easy, try: −2/3 + 1/2 on the line (common denominator as equal jumps). Or: a bank ledger with three transactions, net change, and a sentence about what −$15 meant in that story — still knowing −15 is also just left of zero on a pure line.

Parent sentence bank

"Show me the walk." "Cancel a zero pair." "How far from zero — that's absolute value." "Debt is one story. What's another?" "Your chips. I'll wait."

Use one. Wait. Look at the tape.

[41] National Mathematics Advisory Panel, Critical Foundations of Algebra, and Siegler et al., NCEE 2010-4039, Recommendation 2: number-line representations. Limit line: useful practice design under broader guidance, not a named integers RCT for the kitchen table.

Closing note for the adult

You do not need perfect fluency from your own school years to teach this. You need five minutes with tape before the student sits down, ears tuned for a chant without a picture, and willingness to wait after the locked question. The student walks. You hear. Tomorrow you walk again. Signed numbers stop being a wall and become a tool — for weather, for stairs, for the expressions waiting in the next chapter. When Chapter 4 asks them to evaluate $3n + 2$ at $n = -4$, the walk you practiced this week is what makes the substitution honest instead of magical. Keep the tape for one more week even after you finish the integer focus — a two-minute warm-up walk beats a forgotten slogan on a poster.

That is the whole invitation: meaning on the line, then the sentence. Ask. Wait. Walk. Then write it.

Chapter 4 — Expressions

Figure 7: A kitchen-table still-life: scrap paper with 3n + 2 written beside a tiny table of values for n = 0, 1, 5; a generic phone-plan sketch with flat fee and per-unit marks. No people. No logos.

Why this matters

What is this expression saying?

That is not a request to solve for n on sight. It is not a request to "move the letter." It is the question that turns 3n + 2 from a pile of symbols into a story — three times some number of units, plus a flat two — before anyone chases letters around the page.

Expressions are about structure and equivalence, not only letter-chasing. Write what stays the same as an expression before solving for a letter. Evaluate for several values. Notice like terms. Use the distributive property as a property with a picture, continuing the National Mathematics Advisory Panel's Finding 11 bridge from arithmetic properties toward algebra. Star and colleagues' algebra practice guide (Recommendation 2, *minimal* evidence tier) asks for language that reflects structure. Recommendation 1 (*minimal*) wants solved problems studied — including incorrect ones — before racing blank items. Woodward and colleagues' Recommendation 5 (*moderate*) wants students to articulate concepts and notation and to link arithmetic to algebra and back. Honest limit: some of those tiers are minimal. Use the moves anyway. Do not overclaim them as strong kitchen proof.[42]

The Common Core Grade 6 expressions and equations map — a map, not a statute — wants variables, equivalent forms, tables, and relationships such as 3x = y. Knuth's work on the relational equal sign lives next door in Chapter 5, but it already supports reading expressions on both sides of a true sentence.[43] Age 14

[42]National Mathematics Advisory Panel, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (Washington, DC: U.S. Department of Education, 2008), Finding 11: properties of operations as a bridge toward algebra. Jon R. Star et al., *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students*, NCEE 2015-4010 (also cited as NCEE 2014-4333) (Washington, DC: Institute of Education Sciences, April 2015), Recommendation 1, minimal evidence: use solved problems (including incorrect ones); Recommendation 2, minimal evidence: teach students to use language and structure of algebraic representations. John Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8*, NCEE 2012-4055 (2012), Recommendation 5, moderate evidence: articulate concepts and notation; link arithmetic to algebra. Access date for URLs in these notes: 4 September 2026. https://files.eric.ed.gov/fulltext/ED500695.pdf; https://ies.ed.gov/ncee/wwc/Docs/practiceguide/wwc_algebra_040715.pdf; https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/MPS_PG_043012.pdf.

[43]Eric J. Knuth, Ana C. Stephens, Nicole M. McNeil, and Martha W. Alibali, "Does Understanding the Equal Sign Matter? Evidence from Solving Equations," *Journal for Research in Mathematics Education* 37, no. 4 (2006): 297–312. Relational best definitions 32%/43%/31% in grades 6–8

may touch richer multi-step expressions as edges. This chapter does not dump polynomials, quadratics, or a full Algebra I symbols course because a candle was lit.[44]

What this idea unlocks is phone-plan and gym-fee thinking, store cost rules, sports point structures, and the runway into equations by same-to-both-sides. What it refuses is vocabulary lists first, letter-moving with no situation, and "solve" as the only verb allowed near a letter. You do not need to be a mathematician. You do need to hear letter-moving with no situation as unfinished meaning — and to ask what the expression is saying.

Kitchen, money, and making motivate. They do not replace the written expression or the table of values. A week of only talking about plans with no paper still leaves structure thin. A week of only simplifying with no situation still leaves students asking what the letter stood for. Aim for both in the same week.

This week you can learn to hear that miss. Today the student can write an expression for a flat fee plus a per-unit charge and evaluate it three times.

For the parent: understand it yourself

Many adults feel rusty on why $3(n + 2)$ is not the same as $3n + 2$, and clearer on "get n alone" than on "what is this saying?" That is ordinary. Five minutes here, then the warm-up, is enough. The student still writes the expression.

Everyday picture. A phone plan: \$20 per month plus \$3 per gigabyte. If n is the number of gigabytes, the monthly cost in dollars is $3n + 20$. For $n = 0$, cost = 20. For $n = 1$, cost = 23. For $n = 5$, cost = 35. The expression says the rule. Evaluating fills the table. Solving — "I have \$35; how many gigs?" — is a later verb that needs Chapter 5's balance moves. A store: apples \$2 each and a \$1 bag fee $\rightarrow$ $2a + 1$. Sports: 3 points per three-pointer and 1 per free throw $\rightarrow$ $3t + f$. Situations first. Symbols second. Properties third.

Precise picture. An *expression* is a mathematical phrase with numbers, operations,

(N = 177); foreshadowed here for expressions on both sides; full treatment in Chapter 5.

[44] National Governors Association Center for Best Practices and Council of Chief State School Officers, *Common Core State Standards for Mathematics* (2010), Grade 6 expressions and equations critical area: variables, equivalent forms, tables, relationships such as $3x = y$. Map only, not a homeschool statute. http://www.corestandards.org/Math/. National Mathematics Advisory Panel, Finding 15: place by skill, not birthday; Algebra I edges labelled only.

and maybe variables — no "equals answer" required. A *variable* is a symbol for a number that can change (or that is unknown). To *evaluate* is to replace the variable with a value and compute. *Equivalent expressions* name the same quantity for every value (or identically): 2(n + 3) and 2n + 6. *Like terms* share the same variable part: 3n and 5n combine to 8n; 3n and 5 are not like. The *distributive property*: a(b + c) = ab + ac — show with an area sketch or with chips groups when helpful.

Structure language: "This is three groups of n, plus two," not only "3n plus 2." Order of operations still matters; so do parentheses. Negative inputs from Chapter 3 belong in evaluation: try n = −2 on purpose.

Wrong answers you should be able to hear

1. *Wants to solve before saying what the expression means.* The verb jumped to Chapter 5. Say: "What is this expression saying? Evaluate for 0, 1, and 5 first."

2. *Treats the variable as a decoration.* Writes 3n + 2 but always plugs in the same favorite number, or treats n as "plus n" decoration without substitution. Require three different values.

3. *Cannot evaluate for three values.* Arithmetic slips or order-of-operations slips, especially with negatives. Slow down; one value at a time; use Chapter 3's line for negative inputs.

4. *"Moves letters" with no situation.* Rearranges symbols without knowing what they meant. Ask for the story. Ask for an equivalent form that matches the story.

5. *Treats 3(n + 2) as 3n + 2 without distributing.* Structure miss. Area sketch: a rectangle with sides 3 and (n+2) versus sides broken into n and 2.

Five-minute parent warm-up

Minute 1. Write 3n + 2. Say out loud: "Three times n, plus two." Minute 2. Evaluate for 0, 1, 5. Fill a tiny table. Minute 3. Write a phone-plan sentence that matches. Minute 4. Expand 3(n + 2) beside 3n + 6. Notice equivalence. Minute 5. Sticky: "What is this expression saying?"

How to teach it this week

Warm-up (3–5). Evaluate a known expression for 0 and 1 — maybe 2n + 1. Include one negative input if Chapter 3 is ready.

Short model (5–8). Situation → write 3n + 2 → table of values. Think aloud structure. Stop.

Student attempt (10–18). Write two expressions from situations; evaluate each thrice. You wait.

One good question. "What is this expression saying?" Slow three. Look at the paper.

Mixed practice. Integer evaluation from Chapter 3; one ratio-table row from Chapter 2 if it fits.

Exit ticket. One equivalent-forms item (match or generate). Done-enough: right, or wrong with a named structure miss.

Exact wording

"What is this expression saying?"

"What does the variable stand for?"

"Evaluate for 0, 1, and 5 — what stays the same?"

"Show an equivalent form."

"Does that match the situation?"

"You may change your mind."

When they rush to solve: "Expression first today. Solving waits for both sides and balance."

Age-band moves: 11–12 / 13–14

11–12. Write expressions from situations; evaluate for several values including 0 and 1; simple like terms; distributive property with a sketch; tables; refuse solve-first.

13–14. Richer multi-step expressions; more equivalent forms; evaluate with negatives fluently; edge toward multi-step before equations; still no polynomial dump, no quadratic chapter. A fourteen-year-old who cannot say what 3n + 2 means still does situation → expression work without apology.

Parent learns this week. Hear letter-moving with no situation, and ask what the expression says.

Student tries today. Write an expression for a flat fee plus a per-unit charge and evaluate it three times.

First try-it. "A gym charges $15 per month plus $2 per visit. Write an expression for monthly cost if v is visits. Evaluate for 0, 4, and 10 visits." Fade: co-write → they write you check meaning → they evaluate alone → they invent a parallel plan.

When to stop talking. When you are simplifying while their situation is still unwritten. One question. Stop.

Practice that actually builds learning

Blocked. Day of write-from-situation only. Day of evaluate-only on given expressions. Day of equivalent forms / distribute only.

Mixed. After blocked sits, mix: write, evaluate, match equivalents, one Chapter 3 signed evaluation.

One more out-of-home detail

When you collect store numbers, write units in the variable sentence at home: "Let n be the number of cans." Include the bag fee or not on purpose — two versions of the expression show what changes when the story changes. That contrast teaches structure better than another simplify row.

When you watch sports, pause once: "If they make two more threes, what happens to 3t + f?" Prediction is evaluation in disguise. Keep it light. Keep it kind. Then back to the game.

One more in-home detail

For like-terms tiles, cut paper into long "n" strips and unit squares. Physical grouping makes "you can't add n to 5" obvious without a scold. Photograph only the tiles if you want a record — no faces. Then retire the tiles when the written step is fluent.

Talk box

Opening (locked): “What is this expression saying?”

Follow-ups:

1. What does the variable stand for?
2. Evaluate for 0, 1, and 5 — what stays the same in the rule?
3. Show an equivalent form (distribute, combine like terms, or factor lightly).
4. Does that match the situation?
5. You may change your mind. Try another form.

How to wait. Slow three; wait again; look at the expression and table; do not fill in the simplified form for them.

Stuck silence usually means. Letter-chasing with no meaning; wait-time 1 was zero; they want to “solve” before the expression is clear; order-of-operations fear; negative inputs too new.

Named try-its

1. Phone-plan / gym-fee expression (in-home)

- **Time:** 15 minutes.
- **Materials:** Scrap paper; made-up or real plan numbers.
- **Safety:** Ordinary; no need to critique a real company’s pricing ethics as the math lesson.
- **The fun:** “Flat + per unit” — predict cost for a heavy-use week.
- **The skill:** Write 3n + 2 (or similar) *before* solving. Table of values. Structure talk.

2. Like terms with tiles or sketch (in-home)

- **Time:** 10–15 minutes.
- **Materials:** Paper tiles or sketches (n-tiles vs 1-tiles).
- **Safety:** Ordinary.
- **The fun:** Grouping piles — three n’s and five n’s make eight n’s.

- **The skill:** Structure; then evaluate the simplified form for three values. Tiles motivate. The written equivalent forms finish.

3. Store: write the cost expression (out-of-home)

- **Time:** 5–10 minutes looking during a trip.
- **Materials:** Shelf prices; notebook.
- **Safety:** Aisle courtesy; no cashier quiz; a price is a number.
- **The fun:** "If we buy n of these plus one bag…"
- **The skill:** Expression from a situation; land on paper at home the same day.

4. Sports: points expression (out-of-home / sideline)

- **Time:** During a game you were watching.
- **Materials:** Scrap or phone note.
- **Safety:** Ordinary spectator sense; sports stats are ratios and counts — not identity fights.
- **The fun:** "Three-pointers + free throws" as a structure.
- **The skill:** Expression for a real count; evaluate for a quarter's makes; compare to the box score.

Incorrect example to diagnose (illustration). Given "2 more than 3 times a number," student writes 2n + 3 or jumps to solve for a number never given. Ask what the expression is saying. Rebuild: 3 times a number $\rightarrow$ 3n; two more $\rightarrow$ 3n + 2. Evaluate to check sense.

Structure before letter-chasing — expanded

Star's structure recommendation is *minimal* tier — say so once, then teach. Practical moves:

- Read expressions aloud with grouping: "three times the quantity n plus two" versus "three n plus two."
- Compare 3(n + 2) and 3n + 2 with a table: they differ for most n.
- Combine like terms only after naming why 3n and 5n are alike and 3n and 5 are not.
- Study one *incorrect* simplification (labelled illustration): student turns 2(3 + n) into 6 + n and stops. Autopsy: distributed to 3 but not to n. Fix with area sketch.

Linking arithmetic to algebra

$2 + 2 + 2 = 3 \times 2$. Three groups of n is 3n. The distributive property you used to compute 3×14 as $3\times10 + 3\times4$ is the same property as $3(n + 4) = 3n + 12$. Say the link out loud once per week. Woodward's notation recommendation (*moderate*) is on your side.[45]

A week's shape

Monday: evaluate warm-up; model situation → 3n+2 → table; attempt two writes; talk box; exit equivalent forms.

Tuesday: blocked evaluate including $n = -2$; order of operations light.

Wednesday: phone-plan try-it; like terms tiles.

Thursday: distribute clinic; mix Chapter 3.

Friday: store expression numbers; written finish; sports optional; one diagnostic wrong answer.

What stays the same when n changes

The *rule* stays. The *output* changes. That sentence is early function talk without requiring f(x) — Chapter 6 will name input–output more fully. Here it is enough to say: the expression is the rule; the table shows outputs.

Parent sentence bank

"What is this expression saying?" "What does n stand for in the story?" "Try zero — what does the flat fee look like?" "Show me another form that means the same." "Your pencil. I'll wait."

Practice volume

Four or five math hours beat forty simplify-only problems. Each hour needs at least one situation → expression move. Simplifying with no meaning is letter-chasing

[45]Woodward et al., NCEE 2012-4055, Recommendation 5, moderate: students articulate and write mathematical concepts and notation; connect arithmetic to algebra. Used here for the distributive-property link from 3×14 to $3(n+4)$.

in a tidy shirt.

Equivalent forms clinic (one sitting)

Give: 2(n + 4), 2n + 8, 2n + 4 + 4, n + n + 8. Ask which match. Require evaluation at n = 3 as a check. If two forms disagree at n = 3, they are not equivalent. Checking by evaluation is allowed and wise — not a cheat.

Distributive property without fear

Area model: rectangle width 3, length n+2, split the length into n and 2. Areas 3n and 6. Total 3n+6. Then write 3(n+2) = 3n+6. Factor lightly the other way at 13–14: 3n+6 = 3(n+2). Stop before polynomial factoring theater.

Negative inputs on purpose

Evaluate 2n + 5 for n = −3. Use Chapter 3: 2×(−3) = −6; −6+5 = −1. If they write 2−3+5, they treated juxtaposition as decoration. Repair: "2n means 2 times n." Walk the multiplication on the line if needed.

Out-of-home without theater

Store try-its fail as public quizzes. Whisper. Note two numbers. Write at home. Sports try-its fail as arguments about referees. Stick to the count structure. Courtesy is part of safety.

Connecting toward equations (preview only)

"I paid $35 on a 3n+20 plan — how many gigs?" becomes 3n+20 = 35. That sentence has two sides. Chapter 5 owns same-to-both-sides. This week, you may set up the equation as a preview and *evaluate* guesses for n rather than formally solving — guess-check with the expression keeps the verb "evaluate" central.

Composite illustrations (not reported families)

Illustration 1: Student simplifies 4n − n to 4. They "cancelled" an n. Repair: 4n − 1n = 3n; tiles.

Illustration 2: Student writes “n3” for three n’s from a habit of units notation. Repair: 3n; say “three n.”

Illustration 3: Student insists on solving every expression. Give an expression with no question asked — only “what is it saying?” and three evaluations. Break the reflex.

Continuity pointers (one each, then teach)

Math for Little Thinkers owned early number and equal-sign foundations for ages 5–10 — one pointer. *Mathematics for Homeschooling* compressed middle grades inside a 1–12 arc — one pointer. This chapter owns 11–14 expression structure. Do not reprint those books’ lesson banks here.

Honest limit on evidence

Star’s structure and solved-problem recommendations are minimal tiers. Use them as teaching moves. Do not print them as a promise that every home will see the same result. Then write another phone-plan expression tomorrow.

Extended practice sets (use across the week)

Set Write (blocked). For each, write an expression; define the variable in a sentence. 1. \$8 per ticket plus \$3 service fee. 2. 5 more than twice a number of stickers. 3. Cost of m muffins at \$2.50 each with no fee. 4. Perimeter of a square with side s (remind: 4s). 5. A savings start of \$40 plus \$5 per week for w weeks.

Set Evaluate (blocked). Given $2n + 7$, evaluate $n = 0, 1, 4, -3$. Given $5(n - 1)$, evaluate $n = 1, 3, 0$. Given $n/2 + 3$, evaluate $n = 0, 6, -4$ (Chapter 1/3 support).

Set Equivalent (blocked). Match or rewrite: $3(n+4)$; $2n+2n+8$; $4n+12$; $2(2n+6)$. Check disagreements by evaluating at $n = 2$.

Set Mixed. One write, one evaluate with a negative, one equivalent-forms, one “is this asking you to solve?” sorting card (expression vs equation).

Teaching move: the sorting card

On scrap paper, write five strings: $3n+2$; $3n+2=14$; $2(n+1)$; $n-5=0$; $4+4+4$. Ask: expression or equation? The ones with equals relating two sides are equations —

Chapter 5. This sort prevents the solve reflex better than a lecture.

Teaching move: incorrect solved-expression autopsy

Label as illustration. A page shows: Simplify 2(3+n). Work shown: $2\times3=6$, bring down +n, answer 6+n. Student (or a fictional peer) missed distributing to n. Ask: "What is this expression saying? What did the work forget?" Then fix with area sketch. Star's solved-problem idea (*minimal* tier) applies even before full equations — studying incorrect work teaches structure.[46]

Parent anxiety note

If letters make your stomach drop, do the five-minute warm-up alone every day this week before the student sits. You are installing the miss so you can hear it. You are not required to love algebra nostalgia. You are required to ask what the expression is saying and to wait. A calm video first explanation is allowed; the student still attempts unaided on paper after.

Why three evaluations beat one

One evaluation can be a lucky arithmetic guess. Three — especially including zero and a negative — expose whether the expression matches the story and whether substitution is real. Make three the default, not the enrichment.

Variables that are not n

Use g for gigabytes, v for visits, p for pens. Changing the letter checks that the student did not memorize "n means the answer." The meaning is in the sentence that defines the variable, not in the alphabet choice.

From expression to ratio table (bridge)

Sometimes a plan is proportional with no flat fee: cost = 3n. That is Chapter 2's constant rate in expression clothes. Build a ratio table *and* write 3n. When a flat fee appears, the graph (later) will not go through the origin — Chapter 6 edge.

[46] Star et al., NCEE 2015-4010, Recommendation 1, minimal: study solved problems, including incorrect solved problems, to analyze algebraic reasoning. Tier is minimal — say so, then teach.

Mention lightly if curiosity appears. Slope-intercept theater can wait for Chapter 6.

Session debug table for this chapter

Symptom	Likely cause	Move
Solves immediately	Solve reflex	Sort expression vs equation; evaluate only
Blank at “write an expression”	No situation practice	Phone-plan try-it; sentence first
3(n+2) $\rightarrow$ 3n+2	Distribute miss	Area sketch; compare tables
Fails at negative n	Chapter 3 gap	Walk substitution; repair integers
Combines 3n+5 $\rightarrow$ 8n	Like-terms miss	Tiles; name why not alike
Fast simplify, no meaning	Letter-chasing	Require situation sentence above each

Friday protocol

Pick one diagnostic wrong answer from the week. Classify with the table above. Plan next week: repeat write-from-situation, open Chapter 5 with soft equation preview, or repair negatives. Write the plan in the dated notebook.

More student tries

Try 3. Write two different expressions for “6 more than a number.” Are n+6 and 6+n equivalent? Why?

Try 4. A snack bar sells juice for $3 and bars for $2. Write an expression for j juices and b bars. Evaluate j=2, b=3. Then explain each term.

Challenge plus. Invent a plan with a flat fee and a per-unit fee. Trade with a parent or sibling: they evaluate your expression at three values while you evaluate theirs. Check each other’s story match.

What "done enough" also means for placement

If expression structure is strong and Chapter 1–3 gates are clear, Chapter 5's one-step equations by same-to-both-sides are next — still by skill. If birthday says "Algebra I" but 3n+2 has no meaning, stay here. Publishers' pretests say similar things in their own words. Pride is not a placement instrument.

For the student

An expression is a math phrase. It can have numbers, operations, and letters. The letter stands for a number — sometimes a number that can change. 3n + 2 says: take some number n, multiply by 3, then add 2. It does not yet ask you to find n. Finding n is solving an equation. That comes when there is an equals and another side. First, know what the phrase is saying.

Tiny worked example. Plan: $4 per game plus $6 flat. Expression: 4g + 6. Table: g=0 $\to$ 6; g=1 $\to$ 10; g=5 $\to$ 26. The rule stayed. The total changed.

Try 1. Write an expression for $10 flat plus $3 per hour. Evaluate for 0, 2, and 7 hours.

Try 2. Are 2(n + 5) and 2n + 5 equivalent? Check with n = 3. Explain.

Explain it back. In your own words, what is an expression saying when it has a letter in it?

Challenge. A store sells pens for $2 each and notebooks for $4 each. Write an expression for p pens and k notebooks. Evaluate for p=3, k=2. What does each part mean?

You may change your mind when the table disagrees with a rushed simplify. Showing what it says *is* the work.

If a page tells you to "solve" and there is only an expression, you can still evaluate and explain. Solving needs both sides. You are allowed to wait and ask which verb they want.

If it isn't clicking

Diagnostic 1 — Solve reflex. Ban solving for a week of write + evaluate only. Preview equations later.

Diagnostic 2 — Variable as decoration. Require three evaluations every time; include 0 and a negative.

Diagnostic 3 — Structure / distribute miss. Area sketches; compare tables for 3(n+2) vs 3n+2; autopsy one incorrect worked example.

Slow down if they cannot state the story behind the symbols. **Go ahead** into richer equivalents and negative fluency when tables match stories. **Tutor** if weeks of calm situation→expression work still leave only letter-moving — seek someone who will teach structure, not only tricks.

Also check wait-time. Also check whether Chapter 3 negatives are spilling fear into evaluation — repair with a walk for one substitution.

Composite illustration. A student speeds through simplify worksheets and cannot write an expression for a flat fee plus per-unit charge. The repair is the phone-plan try-it, not more simplifying drills.

Worked parent model you can steal tomorrow

Situation: "Bike rental costs $12 to walk in the door plus $4 per hour." Variable sentence: "Let h be the number of hours rented." Expression: 4h + 12. Table:

h	4h+12
0	12
1	16
3	24
−1	(nonsense in this story — say why the domain of the story matters even without formal domain vocabulary)

Talk: "What is this expression saying?" Hear: "Four dollars for each hour, plus twelve dollars flat." Equivalent form optional: 4(h + 3) — check: 4(h+3)=4h+12. Same rule.

Then stop. Hand them a gym-fee twin. Do not solve for h unless Chapter 5 has begun.

Why refuse vocabulary lists first

Coefficient, constant, term — useful words after the story works. First-day glossary quizzes train look-up, not structure. Introduce names when you need them: "The 12 is the flat fee — we can call that the constant term." Short. Attached to meaning.

Money motivate, equation finish

The rental story motivates. The table finishes the expression work. If you only talk about bikes and never write 4h+12, the Critical Foundation toward algebra did not move. If you only simplify 4h+12+2h into 6h+12 with no story all month, letter-chasing wins. Same week: both.

Final encouragement

You do not need to be a mathematician. You do need to hear a student rearrange letters with no story, smile, and ask what the expression is saying. Then wait. Then point at the table. That is the job. The student can do this. So can you.

Tools, including AI

Point back to The Math Hour for full rules. Unaided first on the written expression and table; parent holds the key; 11–12 parent holds the account; 13–14 still in the room; no photo-to-key; no paste worksheet. After a try, a tool may explain distributive property *to you*, or build isomorphic plan problems with answers on your side. It may not write "the expression is ___ because ___" as their work.

Scrap paper is enough. Algebra tiles (or paper sketches) help. A scan-and-solve of tonight's simplify sheet is not.

If a purchased program is already working, keep it — and keep requiring a situation sentence above each simplification this week. Fit, not rank.

What "done enough" looks like

- ☐ Writes an expression from a flat + per-unit (or similar) situation.
- ☐ Evaluates an expression for at least three values, including 0 and one other (ideally a negative when ready).
- ☐ Explains what the expression is saying in ordinary words.
- ☐ Produces or recognizes an equivalent form (distribute or combine like terms) with a reason.
- ☐ Does not default to "solve" or "move the letter" when no equation was given.
- ☐ Exit tickets usable tomorrow — right, or wrong with a named structure miss.

Place by skill. Title **Mathematics** or **Pre-Algebra** by content; **Algebra I** only if the year's work maps — not because expressions appeared. Carry expression warm-ups into Chapter 5: keep evaluating, then add same-to-both-sides. Age-14 edges (richer multi-step) labelled only — no polynomial dump.

Revisit next Tuesday: one write-from-situation mixed into equation week. Done enough is structure — not a percentile, not a diploma.

Carry forward: when Chapter 6 asks for input–output, the table you built this week is already the idea. When Chapter 5 asks what we did to both sides, the expression on each side still needs to *say* something. You built that habit here.

A parent who finishes this chapter able to hear letter-moving with no situation, and to ask what the expression says, has done the adult half. A student who writes a plan expression and evaluates it three times has done theirs. That is enough for a week. Tomorrow you can compare two plans on purpose — still with expressions first.

More how-to detail: building the table

Always include n = 0 when the situation has a flat fee. Zero reveals the flat piece. Always include a second small value and a larger value. If all three outputs look wrong in the same way, the expression is probably wrong — fix the story match before arithmetic blame.

Have the student predict one output before computing. Estimation from Chapter 1's habit travels: "About thirty dollars for five gigs on a 3n+20 plan?" Then compute. Then compare.

Like terms — slow motion

Write 5n + 3 + 2n + 7. Group: (5n + 2n) + (3 + 7). Say why n-terms group and why constants group. Simplify to 7n + 10. Evaluate before and after at n = 2 to verify equivalence. Verification is structure respect, not distrust.

Factoring lightly (13–14 edge)

From 7n + 14 to 7(n + 2). Check with a value. Stop. Do not open a factoring-trinomials unit. Label richer factoring as Algebra I edge.

Order of operations reminders that matter here

3 + 2n at n = 4 is 3+8=11, not 5×4. Exponents if they appear: $-n^2$ vs $(-n)^2$ — parentheses matter; touch lightly. Fractions: (1/2)n + 3 is fine once Chapter 1 unification works.

Sports expression bank (safe)

Basketball: 3t + 2f + 1s (three-pointers, free throws if counting 1, or adjust to your sport's scoring). Soccer: goals are often just g — too simple; use shots on goal vs goals as a ratio link to Chapter 2 instead if scoring is only +1. Track: points by place if the family meet uses a point system. Keep it numerical and kind.

Phone-plan comparison (extension)

Plan A: 3n + 20. Plan B: 5n + 5. For which n is A cheaper? This *can* become an inequality or equation later. This week: fill tables for n = 0..10 and see where outputs cross. Pattern sniffing is allowed. Formal solve optional preview only.

If the student is ahead

Polynomials stay out of this chapter's spine. Offer richer situations instead: tax as a second expression composed lightly ("compute subtotal expression, then take 1.06 times" as two steps). Keep structure talk. Edges labelled.

If the student is behind on fractions or negatives

Place by skill. Spend two days of Chapter 1/3 repair inside expression evaluation — evaluating 0.5n + 1 and (−2)n + 3 *is* fraction/integer practice with a purpose. No apology for age.

Records grain

Dated notebook: situation sentence, expression, table of three values, one equivalent form, one corrected miss. That sample travels better than an app screenshot of a simplify streak.

Closing invitation

Ask the locked question until it feels ordinary. Wait until the wait feels ordinary. Let the table disagree with a rushed move. Change your mind when the paper says to. That is algebra beginning as sense, not as fear.

Ask what it is saying. Evaluate three times. Then, and only then, decide whether tomorrow opens an equation. Structure first. Letters second. Solving when both sides arrive. That order is the chapter. Keep it. Write the plan. Fill the table. Ask the question again tomorrow.

Tomorrow's expression is already waiting on scrap paper. Fill it in together once, then hand them the pencil.

Chapter 5

Equations

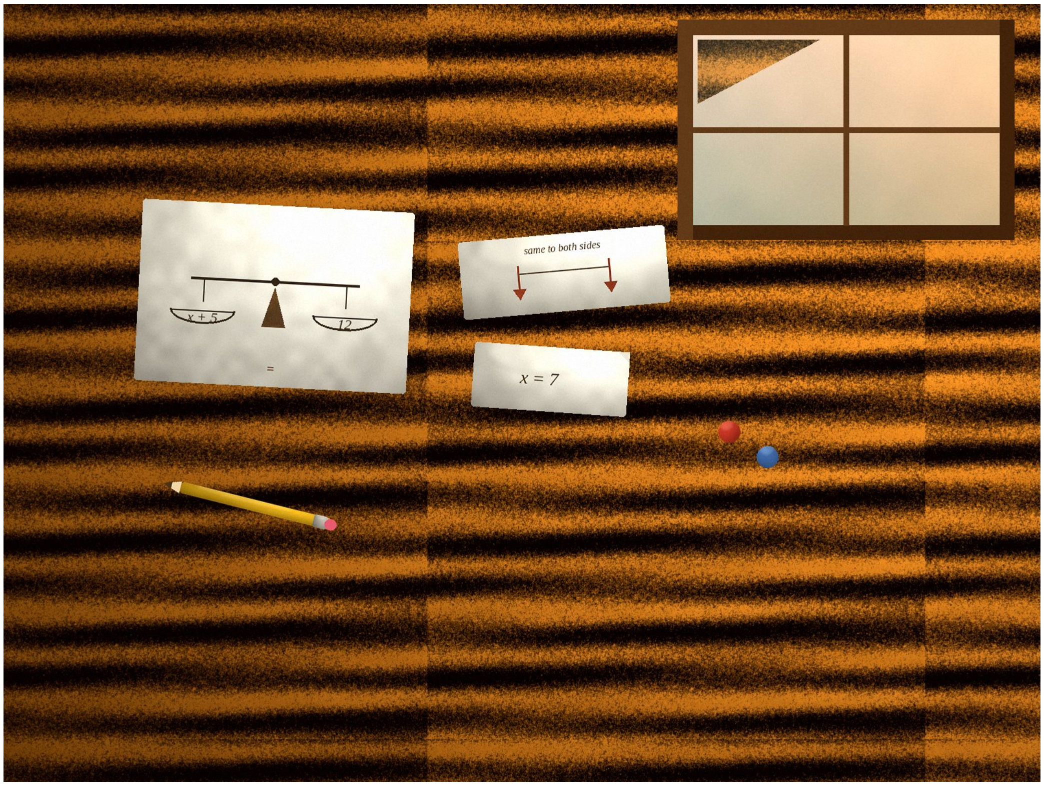

Figure 8: A kitchen-table still-life: a drawn balance with x + 5 on one pan and 12 on the other; arrows labelled “same to both sides”; scrap paper with a checked solution. No people. No logos.

Why this matters

What did we do to both sides?

That question is the whole chapter in one breath. An equation is two sides that name the same amount. When you change one side, you change the other the same way, or the sameness breaks. A balance that stays level is the everyday picture. A check by substitution is the precise one. The work of this week is not hunting for a letter. It is keeping equality true, step by step, out loud.

Many adults remember algebra as "move the 3 to the other side and flip the sign." That speech can arrive later as informal talk for subtracting three from both sides. It is a weak first story. The first story is: same operation, both sides. Subtract 5 from both sides of `x + 5 = 12`. Divide both sides of `3x = 21` by 3. Add the same number. Multiply by the same nonzero number. Name the move. Then check: put the value back into the original equation and see whether both sides still match.

The equal sign's relational meaning — "the same as," not "here comes the answer" — is not an elementary leftover. Eric Knuth, Ana Stephens, Nicole McNeil, and Martha Alibali asked middle-schoolers for their best definition of the equal sign. Relational answers were 32 percent of sixth graders, 43 percent of seventh, and 31 percent of eighth, in a sample of 177 students at one school. There was no linear improvement by grade. Relational understanding predicted correct equation solving even after controlling for mathematics scores, in a subsample.[47] Those percents live inside this book's age band. They are a useful study, not a promise that every home will see the same result, and not your student's score. Steal the implication: keep numbers and expressions on both sides of = while you teach equations, and treat "the answer comes next" as a live middle-grades problem.

What this idea unlocks is everything that follows. Multi-step equations, proportional equations, and linear thinking all assume that equality can be preserved on purpose. If = still means "compute and write the blank," every new letter is a hunt for a box. If both sides stay the same amount, the student can narrate legal moves,

[47] Eric J. Knuth, Ana C. Stephens, Nicole M. McNeil, and Martha W. Alibali, "Does Understanding the Equal Sign Matter? Evidence from Solving Equations," *Journal for Research in Mathematics Education* 37, no. 4 (2006): 297–312. Sample N = 177 (47 sixth, 72 seventh, 58 eighth) at one Midwest middle school. Best definition coded relational ("the same as"): 32% / 43% / 31% by grade; no significant linear or quadratic grade trend. Relational understanding predicted equation-solving success after controlling for standardized mathematics scores (subsample N = 65). These percents are not this student's score. Access date for URLs in these notes: 4 September 2026.

catch illegal ones, and check.

Why teach this *now*, at eleven to fourteen? Because the common map already asks Grade 6 students to maintain equality for one-step equations, and Grades 7–8 to handle multi-step linear work, with systems as an edge.[48] Because Star and colleagues, in a practice guide for algebra in grades 6–12, recommend studying solved problems — including incorrect ones — so students can analyze reasoning before they race blank worksheets.[49] Because expressions from Chapter 4 already taught structure; equations ask that structure to stay true under change.

This book is not a reprint of *Math for Little Thinkers*. That manual owns ages 5–10 and the early equal-sign work. One pointer: the meaning "the same as" continues here. Then we teach 11–14 equation strategy. This book is not a reprint of *Mathematics for Homeschooling*, which compressed middle grades into one chapter and then ran through high-school algebra. One pointer, then teach same-to-both-sides at this grain. Age-14 edges — including systems of two linear equations — are labelled only. They are not this chapter's spine.

You do not need to be a mathematician. You do need to hear "move the 3" with no both-sides story as a missing-meaning move, not a cute slip, and to ask what was done to both sides, without taking the pencil.

This week you can learn to hear an operational equal sign and a "move and flip" shortcut offered as magic. Today the student can solve a one-step equation (11–12) or a two-step equation (13–14), narrate each legal move, and check by substitution.

For the parent: understand it yourself

Many adults feel rusty on why subtracting five from both sides of `x + 5 = 12` is the whole move. That is ordinary. A diet of "get x alone" without saying how trains letter-chasing. Five minutes of this section, then the warm-up at the end, is enough for tomorrow. The student still writes the steps.

[48] National Governors Association Center for Best Practices and Council of Chief State School Officers, *Common Core State Standards for Mathematics* (2010), Grades 6–8 expressions and equations critical areas. Used here as a map, not a homeschool statute. http://www.corestandards.org/Math/

[49] Jon R. Star et al., *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (NCEE 2015-4010 / also cited as NCEE 2014-4333), April 2015. Recommendation 1: use solved problems to engage students in analyzing algebraic reasoning and strategies (minimal evidence). Includes study of incorrect solved problems.

Everyday picture. A coat hanger or a sketch of a balance. One pan holds a bag labelled x and five unit weights. The other pan holds twelve unit weights. The hanger is level. That means the two sides are the same. If you take five units off the left, the hanger tips unless you take five off the right. Same removal, both sides. The bag now equals seven. That is not a trick. That is equality preserved.

A second kitchen picture, still ordinary. Three identical packs of crackers cost twenty-one dollars together. Each pack costs the same. $3x = 21$. Divide both sides by 3. Each pack is seven. Check: three packs at seven is twenty-one. Kitchen motivates. The written equation and the check still happen.

Precise picture. An equation is a claim that two expressions name the same value. Solving means finding the value (or values) that make the claim true, by transforming the equation into an equivalent one — same truth — until the unknown is alone. Every transforming step is an operation applied to both sides, or an equivalence that preserves truth. "Move the 5 and flip" is informal speech for subtracting 5 from both sides. Teach the both-sides story first. Informal speech can arrive later, named as a nickname for the legal move.

Study a solved problem before racing blanks. Here is a correct one, narrated:

$x + 5 = 12$

Subtract 5 from both sides: $x + 5 - 5 = 12 - 5$

Simplify: $x = 7$

Check: $7 + 5 = 12$. True.

Here is an incorrect solved problem you should be able to hear — an autopsy, not a gotcha:

$2x + 3 = 11$

Someone writes: $2x = 11 - 3$ (legal so far if they subtracted 3 from both sides)

Then: $x = 8 \div 2$ wait — they wrote $x = 8$ and then divided by 2 on only one side, or they divided only the 8 and forgot the structure. Or they "moved the 2" and got $x = 8 - 2 = 6$. The illegal step broke equality. The student's job is to find where the sameness broke. Star's first recommendation is exactly this kind of analysis: look at solved work, including wrong work, and talk about the reasoning.[50]

[50] Jon R. Star et al., *Teaching Strategies for Improving Algebra Knowledge in Middle and High*

True/false number sentences with integers and fractions keep the relational equal sign alive. `−3 + 5 = 2` is true. `1/2 + 1/4 = 3/4` is true. `3x = 12` when `x = 5` is false. Checking a solution means substituting back into the original equation, not into the last line you like.

Wrong answers you should be able to hear

1. *"Move the 3 to the other side and flip the sign," offered as the first and only story, with no both-sides narration.* The student may get lucky on one-step items and freeze on two-step. Ask: "What did we do to both sides?" Put it on a balance sketch.
2. *An operational equal sign: treating = as "the answer comes next," so `x + 5 = 12` becomes "x plus 5, answer 12," with no sense that both sides are already a claim.* True/false sentences with expressions on both sides. Check by substitution every time.
3. *Dividing only one side, or subtracting from only one side, mid-solution.* Equality broke. Point at the illegal line. Ask which step was not done to both sides.
4. *Keyword theater on a word equation: "altogether means add," then an equation that does not match the story.* Sort the story type first (Chapter 8). Then write the equation. Then solve.
5. *Refusing to check, or checking only the last line.* Substitution into the original is the discipline. If the check fails, the solution is not done.

A sixth you will also hear: racing to isolate the letter before naming what the expression on each side says. Slow down. Chapter 4's question still helps: "What is this expression saying?" Then: "What did we do to both sides?"

Five-minute parent warm-up

Do this before the lesson, on a scrap of paper, no student in the room.

Minute 1. Draw a balance. Write `x + 5 = 12`. Say out loud: "Same amount on both sides." Subtract 5 from both sides on the sketch. Write `x = 7`. Check: `7 + 5 = 12`.

School Students (NCEE 2015-4010 / also cited as NCEE 2014-4333), April 2015. Recommendation 1: use solved problems to engage students in analyzing algebraic reasoning and strategies (minimal evidence). Includes study of incorrect solved problems.

Minute 2. Solve `3x = 21` by dividing both sides by 3. Narrate: "Same division, both sides." Check.

Minute 3. Look at a wrong solved problem you invent: `2x + 4 = 10`, someone writes `2x = 10`, then `x = 5` — they "moved the 4" without subtracting from both sides consistently, or they subtracted 4 from only the right. Mark the illegal step. You are installing the miss so you can hear it tomorrow.

Minute 4. True or false, said aloud: `−2 + 7 = 5`. `1/4 + 1/4 = 1/2`. If `x = 4`, is `2x + 1 = 9` true? Practice the hearing.

Minute 5. Write the sentence you will actually say: "What did we do to both sides?" Under it: "Show me on the balance. Check by putting the number back in." Put the pencil down. Those sentences are the lesson.

If you can do those five minutes, you are ready to sit down. The student writes the steps. You hear.

How to teach it this week

A good math hour this week has a shape. The math-hour chapter in the front of this book is the full version. Here is the shape scaled to this idea.

Warm-up (2–5 minutes, unaided). Two true/false number sentences with integers or fractions the student already knows. One non-canonical equation such as `12 = x + 5`. Paper. No device. No photo-to-key.

Short model (3–7 minutes). One new idea, one balance sketch, one narrated solution. `x + 5 = 12`. Subtract 5 from both sides. Check. Then show one wrong solved problem for thirty seconds and mark the illegal step together. You talk for a few minutes. Then you stop.

Student attempt (8–15 minutes). One to four items of *today's type* — one-step (11–12) or two-step (13–14). The student writes every line. The student narrates. You wait. Struggle before rescue: ask, wait, hint ("same to both sides — show me"), then a short model on *your* scrap, not as their work.

One good question, then wait. "What did we do to both sides?" A slow three. After they stop, wait again. Look at the balance or the two sides of the equation, not at the student's face, if the silence is hard. Stahl's think-time is a classroom convention about three seconds; Rowe's windows were science class.[51] Use the pause. Neither is a homeschool trial of equations.

Mixed practice (5–10 minutes). Yesterday's expression from Chapter 4 next to today's equation. A true/false next to a one-step. Mixing is how the student learns *when* to solve and when to evaluate.

Exit ticket (2–4 minutes). Solve one equation. Check by substitution. One item from last week. Done-enough is right, or wrong-with-a-reason we can use tomorrow.

Exact wording you can say

"What did we do to both sides?"

"Why was that legal?"

"Show me on the balance."

"Check by putting your number back into the original equation."

"Which step in this solved problem broke equality?"

"You may change your mind."

When they say "move the 3":

"Tell me the both-sides version of that move."

When they skip the check:

"Does that answer make the original equation true?"

When you are about to take over:

"Your pencil. I'll wait."

Age-band moves: 11–12 / 13–14

11–12. One-step equations with whole numbers, then integers, then simple fractions: `x + 5 = 12`, `x — 3 = 10`, `3x = 21`, `x/4 = 5`. Maintain equality: add,

[51] Mary Budd Rowe, "Wait-Time and Rewards as Instructional Variables," NARST, April 1972, ERIC ED061103 — elementary science class, not a mathematics RCT. Robert J. Stahl, "Using Think-Time and Wait-Time Skillfully in the Classroom," ERIC Digest ED370885, 1994 — about three seconds as a convention.

subtract, multiply, or divide both sides by the same number (nonzero for multiply/divide). True/false with expressions on both sides. Balance sketch or hanger. Check every time. Word equations that are already one-step once the story is clear. Systems wait as an age-14 edge. `f(x)` is not required.

13–14. Two-step and multi-step linear equations: `2x + 3 = 11`, `3(x — 2) = 15`, equations with variables on both sides such as `2x + 1 = x + 7`. Narrate each legal move. Study incorrect solved problems weekly. Choose among strategies once some competence exists — Star notes that flexibility lands better after some procedural knowledge is in place.[52] Systems of two linear equations: age-14 edge only, labelled, short. If the year's work is honestly Algebra I by skill, systems can deepen; they are not this chapter's default spine for every fourteen-year-old.

A fourteen-year-old who still treats = as "the answer comes next" is still in the 11–12 band of this chapter for the equal sign, whatever the birthday. An eleven-year-old who can narrate `3x = 21` and check is not "too young" for same-to-both-sides.

First try-it for the student

On scrap paper, or with a hanger if you have one:

Write `x + 5 = 12`.

Say: "What did we do to both sides?" — after they subtract 5 from both sides, or after you model once and they try a twin item.

Wait. If they say "I moved the 5," ask for the both-sides sentence. If they get `x = 7` and shrug, ask them to check. If the check fails, find the broken step together.

Later the same week, the autopsy:

Show a wrong solution to `2x + 4 = 10` that subtracts 4 from only one side or divides incorrectly. Ask which step broke equality. Do not scold. Name the miss. Repair with same-to-both-sides.

How to fade help. First sitting: you draw the balance, you subtract from both sides, they echo the narration. Second: they write the steps, you wait, you hint ("both sides"). Third: they solve and check without the sketch. Fourth: a wrong solved problem autopsy without your mark first. The word *equivalence,* if it arrives at all, arrives after the both-sides habit.

[52] Star et al. (2015), Recommendation 3: intentionally teach students to alternate among different strategies (moderate evidence); panel notes flexibility is supported once some competence exists.

When to stop talking. When you hear yourself filling in the next line. When the sit has become a lecture titled *Properties of Equality*. When the student is mid-check and you have already asked a second and a third question. One good question beats five. Stop while they still have an equation left in them.

Practice that actually builds learning

Blocked, for a new move. The day you introduce subtract-from-both-sides, only that family, small numbers, balance visible. The day you introduce a wrong-solved autopsy, only autopsy. Mixing too early makes the student hunt for "the trick."

Mixed, for when to use it. Later the same week, a one-step next to an expression to evaluate next to a true/false. Mixing is the practice of choosing: solve, evaluate, or judge true/false?

One incorrect example to diagnose. "Move the 3 and flip" with no both-sides story, or dividing only one side. Hear the missing meaning. The repair is a better question, then the balance, then — if needed — a named fact: "Same operation to both sides. Show me." Silence in the face of an illegal step is not kindness.

These are illustrations, not reported families.

Named try-it: Hanger or drawn balance (in-home)

Time. Fifteen to twenty minutes. Then stop.

Materials. A coat hanger and two paper cups, or a sketch of a balance; scrap paper; unit tokens (pennies, cubes).

Safety. Ordinary table. No heavy hanging objects over a head. Hanger as math object, not a weapon.

The fun. Keeping it level. Being the person who names the legal move.

The skill. Same to both sides; name the property; check. Eleven-to-twelve: one-step. Thirteen-to-fourteen: two-step on the sketch, then on paper.

Say: "What did we do to both sides?" Load `x + 5` against `12`. Remove five from both sides. Write the lines. Check. Kitchen table. The equation still gets written.

The check still happens.

Named try-it: Wrong solved problem autopsy (in-home)

Time. Ten to fifteen minutes.

Materials. One incorrect worked example you wrote ahead (or last night's real miss, cropped to the math).

Safety. Tone is curious, not courtroom. No shaming. No screenshot of a classmate.

The fun. Finding the illegal step. Detective work.

The skill. Star's solved-problem habit; structure talk; relational =.

Offer a wrong solution. Ask which step broke equality. Have the student rewrite the legal version and check. Do this once or twice a week while equations are new — not as a daily humiliation.

Named try-it: Two-step equation from a store story (out-of-home $\rightarrow$ table)

Time. Five minutes collecting numbers outside; ten to fifteen minutes solving at home.

Materials. A receipt or shelf prices written on a phone note; scrap paper at the table.

Safety. Ordinary aisle courtesy. No public quiz of a cashier. No shoplifting a "test." Price tags are numbers, not a sermon about a food system.

The fun. "We saw it on the shelf — now it becomes an equation."

The skill. Multi-step (13–14); check by substitution. Eleven-to-twelve can collect a one-step story: three packs for twenty-one dollars.

Example shape (illustration, not a reported trip): a flat fee plus a per-item cost, or a total after a simple discount the student already understands as percent from Chapter 1–2 work. Write the equation at home. Solve. Check. If percent meaning is still shaky, keep the story additive until percent is ready — do not use “tax then discount” as a trap.

Named try-it: Ramp as equation edge (out-of-home)

Time. Ten minutes looking; short write-up at the table.

Materials. Eyes; optional tape measure; scrap.

Safety. No climbing stunts. Sidewalk or park ramp you already use. Roads.

The fun. “What stays equal?”

The skill. Bridge toward linear thinking (Chapter 6 owns slope as rate). Keep slope-formula theater for Chapter 6. Notice that rise and run can sit in a proportion or a simple equation about equal ratios. Hand the steepness graph to Chapter 6.

Talk box

Opening question (locked): “What did we do to both sides?”

Follow-ups

1. Why was that legal?
2. Show me on the balance (or on both sides of the equation).
3. Check by substitution — does that answer make the original true?
4. Which step in this solved problem broke equality?
5. You may change your mind. Try that step another way.

How to wait

Ask. Count a slow three in your head. Look at the two sides, not at the student’s face, if the silence is hard. After they stop talking, wait again. If they are mid-reason, do not cut them off. Three seconds is a convention, not a sacrament.

What a stuck silence usually means

They think = means “the answer comes next.” Or they reached for “move and flip” as magic. Or wait-time after the question was zero. Or they are hunting a keyword on a word equation. Or the numbers or the signed structure are too new. Or they are guessing what you want. Next move: smaller numbers, balance sketch, wait, point back at both sides. "Show me the same move on both sides" is a hint. "x equals 7" said by you is not a hint; it is the grab. Done enough this week: a spoken narration. "I subtracted 5 from both sides. Check: 7 + 5 = 12."

For the student

This page is for you.

What did we do to both sides?

An equation says two sides are the same. When you change one side, you change the other the same way, or the sameness breaks.

`x + 5 = 12` means a number, plus five, is the same as twelve.

Subtract five from both sides. Now `x = 7`.

Check: put 7 back in. `7 + 5 = 12`. True. If the check fails, something broke along the way. Find the step.

"Move the 5 and flip the sign" is a nickname some people use later. The real move is: same subtraction, both sides. Learn that story first.

A tiny worked example

`3x = 21`

Both sides are the same. Divide both sides by 3.

`x = 7`

Check: `3 × 7 = 21`. True.

Someone "moves the 3" and writes `x = 21 — 3 = 18`. That broke the meaning. Division was the legal undo for multiplication, and it had to hit both sides.

Two tries

1. Solve `x — 4 = 9`. Write every line. Say what you did to both sides. Check.
2. Solve `2x + 3 = 11` if you are ready for two steps — or another one-step if two-step is still new. Narrate. Check. Then look at a wrong solution your

parent shows you and mark the illegal step.

Explain it back

Tell someone at the table what "same to both sides" means. Show `x + 5 = 12` on a balance sketch or with two columns. Say what "move and flip" is nicknaming, when it is legal at all.

Challenge

Someone always isolates the letter without checking. What question do you ask them? Someone treats = as "here comes the answer" and will not accept `12 = x + 5` as a real equation. How do you show both sides are already a claim?

You are allowed to struggle. You may use a hanger, a sketch, scrap paper. You write. You talk. If you get stuck, ask for a hint — not the finished number. Then try again.

When you talk, a sentence about both sides is enough for today. You do not have to call yourself an Algebra I student. Today you solve, narrate, and check.

A picture a computer made of a perfect balance is a scene. It is interesting. It is not the equation on your scrap.

If it isn't clicking

Three diagnostics. Each one has a next move. None of them is a verdict on talent, and none of them is a reason to wait for a birthday.

1. The student treats = as "the answer comes next," or solves without ever checking the original equation.

The operational view is doing the work of the relation. Next move: true/false sentences with expressions on both sides. `12 = x + 5` as a normal item. Check by substitution every time for a week of short sits. Stay here if this is still the bottleneck. Multi-step worksheets will not hold on top of a green-light equal sign. A human who will sit with both sides and wait, not a solver app, is a reasonable next step if you have tried the true/false work and the operational reading is still the whole hour.

2. The student says "move and flip" and cannot narrate same-to-both-sides, or divides/subtracts from only one side.

The shortcut arrived without the meaning. Next move: balance or two-column sketch only, for several days. Every line named: "I subtracted 5 from both sides." Autopsy one wrong solved problem. Slow down letter-isolation until the narration is automatic. Go ahead once they can solve a one-step, narrate, and check — then add a second step. A tutor is useful if illegal one-sided moves remain the default after a couple of weeks of daily both-sides work *and* the hour has become a fight.

3. The student freezes on word equations, or every sitting ends in a shrug, and the silence after your question is a wall.

The wait was zero, or they are hunting a keyword, or the story type is unclear, or the arithmetic inside the equation is too new. Next move: shorter sits, numbers they already compute, the opening question from the talk box, wait a slow three twice. Sort the story (Chapter 8) before solving. If silence is hard, you look at the balance, not at them. If you hear yourself writing the next line, you have started doing the work. If the hour has become a fight after a stretch of daily short sits, a human who will wait, not a chatbot that fills the silence, is the release valve.

When to slow down. Operational = still in the room. One-sided moves still the default. Checks skipped. Sits so long that narration never starts. Those are brakes. "Not ready" because of age is the brake this book will not use. "Not ready" because equality is not yet preserved on purpose is a real brake. An Algebra I workbook cover is not a reason to skip the balance.

When to go ahead. The student can solve a one-step (11–12) or two-step (13–14), narrate each legal move, and check by substitution. A wrong solved problem has been autopsied. True/false work has kept relational = alive. Short sits are ordinary. Then linear thinking in Chapter 6 has somewhere to sit. Being "good at algebra" because a student is fast at moving letters is not a reason to skip both-sides narration.

When to get a human tutor. You have run the balance, or the autopsy, or the wait after the question, for a stretch of daily sittings, and the same diagnosis is still the one in the room, and the hour has become a fight. A tutor is a release valve, not a failure of the sitting. Keep yourself as the person who can still hear "move the 3" as a missing both-sides story. Outsourcing the hearing is the thing to avoid, not asking for help.

Correct an illegal step without crushing the attempt. "I hear move-and-flip. Show me that as a both-sides move." Then look. Hearing a wrong answer, asking a better question, then naming a better move if needed, is teaching.

Tools, including AI

Optional helpers for you, the adult. The full rules live in the math-hour box in the front of this book. This chapter is a pointer, not a second policy paper.

The student attempts first. You hold the equation and the question. A tool may explain today's idea *to you* from this chapter, suggest extra isomorphic equations you then vet (answer key held by you), write a short SCRIPT for *you* to say after the student has tried, offer a hint after an attempt, or help you diagnose work the student already produced. Crop to the paper. Do not upload the student's face. The student never sees the key.

Ages 11–12: parent holds the account. Ages 13–14: parent co-holds; still parent in the room. Turning 13 is not a licence. Unaided first. SCRIPT after the try.

Leave these out of the hour: a chatbot as the only partner; a tool that completes the worksheet; a camera pointed at the page so a solution pops up for the exact problem; a detector score; a certificate of algebra-readiness; a cloud agent as a math partner. Photo-to-answer is the ban.

A language model will happily "move the 3." Treat every model-supplied solution as untrusted until the student has narrated both sides and checked. In a high-school math study, an unguarded chatbot made practice look better and left students worse when the window was closed. That paper is high-school mathematics, closer than a 5–10 trial, still not an 11–14 RCT.[53] If the tool produces the 7, the student is a spectator.

A hanger, a balance sketch, scrap paper, and a wrong solved problem you wrote yourself are tools too. Use them, then fade them.

What "done enough" looks like

Placement is by skill, not birthday. A "grade 7 math workbook" is a publisher's scope, not a legal grade, and not a transcript line. The record, when you need one, is a dated notebook, the task named, plus an exit ticket, plus one diagnostic item.

[53] Hamsa Bastani, Osbert Bastani, Alp Sungu, Haosen Ge, Özge Kabakcı, and Rei Mariman, "Generative AI without guardrails can harm learning: Evidence from high school mathematics," *Proceedings of the National Academy of Sciences* 122, no. 26 (2025): e2422633122. High-school mathematics, not an 11–14 RCT. Kitchen-table rule it supports: the model may prepare the adult and the next problem; it may not do the student's problem.

The title a stranger can read is Mathematics, or Pre-Algebra, or Algebra I — only if the year's work was that course. It is not Young Minds Math I. It is not a brand name as the credit.

Checklist before moving on

- You can hear "move the 3" with no both-sides story, and you can ask what was done to both sides.
- The student can solve a one-step equation (11–12) or a two-step equation (13–14), narrating each legal move.
- Every solved item that counts as done has been checked by substitution into the original.
- True/false work with expressions on both sides has appeared in the same week.
- At least one wrong solved problem has been autopsied: the illegal step named and repaired.
- A balance sketch or hanger has appeared; kitchen or store stories motivated, but did not replace the written equation.
- At least one out-of-home number collection (store story or ramp look) landed on paper at the table.
- 13–14, as it holds: variables on both sides or simple multi-step; systems only if labelled as an edge you chose on purpose.
- Sits can be short. A spoken both-sides narration plus a check is enough. You did not require a full Algebra I unit as the proof.
- You can hear operational =, one-sided moves, keyword theater, and skipped checks, and you can ask a good question, without taking the pencil.

If most of that list is true, go on to early functions and linear thinking, even if the birthday says otherwise. If the birthday says "Algebra I" and equality is still not preserved on purpose, stay. The next chapter asks for input and output. It needs same-to-both-sides underneath.

A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

Chapter 6

Early functions and linear thinking

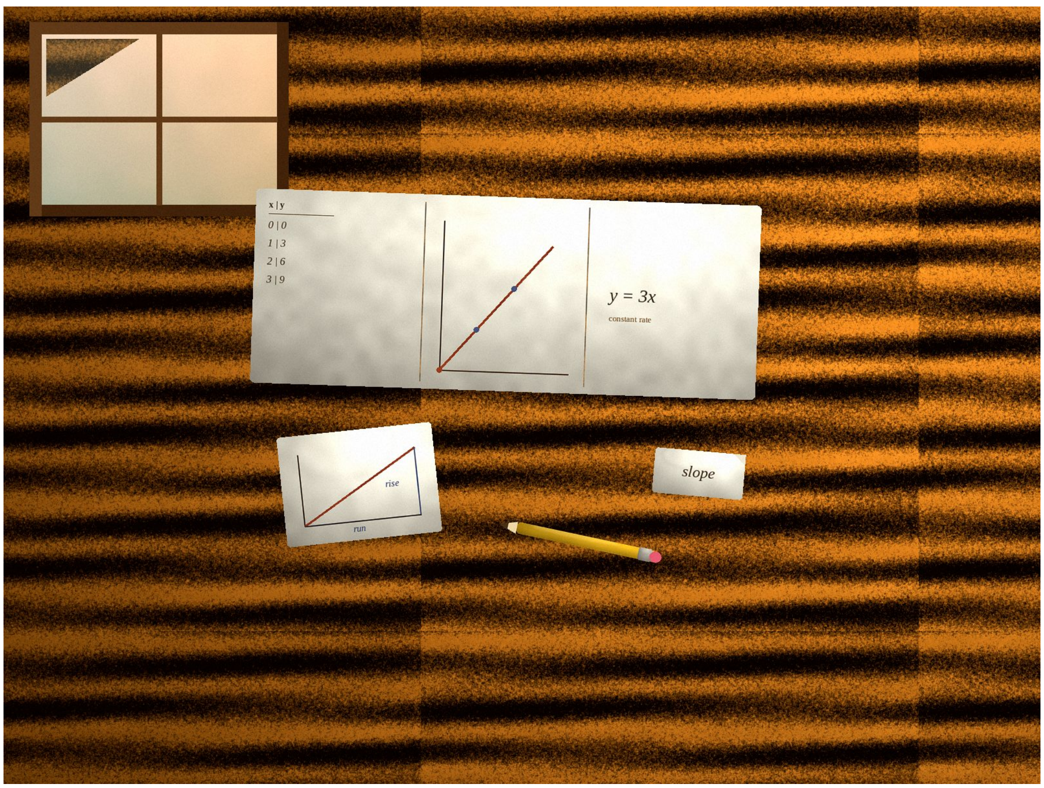

Figure 9: A kitchen-table still-life: three panels on scrap paper — an input–output table, a simple graph through the origin, and y = 3x; a small rise-over-run sketch. No people. No logos.

Why this matters

What is the input, and what comes out?

That question is how functions arrive in this band — as a machine you can fill, not as a vocabulary list. You put a number in. Exactly one number comes out. A table shows the pairs. A graph shows the shape. An equation names the rule. The same relationship can wear all three outfits. The student's job this week is to move among them and say what stays the same.

Constant rate is the bridge from Chapter 2's ratio tables. If every input is multiplied by the same number to get the output, you have a proportional relationship: a line through the origin when you graph it, a unit rate you can read as steepness, a rule that looks like `y = 3x` or "output is three times input." Slope, in ordinary language, is how steep that line is — rise over run, change in output over change in input. You do not need the word *slope* on day one. You need the rate.

Function notation `f(x)` is **not required** in this band. The common Grade 8 map says so plainly: a function assigns to each input exactly one output; students translate among representations; function notation is not required.[54] If your family likes the symbol later, label it as an Algebra I edge — vocabulary after the idea, not instead of the table. A full Algebra I functions unit is not this chapter's spine. Domain-and-range worksheets are not the spine either. The quadratic formula waits for a later course. Age-14 edges are labelled only.

What this idea unlocks is linear thinking with eyes open. Equations from Chapter 5 asked you to keep both sides the same. Functions ask you what depends on what. Proportional relationships sit inside linear ones as the special case that passes through the origin. Sports rates, ramps, phone plans, and recipe scales all become rules you can test: one input, one output, constant rate or not.

Why teach this *now*? Because ratio tables already built the habit of "what stays in the same ratio." Because NMAP wanted more students ready for authentic algebra in the middle grades — and readiness is skill, not birthday.[55] Because

[54] National Governors Association Center for Best Practices and Council of Chief State School Officers, *Common Core State Standards for Mathematics* (2010), Grade 8 critical area on functions: a function assigns to each input exactly one output; translate among representations; "Function notation is not required in Grade 8." Used as a map, not a homeschool statute.

[55] National Mathematics Advisory Panel, *Foundations for Success* (2008): Critical Foundations for algebra; Finding 15 on continuity of topics; readiness for authentic algebra is skill-based.

Star's algebra guide still wants structure and strategy choice on linear work: look at how a representation is built, not only at how fast a letter moves.[56]

This book is not a reprint of *Math for Little Thinkers*. That manual stops at ten. One pointer, then teach input–output here. This book is not a reprint of *Mathematics for Homeschooling*, which ran through high-school functions after a compressed middle-grades chapter. One pointer, then teach early functions at this grain. This book is not a sequel to *Critical Thinking for Young Minds*. One pointer, then teach the math.

You do not need to be a mathematician. You do need to hear `f(x)` pushed as the first move before any table exists, and to ask for input and output on scrap paper, without taking the pencil.

This week you can learn to hear vocabulary-first function talk and "any graph is a function." Today the student can complete a table for a constant-rate rule and sketch whether the relationship could be proportional.

For the parent: understand it yourself

Many adults feel rusty on why a table is already a function story. That is ordinary. School memory often jumps to `f(x) =` and a vertical-line test poster. Five minutes of this section, then the warm-up, is enough for tomorrow. The student still builds the table.

Everyday picture. A snack machine that always charges the same way: pay two dollars, get one pack; pay four dollars, get two packs; pay six dollars, get three packs. Input: dollars (or number of packs). Output: packs (or dollars). Each input has exactly one output under the rule. If somehow "pay four dollars" could spit out either two packs or three packs under the *same* rule, the machine would not be a function. Real machines are more boring than that. Math uses the boring version on purpose.

A second kitchen picture. A recipe: three cups of flour for every two loaves. Input loaves, output flour — or the other way — as long as you stay consistent. The ratio table from Chapter 2 is already an input–output table. Graph the pairs. If the rate

[56] Jon R. Star et al., *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* (NCEE 2015-4010), April 2015. Structure of algebraic representations (Rec 2, minimal evidence); alternative strategies (Rec 3, moderate evidence).

is constant and the graph goes through the origin, you are looking at a proportional relationship.

Precise picture. A function is a rule that assigns to each allowed input exactly one output. In this band, show it as:

- a table of pairs
- a graph of those pairs
- an equation or verbal rule (`y = 3x`, “output is three times input”)

Proportional relationships are linear relationships that pass through `(0, 0)` and have a constant rate of change. That rate is the constant of proportionality — the unit rate — and it is the steepness of the line. Comparing two rates is comparing two steepnesses. A line that does *not* go through the origin can still be linear (constant rate) but is not proportional: `y = 3x + 2` has a flat fee plus a per-unit charge. Name that difference when the student is ready (often 13–14). Do not pretend every line is a proportion.

`f(x)` means “the output when the input is `x`.” It is a naming habit. It is not the idea. If the family insists on seeing it, write `f(x) = 3x` next to the table once — as edge vocabulary — then go back to “input / output.”

Wrong answers you should be able to hear

1. *`f(x)` demanded before any table or rule exists.* Vocabulary theater. Ask: “What is the input, and what comes out?” Build three rows first.

2. *Any squiggle on a graph called a function, or any table called “not a function” because it lacks the letter f.* Test the definition: each input $\rightarrow$ exactly one output. A table with one input paired to two different outputs fails. A graph that fails a vertical-line idea fails — you can say that in plain language without a poster sermon.

3. *Confusing “proportional” with “any line” or with “any ratio word problem.”* Proportional: constant rate *and* through the origin (or equivalent). A phone plan with a signup fee is linear, not proportional. A story that is additive compare is not a proportion (Chapter 8).

4. *Cross-multiply on a rate table with no sense of unit rate or steepness.* Return to Chapter 2’s representation. Then ask what the graph’s steepness means.

5. *Reading a sports average as if it predicts the next play for sure.* Rate language is useful; it is not a promise. Hand deeper “what the average hides”

talk to Chapter 8, but you can plant the question now.

A sixth you will also hear: treating slope as a formula card (`m = (y2 − y1)/(x2 − x1)`) with no rise-over-run meaning. Walk a ramp. Name rise and run. Then write the ratio.

Five-minute parent warm-up

Do this before the lesson, on a scrap of paper, no student in the room.

Minute 1. Make a two-column table: input 0, 1, 2, 3; rule "output = 3 × input." Fill it. Say: "Each input has one output."

Minute 2. Plot the points roughly. Notice the line through the origin. Say: "Unit rate is 3. That is the steepness."

Minute 3. Make a second table: `output = 3 × input + 2`. Plot. Same steepness, not through the origin. Say: "Linear, not proportional."

Minute 4. Invent a broken table: input 2 paired with both 6 and 7. Say: "Not a function — two outs for one in."

Minute 5. Write the sentence you will actually say: "What is the input, and what comes out?" Under it: "Show me the table, the graph, and the rule — same relationship?" Put the pencil down. Those sentences are the lesson.

If you can do those five minutes, you are ready to sit down. The student fills the table. You hear.

How to teach it this week

A good math hour this week has a shape. The math-hour chapter in the front of this book is the full version. Here is the shape scaled to this idea.

Warm-up (2–5 minutes, unaided). Complete two columns of a ratio table the student already knows. One question: "What stays in the same ratio?" Paper. No device.

Short model (3–7 minutes). One input–output rule. Fill three rows. Sketch the points. Name the unit rate as steepness. If 13–14, contrast a through-origin rule with a plus-flat-fee rule for one minute. You talk briefly. Then you stop.

Student attempt (8–15 minutes). Complete a table for a constant-rate rule. Sketch. Decide whether it could be proportional. You wait. Struggle before rescue: ask, wait, hint ("one output for each input — show me"), then a short model on *your* scrap.

One good question, then wait. "What is the input, and what comes out?" A slow three. After they stop, wait again. Look at the table or the graph, not at the student's face, if the silence is hard.

Mixed practice (5–10 minutes). An equation check from Chapter 5 next to today's table. A ratio-table item next to a "is this a function?" table. Mixing is how they learn *when* the rule is a function and when the rate is constant.

Exit ticket (2–4 minutes). One "is this a function?" table. One constant-rate sketch. Done-enough is right, or wrong-with-a-reason we can use tomorrow.

Exact wording you can say

"What is the input, and what comes out?"

"Is there exactly one output for each input?"

"Show the table, the graph, and the equation — same relationship?"

"What is the unit rate / steepness?"

"Does this graph go through the origin? What does that tell you?"

"You may change your mind."

When they demand `f(x)` first:

"We'll name that later if we want. Table first."

When they call every line proportional:

"Same steepness — does it pass through zero-zero?"

When you are about to take over:

"Your table. I'll wait."

Age-band moves: 11–12 / 13–14

11–12. Light functional language via ratio tables. Input–output machine tables with whole-number rules. "Each input $\rightarrow$ one output." Graph a few points on a coordinate grid they already know. Constant rate as unit rate. Proportional: through the origin, same ratio. `f(x)` is not required. Domain/range drills wait. Keep rules simple: `y = 2x`, `y = 5x`, "add 4 each time" as a pattern that is *not* the same as multiply-by-a-constant — name the difference when it appears.

13–14. Serious work: identify proportional vs not; constant of proportionality; slope as rate of change in plain language; translate among table, graph, equation; compare steepness; linear rules with a starting value (`y = mx + b` language as edge if useful — the idea matters more than the letters). "Is it a function?" from tables and simple graphs. Formal `f(x)`, formal domain/range batteries, and quadratics: Algebra I edges labelled only — a peek, not a course dump. Systems and function notation can appear as short labelled edges if skill placement already supports Algebra I; birthday alone does not.

A fourteen-year-old who cannot complete a constant-rate table is still in the 11–12 band of this chapter for functions, whatever the birthday. An eleven-year-old who can fill a ratio table and say what comes out is already doing the work.

First try-it for the student

On scrap paper:

Make a table. Input: 0, 1, 2, 3, 4. Rule: output is three times input.

Say: "What is the input, and what comes out?"

Wait. If they fill correctly, ask whether the relationship could be proportional and why. If they want to write `f(x)` first, redirect to the table. If two different outputs appear for one input, ask whether that rule is a function.

Later the same week: a table that fails the "exactly one output" test, and a ramp or stairs rise-over-run look (below).

How to fade help. First sitting: you name input and output columns, they fill. Second: they choose the rule, you wait. Third: table plus sketch without your axes drawn. Fourth: "function or not?" without a hint. The symbol `f(x)`, if it arrives, arrives after three clean tables.

When to stop talking. When you hear yourself lecturing on the vertical-line test for ten minutes. When the sit has become a vocabulary quiz. When the student is mid-row and you have already asked a second and a third question. One good question beats five. Stop while they still have a row left to fill.

Practice that actually builds learning

Blocked, for a new move. The day you introduce input–output, only fill-the-table. The day you introduce "function or not," only that judgment. Mixing too early makes them hunt for the letter f.

Mixed, for when to use it. Later the same week, a ratio table next to a function-or-not table next to an equation check. Mixing is the practice of choosing: rate, function test, or solve?

One incorrect example to diagnose. A student writes `f(x)` on a blank page with no pairs, or calls `y = 3x + 2` proportional because it is a line. Hear the miss. Ask for the table. Ask about the origin. Silence in the face of vocabulary theater is not kindness.

These are illustrations, not reported families.

Named try-it: Input–output machine table (in-home)

Time. Fifteen minutes. Then stop.

Materials. Scrap paper; ruler optional.

Safety. Ordinary table.

The fun. "What comes out?" Guessing the rule from three pairs, then testing a fourth.

The skill. Each input $\to$ exactly one output; not a function if two outs. Eleven-to-twelve: whole-number multiply rules. Thirteen-to-fourteen: include a non-example and a linear-with-intercept rule.

Say: "What is the input, and what comes out?" Fill. Test one new input. Then offer a broken table (one in, two outs) and ask whether it is a function. Kitchen table. The table still gets written. The graph can wait five minutes or come the next day.

Named try-it: Graph from a ratio table (in-home)

Time. Fifteen to twenty minutes.

Materials. Ratio table already filled; grid paper or hand-drawn axes.

Safety. Ordinary.

The fun. Watching steepness appear. Comparing two rates on the same axes.

The skill. Constant rate; line through origin vs not (13–14). Unit rate as steepness.

Plot. Ask what the steepness means in the situation. If two recipes or two plans are on the table, ask which is steeper and what that says about the unit rate. Kitchen motivates. The graph and the equation still appear.

Named try-it: Ramp or stairs rise-over-run (out-of-home)

Time. Ten to fifteen minutes on site; short write-up at home.

Materials. Optional tape measure; scrap; shoes you already walk in.

Safety. No climbing stunts. No standing in traffic. Use a ramp or stair flight you already use. Measure from a safe stance.

The fun. "How steep?" Comparing the porch ramp to the playground ramp (or two stair flights).

The skill. Slope as ratio toward linear expressions; rise over run as a rate.

Record rise and run. Write the ratio. Ask: if this were a graph, what would the steepness be? Hand formal slope-intercept theater to an Algebra I edge later if skill placement earns it. Leave formula cards off the sidewalk.

Named try-it: Sports rate as function talk (out-of-home)

Time. During a game or meet you were already watching; five minutes of talk; optional table at home.

Materials. Box score, scoreboard, or a simple tally you keep.

Safety. Ordinary sideline courtesy. No lecturing a stranger's adolescent. Sports stats are ratios and rates — not identity fights.

The fun. Points per game, yards per carry, goals per match as a rule.

The skill. Rate language; input (games) and output (points). What the average hides — plant the question; Chapter 8 deepens data talk.

Ask: "If games are the input, what comes out?" Build two rows from real numbers. Ask whether the season average promises the next game. Keep it light. Land a tiny table at home if the sideline is loud.

Friday mix that earns the exit ticket. Three items only: (1) complete two missing cells in a constant-rate table; (2) mark function or not on a tiny table; (3) circle which of two sketched lines is steeper and write one sentence about unit rate. No vocabulary quiz. No photo of a worksheet sent to a solver. Date the scrap. That Friday mix is enough assessment for this idea.

When formal slope language arrives. If the student is 13–14 and already says "rise over run" with meaning, you may write `m = rise/run` once as a summary of what they already did on the ramp. If they cannot yet point to rise and run on a sketch, the formula card is early. Meaning first; letters second — the same discipline as Chapters 4 and 5.

Talk box

Opening question (locked): "What is the input, and what comes out?"

Follow-ups

1. Is there exactly one output for each input?
2. Show table, graph, and equation — same relationship?
3. What is the unit rate / steepness?
4. Does that answer make sense for the situation?
5. You may change your mind. Try another representation.

How to wait

Ask. Count a slow three. Look at the table or the graph, not at the student's face, if the silence is hard. After they stop, wait again. If they are mid-reason, do not cut them off.

What a stuck silence usually means

They want `f(x)` vocabulary before the idea. Or wait-time after the question was zero. Or they confuse any graph with a function. Or they are hunting a keyword. Or the coordinate grid is still new. Or they are guessing what you want. Next move: smaller numbers, three-row table, wait, point back at input and output columns. "Show me one in and one out" is a hint. "The function is y = 3x" said by you before they build is not a hint; it is the grab. Done enough this week: a completed table and a sentence. "Input is packs; output is dollars; each input has one output."

Comparing two rates on purpose. Put two phone plans or two walking speeds on the same axes — as an illustration, not a brand pitch. Plan A: $3 per gigabyte, no fee. Plan B: $2 per gigabyte plus a $5 monthly fee. Same input column (gigabytes). Different outputs. Ask which is steeper. Ask at what input they cost the same — that crossing is a Chapter 5 equation in disguise: solve `3x = 2x + 5`. Functions and equations talk to each other. You do not need a full systems unit to notice one crossing.

Patterns that are not proportional. "Add 4 each time" starting from 1 gives 1, 5, 9, 13… That is a constant *difference*, and it is linear if you graph term number against value — but the ratio of output to input is not constant. Students who only chant "constant rate" without checking ratios will call everything proportional. Have them compute output $\div$ input for a few rows. If those quotients wander, it is not a proportional relationship even if the graph is a straight line that misses the origin, or even if the pattern feels steady.

Coordinate readiness. If plotting points still fights the sitting, spend two short warm-ups on `(x, y)` pairs from Chapter 7's coordinate work or from any grid you already use. Functions do not require artistic graphs. Three plotted points and a sentence about steepness beat a perfect grid with no meaning.

Bridge sentence you can reuse. "Yesterday's ratio table is today's input–output table. Tomorrow's graph is the same relationship standing up." Say it once. Point at the paper. Move on.

For the student

This page is for you.

What is the input, and what comes out?

A function is a rule. You put a number in. Exactly one number comes out.

A table shows the pairs. A graph shows the shape. An equation names the rule. Same relationship, three views.

If one input somehow gets two different outputs under the same rule, that rule is not a function.

Constant rate means the output grows by the same amount for each equal step in the input — like "always three times as much." If the graph of that rule goes through the origin, the relationship is proportional. Steepness is the unit rate.

You do **not** need `f(x)` to do this. If you see that symbol later, it is just a name for "the output when the input is x." Table first.

A tiny worked example

Rule: output = 3 $\times$ input.

input	output
0	0
1	3
2	6
3	9

Each input has one output. The points line up through the origin. Unit rate is 3 — that is the steepness.

Now a broken table: input 2 $\rightarrow$ 6 and also input 2 $\rightarrow$ 7. Two outs for one in. Not a function.

Two tries

1. Make a table for "output is 5 times input" for inputs 0 through 4. Sketch the points. Is it proportional? Why?
2. Make a table for "output is 5 times input, plus 2." Sketch. Same steepness? Through the origin? Function or not?

Explain it back

Tell someone at the table what input and output mean for your table. Say how you know a rule is a function. Say what steepness means without only reciting a formula.

Challenge

Someone says you cannot talk about functions until you write `f(x)`. What do you show them first? Someone calls every straight line proportional. What question do you ask about the origin?

You are allowed to struggle. You may use a ratio table, a grid, a ramp measurement. You build. You talk. If you get stuck, ask for a hint — not the finished rule typed by a machine. Then try again.

When you talk, a sentence about input and output is enough for today. You do not have to finish Algebra I. Today you fill a table and say what comes out.

A perfect graph a computer drew is a scene. It is interesting. It is not the pairs on your scrap.

If it isn't clicking

Three diagnostics. Each one has a next move. None of them is a verdict on talent, and none of them is a reason to wait for a birthday.

1. The student wants `f(x)` or a vocabulary list before any table exists, or calls every graph a function.

Vocabulary is doing the work of the idea. Next move: three-row tables only, for several days. "Function or not?" with one clear non-example (one in, two outs). No `f(x)` until three clean tables exist. Stay here if this is still the bottleneck. An Algebra I packet will not hold on top of empty notation. A human who will sit with a table and wait, not a video that starts with `f(x)`, is a reasonable next step if vocabulary theater is still the whole hour.

2. The student fills ratio tables but cannot connect them to a graph or to steepness, or confuses proportional with any line.

The representation bridge is missing. Next move: plot the ratio table they already trust. Name unit rate as steepness. Contrast `y = 3x` with `y = 3x + 2` on the same axes (13–14). Walk a ramp for rise/run. Slow down formula cards until the graph means something. Go ahead once they can complete a constant-rate table, sketch it, and say whether it goes through the origin on purpose. A tutor is useful if the graph remains a mystery after a couple of weeks of short table-to-graph sits *and* the hour has become a fight.

3. The student freezes when numbers are messy, or every sitting ends in a shrug, and the silence after your question is a wall.

The wait was zero, or the grid is new, or they are guessing what you want, or they think this is “algebra theater” unrelated to Chapter 2. Next move: smaller whole numbers, the opening question from the talk box, wait a slow three twice. Point back at the ratio table they already own. If silence is hard, you look at the table, not at them. If you hear yourself filling the output column, you have started doing the work. If the hour has become a fight after a stretch of daily short sits, a human who will wait, not a chatbot that fills the silence, is the release valve.

When to slow down. Vocabulary-first still the default. Any-graph-is-a-function still in the room. Proportional confused with every line. Sits so long that no table gets finished. Those are brakes. “Not ready” because of age is the brake this book will not use. “Not ready” because input–output is not yet visible is a real brake. A birthday cake with “Algebra I” icing is not a placement instrument.

When to go ahead. The student can complete a constant-rate table, sketch whether it could be proportional, and judge a simple function-or-not table. Rise/run has appeared at least once. Short sits are ordinary. Then geometry with reasons (similarity ↔ slope) and data stories have somewhere to sit. Being “good at algebra” because a student can chant `f(x)` is not a reason to skip the table.

When to get a human tutor. You have run the machine table, or the ramp, or the wait after the question, for a stretch of daily sittings, and the same diagnosis is still the one in the room, and the hour has become a fight. A tutor is a release valve, not a failure of the sitting. Keep yourself as the person who can still hear `f(x)`-first as a missing-table move. Outsourcing the hearing is the thing to avoid, not asking for help.

Correct a wrong function claim without crushing the attempt. “I hear that every line is proportional. Let’s look at whether it goes through zero-zero.” Then look. Hearing a wrong answer, asking a better question, then naming a better move if needed, is teaching.

Tools, including AI

Optional helpers for you, the adult. The full rules live in the math-hour box in the front of this book. This chapter is a pointer, not a second policy paper.

The student builds the table first. You hold the question. A tool may explain today's idea *to you* from this chapter, suggest extra isomorphic tables you then vet, write a short SCRIPT for *you* to say after the student has tried, offer a hint after an attempt, or help you diagnose a table or sketch the student already made. Crop to the paper. Do not upload the student's face. The student never sees the key.

Ages 11–12: parent holds the account. Ages 13–14: parent co-holds; still parent in the room. Unaided first. SCRIPT after the try.

Leave these out of the hour: a chatbot as the only partner; a tool that completes the worksheet; photo-to-key on a graphing page; a detector score; a certificate of function-fluency; an agent as a math partner. Photo-to-answer is the ban.

A language model will happily write `f(x) =` and a polished graph description. Treat every model-supplied rule as untrusted until the student's table exists. The high-school chatbot study named in the math hour still supports the same kitchen rule: the model may prepare the adult; it may not do the student's pairs.[57]

Grid paper, a tape measure on a ramp, a ratio table, and a sports tally are tools too. Use them, then fade them.

What "done enough" looks like

Placement is by skill, not birthday. A "grade 8 functions unit" is a publisher's scope, not a legal grade, and not a transcript line. The record, when you need one, is a dated notebook, the task named, plus an exit ticket, plus one diagnostic item. The title a stranger can read is Mathematics, or Pre-Algebra, or Algebra I — only if the year's work was that course. It is not Young Minds Math I.

Checklist before moving on

- You can hear `f(x)` pushed as the first move, and you can ask for input and output in a table.

[57] Hamsa Bastani et al. (2025), high-school math generative-AI tutoring trial — not an 11–14 RCT. See math-hour AI box for the kitchen rule.

- The student can complete a table for a constant-rate rule and sketch whether it could be proportional.
- "Function or not?" has been judged from at least one clean example and one non-example (one in, two outs).
- Unit rate has been named as steepness at least once, on a graph or a ramp.
- A ratio table from Chapter 2 has been reused as an input–output table in the same week.
- Kitchen, sports, or ramp motivated; the written table and sketch still appeared.
- At least one out-of-home try-it (ramp/stairs or sports rate) happened.
- 13–14, as it holds: proportional vs linear-with-intercept distinguished; `f(x)` only if labelled as edge vocabulary you chose on purpose — not as the spine.
- Sits can be short. A completed table plus a sentence is enough. You did not require a full Algebra I functions course as the proof.
- You can hear vocabulary theater, any-graph-is-a-function, and proportional/line confusion, and you can ask a good question, without taking the pencil.

If most of that list is true, go on to geometry with reasons, even if the birthday says otherwise. If the birthday says "Algebra I" and the table is still empty, stay. The next chapter asks what must be true and how you know. It needs rates and representations underneath.

A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

Chapter 7

Geometry with reasons

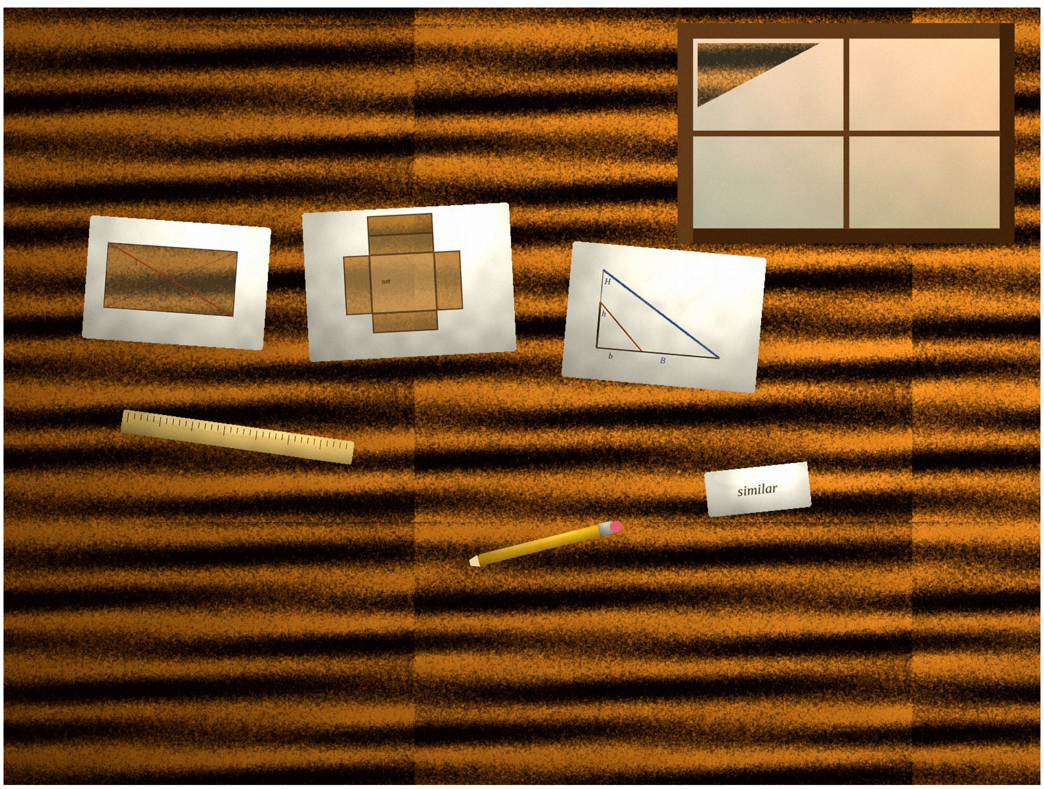

Figure 10: A kitchen-table still-life: a rectangle decomposed into triangles on scrap paper; a cereal-box net unfolded; a shadow similar-triangles sketch with corresponding sides marked. No people. No logos.

Why this matters

What must be true, and how do you know?

That question turns geometry from a packet of formula cards into a chain of reasons. Area is not only "length times width." Area is covering without gaps or overlaps — and sometimes you prove a formula by cutting a shape into pieces you already trust. Surface area is the net you can hold. Volume is packing space, including with fractional edges. Similarity is "same shape, scaled," with corresponding sides in the same ratio. The Pythagorean relationship is a claim about right triangles you can explain, not only a chant.

Formulas arrive as justified summaries of reasoning. They are welcome. They are not the first sentence of the lesson. A parent who only drills `A = ℓw` without "why these lengths" is selling a fragment. NMAP named particular aspects of geometry and measurement — properties of shapes, area and volume with understanding, similar triangles related to slope — as foundations for algebra, not as decoration.[58] The common map for grades 6–8 asks for decomposition, nets, scale drawings, angle relationships, similarity through transformations, and (at the Grade 8 edge) Pythagorean explanation plus volumes of cylinders, cones, and spheres.[59] That is this chapter's home. It is not a full high-school geometry course.

What this idea unlocks is measurement you can defend, scale drawings you can trust, and a bridge from similar triangles to the steepness work in Chapter 6. Kitchen floor plans and cardboard nets motivate. The written justification and the diagram still have to appear.

Why teach this *now*? Because ages 11–12 can already decompose a rectangle and find surface area from a net. Because ages 13–14 can connect similar triangles to slope and explain why `a² + b² = c²` holds for a right triangle before they memorize the letters. Because Woodward's problem-solving guide rates visual representations as strong evidence for grades 4–8 — diagrams that show relationships, not only pretty pictures.[60] There is no named kitchen-table geometry RCT

[58] National Mathematics Advisory Panel, *Foundations for Success* (2008), Critical Foundation 3 (geometry and measurement) and related Grade 6–7 benchmarks on properties, area/volume, and similar triangles related to slope.

[59] *Common Core State Standards for Mathematics* (2010), Grades 6–8 geometry critical areas: decomposition and nets (6); scale, circles, angles, composition (7); transformations, Pythagorean Theorem, volumes of cylinders/cones/spheres (8). Map, not statute.

[60] John Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8*

to promise you a score. Steal the moves: reason, draw, then compute.

This book is not a reprint of *Math for Little Thinkers*. Early shape names live there. One pointer, then teach middle-grades geometry with reasons. This book is not a reprint of *Mathematics for Homeschooling*, which ran through high-school geometry after a compressed middle chapter. One pointer, then teach 11–14 at this grain. Age-14 edges are labelled. Full proof-course geometry is not the spine.

You do not need to be a mathematician. You do need to hear a formula offered with no reason, and to ask what must be true, without taking the pencil.

This week you can learn to hear formula-card geometry and rotated-figure panic. Today the student can justify an area or a similarity statement with a diagram and a sentence.

For the parent: understand it yourself

Many adults feel rusty on why a triangle's area is half a related parallelogram. That is ordinary. School memory often jumps to a wall of formulas. Five minutes of this section, then the warm-up, is enough for tomorrow. The student still draws the diagram.

Everyday picture. A rectangular tabletop. You can tile it with unit squares in your mind: rows times columns. That is area by covering. Now cut the rectangle along a diagonal. Two congruent right triangles. Each has area half the rectangle — so `(1/2) × base × height` is not magic; it is the rectangle's area shared by two equal pieces. Say that out loud once. Then write the formula as a summary.

A second kitchen picture. An empty cereal box. Unfold it into a net — six rectangles if it is a right rectangular prism. Surface area is the sum of the faces you can see on the net. Volume is how much it holds: length times width times height, including when an edge is a fraction. Measure. Compute. Write one "because" sentence.

Precise picture — 11–12 grain. Find areas of triangles and special quadrilaterals by decomposing into rectangles and triangles you trust. Find surface area from nets. Find volume of right rectangular prisms with fractional edge lengths. Place polygons in the coordinate plane and find lengths of horizontal and vertical sides.

(NCEE 2012-4055), May 2012 / revised 2018. Recommendation 3: teach visual representations — strong evidence.

Every computation wears a reason: "I split the L-shape into two rectangles because…"

Precise picture — 13–14 grain. Angle relationships (complementary, supplementary, vertical, angles formed by parallel lines and a transversal) with a "because." Scale drawings. Circles: circumference and area with π as a reasoned constant, not a mystery button. Similarity and congruence through transformations — slide, flip, turn, enlarge — so "same shape" has a motion story. Similar triangles share angle measures; corresponding sides scale by the same factor; that scale factor is a ratio table's constant. Similar right triangles connect to slope: rise/run repeats. Pythagorean Theorem: in a right triangle, the square on the hypotenuse equals the sum of the squares on the legs — with an explanation (area argument, similarity argument, or a vetted visual proof you can narrate), not chant-only. Volumes of cylinders, cones, and spheres as Grade 8 completion edges: formulas as summaries after a comparison argument (cone related to cylinder, and so on), labelled when you reach them.

Wrong answers you should be able to hear

1. *Formula with no reason: "Area is length times width" on a triangle, or "just multiply" on a volume with no packing meaning.* Ask: "What must be true, and how do you know?" Require a diagram sentence.

2. *Rotated-figure panic: a triangle on a slant "doesn't have a base."* Any side can be a base; the height is perpendicular to that base. Turn the paper. Draw the height.

3. *Similarity claimed because two shapes "look alike," with no corresponding sides or scale factor.* Build a ratio table of corresponding sides. Check angles.

4. *Pythagorean chant $a^2 + b^2 = c^2$ applied to a non-right triangle, or with c not the hypotenuse.* Ask what must be true first: is there a right angle? Which side is opposite it?

5. *Net that overlaps or omits a face, then a surface-area number that floats free of the cardboard.* Rebuild the net. Count faces.

A sixth you will also hear: treating π as "3.14 always, forever, no meaning." π is the constant ratio of circumference to diameter. Compute with a sensible approximation once the meaning sits.

Five-minute parent warm-up

Do this before the lesson, on a scrap of paper, no student in the room.

Minute 1. Draw a rectangle 6 by 4. State its area with a covering sentence. Cut a diagonal. State each triangle's area as half, with a "because."

Minute 2. Sketch a cereal-box net. Label dimensions. Write surface area as a sum of faces.

Minute 3. Draw two similar right triangles that share an angle. Mark corresponding sides. Write a ratio table with one scale factor.

Minute 4 (13–14). Sketch a right triangle with legs 3 and 4. Explain why the hypotenuse should be 5 using squares on the sides (even a rough area sketch). If 11–12, skip to a coordinate rectangle instead.

Minute 5. Write the sentence you will actually say: "What must be true, and how do you know?" Under it: "Show me on the diagram. Write one because-sentence." Put the pencil down. Those sentences are the lesson.

If you can do those five minutes, you are ready to sit down. The student draws. You hear.

How to teach it this week

A good math hour this week has a shape. The math-hour chapter in the front of this book is the full version. Here is the shape scaled to this idea.

Warm-up (2–5 minutes, unaided). Name why a shape is a rectangle (or why two angles are equal) in one sentence. One quick sketch. Paper. No device.

Short model (3–7 minutes). Area by decomposition with a spoken "because," or a net unfolded, or (13–14) similar triangles with a ratio table of sides. One new idea. You talk briefly. Then you stop.

Student attempt (8–15 minutes). One area/surface item with a reason sentence, or similarity ratios on a table (13–14). Diagram required. You wait. Struggle before rescue: ask, wait, hint ("split it into shapes you know"), then a short model on *your* scrap.

One good question, then wait. "What must be true, and how do you know?" A slow three. After they stop, wait again. Look at the diagram, not at the student's face, if the silence is hard.

Mixed practice (5–10 minutes). A ratio-table skill from Chapter 2 next to today's area. An equation check next to a scale factor. Mixing is how they learn *when* geometry needs a ratio and when it needs a decomposition.

Exit ticket (2–4 minutes). Reason sentence plus compute. One item from last week. Done-enough is right, or wrong-with-a-reason we can use tomorrow.

Exact wording you can say

"What must be true, and how do you know?"

"Which lengths correspond?"

"Show the ratio table of sides."

"Why multiply these for area?"

"Walk the angle — what matches when you turn it?"

"You may change your mind."

When they recite a formula with no diagram:

"Write the because-sentence first. Then the number."

When a rotated triangle freezes them:

"Choose a base. Draw the height perpendicular to it."

When you are about to take over:

"Your diagram. I'll wait."

Age-band moves: 11–12 / 13–14

11–12. Decompose area. Nets and surface area. Volume of right rectangular prisms with fractional edges. Coordinate polygons with horizontal/vertical sides. Reason sentences on every exit ticket. Circles can appear lightly if circumference meaning is ready; keep the formula sheet off the table until a because-sentence exists. Similarity can wait for clear 13–14 work unless a scale drawing appears naturally.

13–14. Angle relationships with reasons. Scale drawings. Circles with meaning. Similarity and congruence through transformations. Similar triangles ↔ slope (connect to Chapter 6). Pythagorean Theorem with explanation — edge labelled as Grade 8 completion, not chant-only. Volumes of cylinders, cones, spheres as completion edges: compare, then summarize with a formula. Full high-school proof geometry is not this week's object. There is no kitchen-table geometry RCT to promise you a score.

A fourteen-year-old who multiplies base and height for every shape without looking is still in the 11–12 band of this chapter for area meaning. An eleven-year-old who can decompose an L-shape and write a because-sentence is doing real geometry.

First try-it for the student

Measure a tabletop or a clear floor rectangle (or use grid paper as a stand-in).

Find the area two ways if you can: covering count / length times width, and (if you draw a diagonal) half for a triangle.

Say: "What must be true, and how do you know?"

Wait. If they only recite a formula, ask for the covering sentence. If the number is right and the reason is missing, the reason is still the work.

Later the same week: unfold a box net, or (13–14) compare shadows / walk angles (below).

How to fade help. First sitting: you decompose, they echo the because. Second: they cut the shape, you wait. Third: reason sentence without your outline. Fourth: similarity table or Pythagorean explanation without a chant sheet. Formulas, when they appear, appear as summaries they can defend.

When to stop talking. When you hear yourself delivering a lecture titled *The History of* π. When the sit has become a formula quiz. When the student is mid-diagram and you have already asked a second and a third question. One good question beats five. Stop while they still have a shape left to explain.

Practice that actually builds learning

Blocked, for a new move. The day you introduce decomposition, only decompose-and-explain. The day you introduce nets, only nets. Mixing too early makes them grab the formula sheet.

Mixed, for when to use it. Later the same week, an area item next to a ratio table next to a scale factor. Mixing is the practice of choosing: decompose, scale, or angle reason?

One incorrect example to diagnose. Formula without reason — "area is length times width" on a triangle — or Pythagorean on a non-right triangle. Hear the miss. Ask what must be true. Require the diagram. Silence in the face of a formula card is not kindness.

These are illustrations, not reported families.

Conceptual and procedural together. NMAP Finding 10 says conceptual understanding, computational fluency, and problem solving reinforce each other; ranking them as enemies is misguided.[61] So: the student still computes. The student also explains. A perfect paragraph with a wrong area is incomplete. A correct area with no because-sentence is incomplete too. Aim for both in short sits, not for an essay contest.

Named try-it: Decompose a room or tabletop area (in-home)

Time. Fifteen to twenty minutes. Then stop.

Materials. Measuring tape or ruler; scrap paper; optional grid.

Safety. Ordinary. No standing on unstable furniture to "measure the ceiling for fun." Floor and tabletops are enough.

The fun. "How much flooring / wrapping / paint-for-a-panel?" — as a number problem, not a shopping sermon.

The skill. Area by decomposition; write why. Eleven-to-twelve: rectangles and L-shapes. Thirteen-to-fourteen: add a triangular region or a scale drawing of the room.

Measure. Sketch. Split into shapes you know. Compute. Write: "The area is ___ because ___." Kitchen motivates. The diagram and the because-sentence still happen.

[61] NMAP (2008), Finding 10: conceptual understanding, computational fluency, and problem solving are mutually reinforcing.

Named try-it: Net of a box (in-home)

Time. Fifteen minutes.

Materials. Empty cereal or shipping box; scissors; scrap for the sum.

Safety. Scissors courtesy. No cutting toward a hand. Recycle the cardboard after.

The fun. Unfolding. Seeing all six faces at once.

The skill. Surface area; volume with fractional edges if you measure carefully (11–12).

Unfold into a net. Label each face. Add for surface area. Measure edges; compute volume. Ask what would change if one edge were half as long. The cardboard is the representation. The written sum is required.

Named try-it: Walk angles or similar shadows (out-of-home)

Time. Fifteen to twenty minutes.

Materials. Stick or meterstick; tape optional; sunny day or a lamp at home as backup; scrap for ratios.

Safety. Roads; sun-safe (shade breaks, no staring at the sun). Sidewalk courtesy. No climbing for a “similarity photo.”

The fun. Shadow twins. Matching angles with your arms as rays.

The skill. Corresponding angles; similarity ratios on a table (13–14). Eleven-to-twelve can walk complementary/supplementary estimates with a right angle reference (book corner) without full similarity.

Compare a stick’s shadow to a taller object’s shadow when the sun is steady. Corresponding sides; scale factor on a ratio table. Ask: “What must be true, and how do you know?” Land the table on paper before you go inside for lemonade.

Named try-it: Ramp or stairs as similar-right-triangle edge (out-of-home)

Time. Ten minutes on site; short write-up at home.

Materials. Optional tape measure; scrap.

Safety. No climbing stunts. Safe stance on stairs you already use.

The fun. Rise/run again — now as similar right triangles stacked along a slope.

The skill. Reason before formula; Pythagorean edge labelled (13–14). Connect to Chapter 6 steepness.

Measure rise and run for one step or a ramp segment. Sketch the right triangle. Ask what must be true about the hypotenuse length — estimate, then (13–14) check with Pythagorean explanation. Do not chant first. Reason first. Formula as summary.

Incorrect example to keep on a sticky note. A worksheet shows a tilted triangle with sides labelled 5, 5, and 6. A student multiplies 5×6 and halves it without drawing a height — or multiplies all three numbers. The miss is missing height, or treating every length as a base-height pair. Repair: redraw, choose a base, drop a perpendicular, then compute. Keep that sticky note for Friday's diagnostic.

Blocked week sketch (illustration, not a mandate). Monday: decompose area. Tuesday: net and surface area. Wednesday: one out-of-home angle or shadow look. Thursday: mixed — area next to a ratio table. Friday: exit ticket with reason + compute, plus the sticky-note diagnostic. Adjust the days. Keep the shape. Not 180 worksheets.

Talk box

Opening question (locked): "What must be true, and how do you know?"

Follow-ups

1. Which lengths correspond?
2. Show the ratio table of sides (or the decomposed pieces).
3. Why multiply these for area (or how do the faces make surface area)?
4. Walk the angle — what matches when you turn or flip it?

5. You may change your mind. Try another decomposition or another corresponding pair.

How to wait

Ask. Count a slow three. Look at the diagram, not at the student's face, if the silence is hard. After they stop, wait again. If they are mid-reason, do not cut them off.

What a stuck silence usually means

They reached for a formula card only. Or a rotated figure panicked them. Or wait-time after the question was zero. Or corresponding sides are unmarked. Or the numbers are too messy. Or they are guessing what you want. Next move: simpler shape, draw the height, wait, point back at the diagram. "Split it into two rectangles" is a hint. "The area is 24" said by you is not a hint; it is the grab. Done enough this week: a diagram plus a because-sentence. "The area is 24 because I split the L into a 4-by-4 and a 2-by-4."

Angles you can walk. Stand on a sidewalk seam. Your two arms can be rays. A right angle matches a book corner or a phone corner. Complementary angles add to 90°; supplementary add to 180°. Vertical angles match when two lines cross — not because a poster said so, but because a turn lands on the same opening. Have the student show the match with a walk or a tracing, then write the because-sentence. Degrees are a unit; the reason is the relationship.

Circles without mystery buttons. Circumference is the distance around; it grows in proportion to the diameter. That constant ratio is π. Area of a circle can wait until circumference meaning is steady — then a dissection argument or a vetted visual (sectors rearranged toward a parallelogram) can justify πr^2 as a summary. If the student only punches π on a calculator with no diameter talk, slow down. A bicycle wheel on a walk is a circumference story; the calculator is optional.

Similarity and slope in one glance. Two similar right triangles stacked on a ramp share the same rise/run. That repeated ratio *is* the steepness from Chapter 6. You do not need a separate "slope unit" if the triangles already carry the scale factor on a ratio table. Point at both papers in the same week once. Then stop talking and let the student say the connection.

What "Grade 8 completion edge" means at home. Cylinders, cones, and spheres appear when area and volume meaning for prisms is already honest. Compare a cone to a cylinder with the same base and height (the cone's volume is one-third the cylinder's — after a comparison argument you trust, not as a floating fraction). Spheres wait until you have a reason you can narrate, even briefly. If those edges turn the hour into fog, label them "later," keep nets and similar triangles, and move on. Edges are invitations, not obligations tied to a birthday.

Coordinate plane as geometry, not only graphing. Place a rectangle on the grid. Horizontal and vertical side lengths are differences of coordinates. Area still needs a covering reason. This is a gentle bridge from Chapter 6's plotted pairs to shapes with measurable sides — same grid, new question: what must be true about this polygon?

For the student

This page is for you.

What must be true, and how do you know?

Geometry is reasons, not only formulas.

Area means covering a shape without gaps or overlaps. Sometimes you cut a shape into pieces you already understand. The formula is a short way to say what you already showed.

A net is a box unfolded. Surface area is the total of the faces. Volume is how much space is inside — including when a side is a fraction.

Similar shapes are the same shape at different sizes. Matching angles; sides in the same ratio. You can put those sides in a ratio table.

For a right triangle, the Pythagorean relationship connects the three sides — with an explanation, not only a chant. Check that there is a right angle first. The longest side is opposite the right angle.

A tiny worked example

A rectangle is 6 units by 4 units.

Area is 24 because I can cover it with 6 columns of 4 (or 4 rows of 6).

Cut along a diagonal. Each right triangle has area 12 because the two triangles match and share the 24.

Someone says a triangle "has no base" because it is tilted. Choose any side as base. Draw the height perpendicular to that side. The formula still means half of base times height — after you know why.

Two tries

1. Find the area of an L-shape (or a room sketch) by decomposing. Write: "The area is ___ because ___."
2. Unfold a box and find surface area from the net — or, if you are ready, set up a similarity ratio table from two shadows or two triangles.

Explain it back

Tell someone at the table why a triangle's area is half of a related rectangle's area (or why your L-shape split works). Point at the diagram while you talk.

Challenge

Someone always multiplies two numbers they see on a picture and calls it area. What question do you ask? Someone chants `a² + b² = c²` on a triangle with no right angle. What must be true first?

You are allowed to struggle. You may use scrap paper, a box, a measuring tape, a shadow. You draw. You talk. If you get stuck, ask for a hint — not the finished area from a photo app. Then try again.

When you talk, a because-sentence is enough for today. You do not have to finish high-school geometry. Today you justify one measurement.

A perfect 3-D render a computer made is a scene. It is interesting. It is not the net on your table.

If it isn't clicking

Three diagnostics. Each one has a next move. None of them is a verdict on talent, and none of them is a reason to wait for a birthday.

1. The student recites formulas with no diagram reason, or multiplies random side lengths for area.

The formula card is doing the work of the geometry. Next move: covering and decomposition only, for several days. Every answer needs a because-sentence before the number counts as done. Stay here if this is still the bottleneck. A similarity packet will not hold on top of empty area meaning. A human who will sit with grid paper and wait, not an app that prints `A =`, is a reasonable next step if formula-only is still the whole hour.

2. The student panics when a figure is rotated, or claims similarity by "looks like" with no scale factor.

Orientation or vague resemblance is doing the work of the definition. Next move: redraw with a chosen base and height; mark corresponding sides in color; build a ratio table. For similarity, require at least one scale factor checked on two pairs. Slow down until "looks like" is replaced by correspondence. Go ahead once a decomposed area and one similarity or scale item both carry reasons. A tutor is useful if rotated panic or vague similarity remain the default after a couple of weeks of short diagram sits *and* the hour has become a fight.

3. The student freezes on Pythagorean or volume edges, or every sitting ends in a shrug, and the silence after your question is a wall.

The wait was zero, or the edge was dumped too early, or they are guessing what you want, or measurement arithmetic is shaky. Next move: shorter sits, return to 11–12 decomposition if needed, the opening question from the talk box, wait a slow three twice. Label Pythagorean and sphere volumes as edges — peek with explanation; keep them short. If silence is hard, you look at the diagram, not at them. If you hear yourself filling in the area, you have started doing the work. If the hour has become a fight after a stretch of daily short sits, a human who will wait, not a chatbot that fills the silence, is the release valve.

When to slow down. Formula-only still the default. Rotated panic still in the room. Similarity without ratios. Pythagorean chant on non-right triangles. Sits so long that no diagram gets finished. Those are brakes. "Not ready" because of age is the brake this book will not use. "Not ready" because reasons are not yet attached to measurements is a real brake. A high-school geometry book cover is not a reason to skip the cereal-box net.

When to go ahead. The student can justify an area or surface-area result with a diagram and a sentence. At 13–14, a similarity statement with a ratio table, or a Pythagorean explanation on a true right triangle, has appeared. Short sits are ordinary. Then data and story-problem schemas have somewhere to sit. Being "good at geometry" because a student memorized a formula sheet is not a reason to skip the because-sentence.

When to get a human tutor. You have run the decomposition, or the net, or the wait after the question, for a stretch of daily sittings, and the same diagnosis is still the one in the room, and the hour has become a fight. A tutor is a release valve, not a failure of the sitting. Keep yourself as the person who can still hear a formula without a reason. Outsourcing the hearing is the thing to avoid, not asking for help.

Correct a formula-only answer without crushing the attempt. “I hear length times width. Show me what that means on this shape.” Then look. Hearing a wrong answer, asking a better question, then naming a better move if needed, is teaching.

Tools, including AI

Optional helpers for you, the adult. The full rules live in the math-hour box in the front of this book. This chapter is a pointer, not a second policy paper.

The student draws first. You hold the question. A tool may explain today’s idea *to you* from this chapter, suggest extra isomorphic shapes you then vet, write a short SCRIPT for *you* to say after the student has tried, offer a hint after an attempt, or help you diagnose a diagram the student already drew. Crop to the paper. Do not upload the student’s face. The student never sees the key.

Ages 11–12: parent holds the account. Ages 13–14: parent co-holds; still parent in the room. Unaided first. SCRIPT after the try.

Leave these out of the hour: a chatbot as the only partner; a tool that completes the worksheet; photo-to-key on a textbook diagram so a solution pops up; a detector score; a certificate of geometry-fluency; an agent as a math partner. Photo-to-answer is the ban.

A language model will happily dump a formula sheet. Treat every model-supplied area as untrusted until the student’s diagram and because-sentence exist. The math-hour AI box still governs.[62]

Measuring tape, cardboard nets, scrap diagrams, and a sunny sidewalk are tools too. Use them, then fade them.

[62] See the math-hour AI box; Bastani et al. (2025) high-school math trial is a crutch sentence for adult use of tools, not an 11–14 geometry RCT.

What "done enough" looks like

Placement is by skill, not birthday. A "grade 7 geometry workbook" is a publisher's scope, not a legal grade, and not a transcript line. The record, when you need one, is a dated notebook, the task named, plus an exit ticket, plus one diagnostic item. The title a stranger can read is Mathematics, or Pre-Algebra, or Algebra I — only if the year's work was that course. It is not Young Minds Math I.

Checklist before moving on

- You can hear a formula without a reason, and you can ask what must be true.
- The student can justify an area or surface-area result with a diagram and a because-sentence.
- A net or a decomposition has appeared in the same week as a written computation.
- At least one out-of-home try-it (shadow similarity, walk angles, or ramp/stairs) happened.
- Kitchen floor plans motivated; they did not replace the diagram or the reason.
- 11–12, as it holds: fractional-edge volume or coordinate polygon length has appeared.
- 13–14, as it holds: similarity with a ratio table, and/or Pythagorean with explanation on a right triangle; cylinder/cone/sphere volumes only as labelled completion edges you chose on purpose.
- Sits can be short. A diagram plus a because-sentence is enough. You did not require a full high-school proof course as the proof of readiness.
- You can hear formula-card answers, rotated panic, vague similarity, and chant-only Pythagorean, and you can ask a good question, without taking the pencil.

If most of that list is true, go on to data, chance, and story-problem schemas, even if the birthday says otherwise. If the birthday says "geometry done" and reasons are still missing, stay. The next chapter asks what kind of story a problem is. It needs representations and honesty about what a number claims.

A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

Chapter 8

Data, chance, and story problems

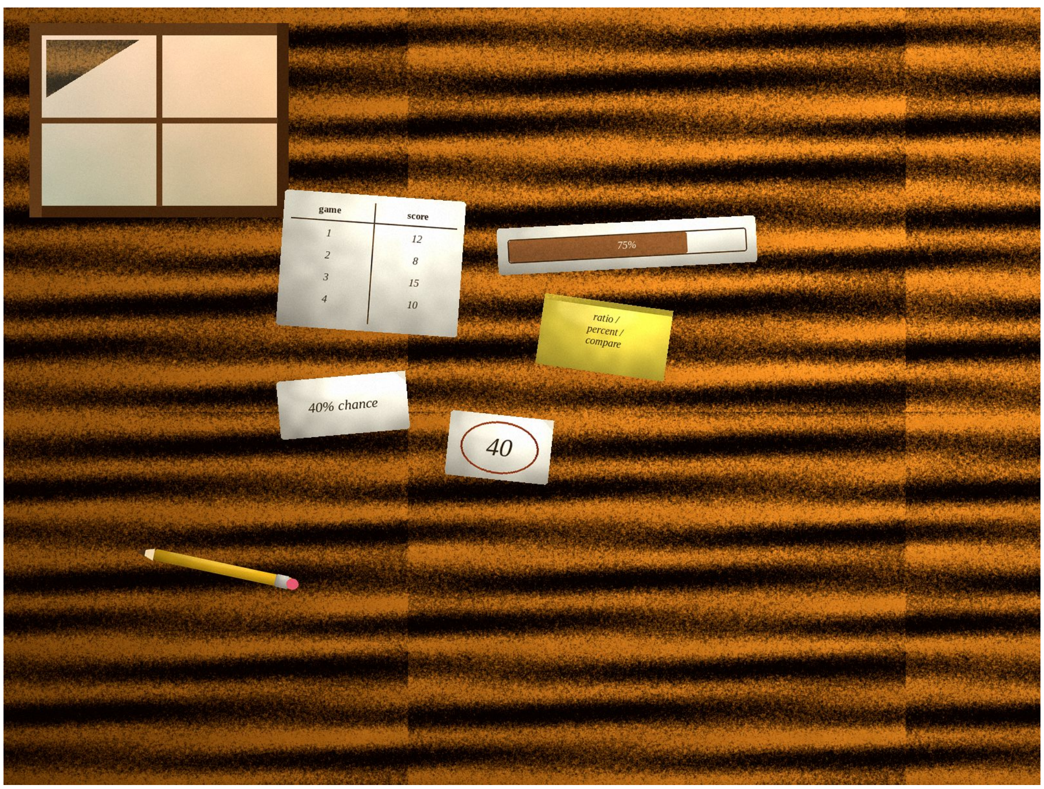

Figure 11: A kitchen-table still-life: a simple table of scores; a percent bar; a sticky note reading ratio / percent / compare; a weather forecast number circled on scrap. No people. No logos.

Why this matters

What kind of story is this?

That question is how word problems become mathematics instead of a keyword scavenger hunt. In this band the durable story types are **ratio**, **percent**, and **compare** — multiplicative compare and additive compare both deserve names. "Of means multiply" is not a type. "Altogether means add" is not a type. Circling cue words is not a schema. The student sorts the story, draws the quantities, then computes. Data and chance belong in the same week as those stories: a mean that hides a wild day, a forecast number that does not promise your block, a sports rate that claims something and not something else.

This chapter also carries the life-of-the-habit *tips that are moves* — in "done enough," not as a ninth teaching chapter. Revisit next Tuesday. Mix last week's type with this week's. Keep a dated notebook. Stop when the hour is honest. A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

What this idea unlocks is independence on mixed pages. Chapters 1–7 built numbers on a line, ratio tables, signed meaning, expressions, equations, input–output, and geometry with reasons. Story problems and data ask the student to choose which tool fits. Monitoring — "does this make sense?" — is not a soft add-on. Woodward's problem-solving guide rates monitoring and reflecting as strong evidence for grades 4–8, and visual representations as strong evidence too.[63] Schemas based on underlying structure, not keywords, sit in the same family of recommendations across intervention and fractions guides.[64]

Why teach this *now*? Because Grade 6 statistics is already a critical area on the common map: center, spread, and describing distributions with meaning. Grades

[63] John Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8* (NCEE 2012-4055), May 2012 / revised 2018. Recommendation 2 (monitor and reflect) strong evidence; Recommendation 3 (visual representations) strong evidence.

[64] Russell Gersten et al., *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (NCEE 2009-4060), 2009, Recommendation 4 strong — word problems based on common underlying structures. Siegler et al., *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (NCEE 2010-4039), 2010, Recommendation 4 — ratio/rate/proportion meaning before cross-multiply. Fuchs et al., *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* (WWC 2021006), 2021, Table 5.2 — keyword traps (elementary-intervention grain; still the right anti-finding for keyword theater).

7–8 add chance models and bivariate association edges.[65] Because NAEP Grade 8 mathematics in 2024 averaged **272** for the nation's public schools, with **27%** at or above NAEP Proficient — sobering scale difficulty in U.S. schools, labelled NAEP, not a homeschool census, and not your student's destiny.[66] Because a parent who only drills keyword posters will hear "of means multiply" on stories that are not multiplicative at all.

This book is not a reprint of *Math for Little Thinkers*. That manual owns early join/separate/compare types for ages 5–10. One pointer: types not keywords continues. Then we teach middle-grades schemas — ratio, percent, compare — plus data and chance in the week. This book is not a reprint of *Mathematics for Homeschooling*. One pointer, then teach 11–14 at this grain. This book is not a sequel to *Critical Thinking for Young Minds*. Reasons and testimony live there. Writing scores from another subject are a different conversation; NAEP Grade 8 mathematics stands alone here. The talk box asks about story type and data claims — it is not a thinking-skills brand transplanted onto ratio tables. One pointer, then teach the math. Philosophy-for-children trials that are not maths trials stay out of this chapter.[67]

You do not need to be a mathematician. You do need to hear "of means multiply," and to ask what kind of story this is, without taking the pencil.

This week you can learn to hear keyword theater and a mean treated as the whole truth. Today the student can sort three problems by type, draw one, and write one sentence about what an average hides.

For the parent: understand it yourself

Many adults feel rusty on why sorting stories beats underlining words. That is ordinary. School memory often includes a poster: *altogether* $\rightarrow$ *add*, *left* $\rightarrow$ *sub-*

[65] *Common Core State Standards for Mathematics* (2010), Grade 6 statistics critical area; Grades 7–8 probability and bivariate association. Map, not statute.

[66] NAEP Grade 8 mathematics 2024, nation (public): average scale score 272; 41% below Basic, 32% Basic, 19% Proficient, 8% Advanced — 27% at or above Proficient. NAEP Proficient does not represent grade-level proficiency as determined by other standards. Not a nationally representative homeschool score. nationsreportcard.gov materials opened for this project 4 September 2026.

[67] EEF Philosophy for Children (P4C) is not a mathematics trial; it does not belong in chance or data teaching claims.

tract, *of* → *multiply*. Five minutes of this section, then the warm-up, is enough for tomorrow. The student still sorts and draws.

Everyday picture. Three short stories on scrap paper (illustrations, not reported homework):

1. A recipe uses 2 cups of oats for every 3 cups of milk. You want enough for 9 cups of milk. How much oats? — **Ratio** (scale a relationship).
2. A jacket costs $40. It is 25% off. What is the sale price? — **Percent** (a special ratio to 100, with a whole and a part).
3. Maya has 12 cards. Jordan has 5 more than Maya. How many does Jordan have? — **Compare** (additive difference). Or: Jordan has twice as many — **Compare** (multiplicative).

Same page. Three structures. Keyword "of" might appear in none, or in a red herring. The type is the structure. Draw strips, a ratio table, or a percent bar. Then compute.

Data picture. Seven days of high temperatures, including one spike. The mean is one number. It can hide the spike. The student computes the mean *and* says what it hides. A board-game score table works the same way: who won, what the average hides, one true sentence and one false sentence about the table.

Chance picture. A forecast says "30% chance of rain." That is a model number about conditions like these, not a promise that your street gets rain on three of ten specific Tuesdays in a row you personally track — and not a guarantee about your block today. Ask what the number claims. Ask what it does not claim. Keep jargon light. Experimental probability from a simple chance experiment (coins, number cubes) can sit beside the forecast talk for 13–14.

Precise picture. Schemas for this chapter:

- **Ratio / rate / proportion stories.** Quantities in a constant relationship; scale up or down; unit rate; maybe a missing value in a table. Representation first (Chapter 2), then compute. Cross-multiply only after meaning.
- **Percent stories.** Part-whole with a whole of 100 as reference; percent of a number; percent change; simple markups/discounts when percent meaning from Chapter 1 is ready. Percent bar or double number line.
- **Compare stories.** Additive compare ("how many more") or multiplicative compare ("how many times as many"). Strip diagrams help. "More" is not a cue to subtract automatically — check what is unknown.

Data moves: read a table; compute mean (and median when useful) with meaning; talk about spread in ordinary language; write one sentence that is true and one that overclaims. Chance moves: probability as a number between 0 and 1 (or a percent); simple experiments; question a forecast's claim. Bivariate association (scatterplot "does this trend look positive?") is a 13–14 edge — labelled, not an AP Statistics dump.

Wrong answers you should be able to hear

1. *"Of means multiply" on any sentence containing of.* Keyword grab. Ask the story type. Draw the quantities.
2. *Treating every fraction story as a proportion* (a Lamon reminder from ratio research: some structures are solvable by less sophisticated methods; stretcher/shrinker multiplicative structure is the hard one).[68] Sort carefully. Not every fraction word problem is a scale-the-ratio story.
3. *Computing a mean and treating it as the whole description.* Ask what it hides. Point at the extreme value.
4. *Reading "30% chance" as "it will rain on our block for sure / for sure not."* Ask what the model claims.
5. *Circling "altogether" and adding on a ratio story.* Schema miss. Sort first.

A sixth you will also hear: dumping AP Stats vocabulary (z-scores, formal inference) into a Thursday sit because a video said "data literacy." Prefer a small real data set and a computed mean. Name formal inference as a later edge.

Five-minute parent warm-up

Do this before the lesson, on a scrap of paper, no student in the room.

Minute 1. Write three one-line stories: ratio, percent, additive compare. Sort them yourself. Draw one strip or table for each.

Minute 2. Make a tiny list: 4, 5, 5, 6, 20. Compute the mean. Say out loud what it hides.

[68] Susan J. Lamon, "Ratio and Proportion: Connecting Content and Children's Thinking," *Journal for Research in Mathematics Education* 24, no. 1 (1993): relative thinking and unitizing; part-part-whole solvable by less sophisticated methods; stretcher/shrinker hardest. Steal the caution: not every fraction story is a proportion story.

Minute 3. Write "30% chance of rain." Write one sentence about what it claims and one about what it does not.

Minute 4. Invent a keyword trap: a sentence with "of" that is *not* multiply-the-two-numbers. Mark why.

Minute 5. Write the sentence you will actually say: "What kind of story is this?" Under it: "Draw the quantities. What does this average hide?" Put the pencil down. Those sentences are the lesson.

If you can do those five minutes, you are ready to sit down. The student sorts. You hear.

How to teach it this week

A good math hour this week has a shape. The math-hour chapter in the front of this book is the full version. Here is the shape scaled to this idea.

Warm-up (2–5 minutes, unaided). One mean from a tiny list; ask what it hides. Or one quick sort: ratio / percent / compare labels only. Paper. No device.

Short model (3–7 minutes). Sort three stories. Draw one. Solve one. Name why the keyword poster would have lied on at least one of them. You talk briefly. Then you stop.

Student attempt (8–15 minutes). Three mixed stories; no keyword circling. Sort all three. Draw at least one. Solve at least one. You wait. Struggle before rescue: ask, wait, hint ("what kind of story — ratio, percent, or compare?"), then a short model on *your* scrap.

One good question, then wait. "What kind of story is this?" A slow three. After they stop, wait again. Look at the diagram or the table, not at the student's face, if the silence is hard.

Mixed practice (5–10 minutes). Any prior chapter skill next to today's schema: an equation check, a ratio table, a percent bar, a signed number from a temperature list. Mixing is how they learn *when* to use which tool.

Exit ticket (2–4 minutes). Sort + solve one + one data sentence. Done-enough is right, or wrong-with-a-reason we can use tomorrow — and next Tuesday's revisit.

Exact wording you can say

“What kind of story is this?”

“Draw the quantities.”

“Is this multiplicative compare or additive compare?”

“What does this average hide?”

“Does the chance number promise your block?”

“You may change your mind.”

When they underline “of” and multiply:

“Sort first. What are the quantities doing?”

When they treat the mean as the whole truth:

“Point at the number that pulls the mean. What changes if we set it aside for a moment?”

When you are about to take over:

“Your sort. I’ll wait.”

Age-band moves: 11–12 / 13–14

11–12. Center and variability with meaning on tiny lists. Mean as “fair share” / balance point talk in ordinary language. Simple tables. Sort ratio / percent / compare with drawings. Percent stories only as far as Chapter 1 percent meaning supports. Experimental chance: coins or cubes, record frequencies. No keyword posters. No AP Stats.

13–14. Multi-step percent with meaning. Sampling talk: what a sample can and cannot claim. Probability models: theoretical vs experimental on simple spaces. Bivariate association edge: “does this scatter look like it rises together?” — labelled, short. Richer compare stories with unknown in different slots. Keyword lists stay off the wall. A diploma chase is not this week’s object.

A fourteen-year-old who still circles “altogether” is still in the 11–12 band of this chapter for schemas, whatever the birthday. An eleven-year-old who can sort three stories and say what a mean hides is doing the work.

First try-it for the student

Three short stories on one scrap (you write them ahead).

Say: “What kind of story is this?” for each — ratio, percent, or compare.

Wait. If they grab a keyword, cover the cue words and ask what the quantities are doing. If they sort correctly, have them draw one and solve one.

Later the same week: a temperatures list or board-game table (below), and one forecast question.

How to fade help. First sitting: you sort one, they sort two. Second: they sort all three, you wait. Third: draw without your outline. Fourth: Friday mixed exit without labels printed on the page. Data sentences start oral; then one written sentence.

When to stop talking. When you hear yourself delivering a lecture titled *The Philosophy of Randomness*. When the sit has become a vocabulary quiz about “bimodal distributions.” When the student is mid-sort and you have already asked a second and a third question. One good question beats five. Stop while they still have a story left to sort.

Practice that actually builds learning

Blocked, for a new move. The day you introduce percent bars for percent stories, only percent stories. The day you introduce “what the mean hides,” only tiny data lists. Mixing too early makes them hunt for a cue word.

Mixed, for when to use it. Later the same week — and especially next Tuesday — ratio next to percent next to compare next to a data sentence. Mixing is the practice of choosing the schema and the representation.

One incorrect example to diagnose. “Of means multiply” on a non-multiplicative story, or a mean treated as destiny. Hear the miss. Ask the type. Ask what the average hides. Silence in the face of a keyword grab is not kindness.

These are illustrations, not reported families.

Monitoring is a teachable move. After a solution, ask: “Does that answer make sense?” Require a look back at the diagram and the units. Woodward rates this kind of monitoring strong in the grades 4–8 problem-solving guide.[69] Steal the

[69] John Woodward et al., *Improving Mathematical Problem Solving in Grades 4 Through 8*

move. You do not need a school RCT at your table to ask whether 250 pounds of oats for a family breakfast makes sense.

Named try-it: Week of temperatures as data (in-home)

Time. Five minutes a day recording; ten to fifteen minutes Friday summary.

Materials. Thermometer, weather app numbers you already check, or a published forecast high; dated notebook.

Safety. Ordinary. No climbing on a roof for a "better reading." Indoor/outdoor thermometer at a safe height.

The fun. Spotting the weird day. Arguing (kindly) about whether the mean is "fair."

The skill. Center; spread talk; integers + data (temperatures through zero welcome from Chapter 3).

Record daily highs. Friday: compute the mean. Write one sentence about what it hides. Optional: mark the median. Kitchen-table weather is motivation. The table and the sentence still happen.

Named try-it: Board-game scores table (in-home)

Time. Ten to fifteen minutes after a game you were already playing.

Materials. Score pad; scrap for true/false sentences.

Safety. Ordinary family tone. No using scores to shame a player. The math object is the table.

The fun. Who won — and what the average hides.

The skill. Read a table; monitoring: one true sentence, one false sentence about the data.

(NCEE 2012-4055), May 2012 / revised 2018. Recommendation 2 (monitor and reflect) strong evidence; Recommendation 3 (visual representations) strong evidence.

Make a small table of round scores. Compute a mean for one player. Write: "True: ___ / False: ___." Ask what kind of story a "how many more points" question would be — compare.

Named try-it: Sports box score (out-of-home)

Time. During a game or meet you were already watching; five to ten minutes of talk; optional write-up at home.

Materials. Box score, scoreboard, or tally.

Safety. Sideline courtesy. No lecturing strangers. Sports stats are ratios and percents — not identity fights.

The fun. Batting average, shooting percent, save percentage as a ratio claiming something specific.

The skill. What the number claims and does not claim; rate language meeting schema talk.

Ask: "Is this a ratio story, a percent story, or both?" Build the fraction. Ask whether it promises the next play. Land a sentence at home: "This number claims **. *It does not claim*** ."

Named try-it: Weather chance language (out-of-home)

Time. Morning forecast; three to five minutes.

Materials. Forecast you already hear or see; scrap optional.

Safety. Ordinary. No fear talk. No mocking a meteorologist as a personality contest.

The fun. Catching overclaim language in everyday speech — including your own.

The skill. Model probability talk without jargon theater; not a promise about your street.

Point at "30% chance" (or whatever the number is). Ask what it claims. Ask whether it promises your block. If 13–14, compare to a simple experiment's relative frequency later the same week. Keep it short. The forecast is motivation. The questioning sentence is the math.

Named try-it: Mixed schema exit (in-home)

Time. Ten minutes Friday.

Materials. Three short stories (ratio / percent / compare) you wrote; scrap; no keyword underlining allowed.

Safety. Ordinary table.

The fun. Sorting like a detective. Being right about the type even before the arithmetic.

The skill. "What kind of story is this?" — never a keyword.

Sort all three. Draw one. Solve one. Add one data sentence from the week's temperatures or scores. Date the page. This exit ticket *is* the assessment object — not a percentile battery.

Talk box

Opening question (locked): "What kind of story is this?" (ratio / percent / compare — never a keyword)

Follow-ups

1. Draw the quantities.
2. Is this multiplicative compare or additive compare? (Or: is this a scale-the-ratio story?)
3. What does this average hide?
4. Does the chance number promise your block? Does that answer make sense?
5. You may change your mind. Try another representation.

How to wait

Ask. Count a slow three. Look at the diagram, the table, or the percent bar, not at the student's face, if the silence is hard. After they stop, wait again. If they are mid-reason, do not cut them off.

What a stuck silence usually means

They are hunting a keyword. Or wait-time after the question was zero. Or they treat every fraction story as a proportion. Or the numbers are too new. Or they are guessing what you want. Or data talk feels like a different subject than "math." Next move: cover cue words, smaller numbers, wait, point back at the strip or table. "Is it ratio, percent, or compare?" is a hint. "Multiply because of 'of' " said by you is not a hint; it is the grab. Done enough this week: a sort plus a drawing. "This is a percent story because there is a whole and a part named out of 100."

For the student

This page is for you.

What kind of story is this?

Before you compute, sort.

Ratio stories scale a relationship: for every **, *there are*** . A ratio table or double number line helps.

Percent stories use parts out of 100 (or a percent of a whole). A percent bar helps.

Compare stories ask how much more or how many times as many. A strip diagram helps.

"Of," "altogether," and "left" are words in English. They are not automatic math buttons. Sort the structure. Draw. Then compute.

Data: a mean is one useful number. It can hide an extreme day. Say what it hides.

Chance: "30% chance" is a model number. It does not automatically promise rain on your block today.

A tiny worked example

Story A: 2 packs for \$5. How much for 8 packs? — Ratio. Table: $2\to5$, $4\to10$, $8\to20$.

Story B: \$50 game, 20% off. Sale price? — Percent. 20% of 50 is 10; sale price 40. Bar: whole 50, shade 20%.

Story C: Sam has 9 points. Alex has 4 more. Alex has? — Compare (additive). Strip longer by 4.

Someone sees “of” in a sentence and multiplies two numbers that should have been compared additively. They used a keyword. Ask what kind of story it is. Draw again.

List: 3, 4, 4, 5, 20. Mean is 7.2. What it hides: the 20 pulls the mean up; most numbers sit near 4.

Two tries

1. Sort three stories your parent gives you (ratio / percent / compare). Draw one. Solve one.
2. From a tiny data list (temperatures, scores, or steps), compute the mean and write one sentence about what it hides. Optional: find a forecast percent and write what it does not claim.

Explain it back

Tell someone at the table how you sort stories without circling keywords. Explain what a mean can hide. Explain what a chance percent does not promise.

Challenge

Someone always multiplies when they see “of.” How do you answer them with a story type and a drawing? Someone says the team with the higher average always wins the next game. What does the average claim, and what does it not claim?

You are allowed to struggle. You may use tables, bars, strips, a score pad. You sort. You talk. If you get stuck, ask for a hint — not the finished answer from a photo solver. Then try again.

When you talk, a sort plus a sentence is enough for today. You do not have to finish a statistics course. Today you name the kind of story and say what a number hides.

A glossy infographic a computer made is a scene. It is interesting. It is not the three stories on your scrap.

If it isn’t clicking

Three diagnostics. Each one has a next move. None of them is a verdict on talent, and none of them is a reason to wait for a birthday.

1. The student hunts keywords (“of means multiply,” “altogether means add”) and skips sorting.

The poster is doing the work of the schema. Next move: cover cue words; sort only, for several days; draw before any computation. Stay here if this is still the bottleneck. Mixed worksheets will not hold on top of keyword theater. A human who will sit with three stories and wait, not a video of cue-word tricks, is a reasonable next step if keyword grabs are still the whole hour.

2. The student computes means correctly but never says what they hide, or treats forecast percents as promises about the block.

The number is doing the work of the claim. Next move: every mean needs a “hides ***” sentence before it counts as done. Every forecast percent needs a “claims /*** does not claim ___” pair. Slow down fancy vocabulary until honesty about claims is ordinary. Go ahead once sorts are clean and one data sentence plus one chance sentence have appeared in the same week. A tutor is useful if overclaim language remains the default after a couple of weeks of short monitoring sits *and* the hour has become a fight.

3. The student freezes on mixed pages, or every sitting ends in a shrug, and the silence after your question is a wall.

The wait was zero, or too many skills landed on one page too soon, or they are guessing what you want, or fraction/percent meaning from earlier chapters is shaky. Next move: shorter sits, return to one schema at a time, then remix next Tuesday; opening question from the talk box; wait a slow three twice. If silence is hard, you look at the strip or table, not at them. If you hear yourself sorting for them, you have started doing the work. If the hour has become a fight after a stretch of daily short sits, a human who will wait, not a chatbot that fills the silence, is the release valve.

When to slow down. Keyword theater still the default. Means without “what it hides.” Chance numbers as promises. Sits so long that no sort gets finished. Those are brakes. “Not ready” because of age is the brake this book will not use. “Not ready” because schemas are not yet chosen on purpose is a real brake. An AP

Statistics trailer is not a reason to skip three short stories.

When to go ahead. The student can sort three problems by type, draw one, solve one, and write one sentence about what an average hides. A forecast or sports-rate claim has been questioned once. Short sits are ordinary. Then records and resources chapters help you keep the file — they are not a ninth content dump. Being "good at word problems" because a student is fast at cue words is not a reason to skip the sort.

When to get a human tutor. You have run the mixed sort, or the temperatures week, or the wait after the question, for a stretch of daily sittings, and the same diagnosis is still the one in the room, and the hour has become a fight. A tutor is a release valve, not a failure of the sitting. Keep yourself as the person who can still hear "of means multiply." Outsourcing the hearing is the thing to avoid, not asking for help.

Correct a keyword grab without crushing the attempt. "I hear multiply because of 'of.' What kind of story is this if we hide that word?" Then look. Hearing a wrong answer, asking a better question, then naming a better move if needed, is teaching.

Tools, including AI

Optional helpers for you, the adult. The full rules live in the math-hour box in the front of this book. This chapter is a pointer, not a second policy paper.

The student sorts and attempts first. You hold the question. A tool may explain today's idea *to you* from this chapter, suggest three isomorphic stories you then vet (key held by you), write a short SCRIPT for *you* to say after the student has tried, offer a hint after an attempt, or help you diagnose work the student already produced. Crop to the paper. Do not upload the student's face. The student never sees the key.

Ages 11–12: parent holds the account. Ages 13–14: parent co-holds; still parent in the room. Unaided first. SCRIPT after the try.

Leave these out of the hour: a chatbot as the only partner; a tool that completes the worksheet; photo-to-key so a solution pops up for the exact problem; a detector score; a certificate of data-literacy; an agent as a math partner. Photo-to-answer is the ban.

A language model will happily invent keyword rules and polished "statistics essays." Treat every model-supplied solution as untrusted until the student's sort and drawing exist. The math-hour AI box still governs.[70]

A dated notebook, a score pad, a forecast, and three short stories on scrap are tools too. Use them, then fade them.

What "done enough" looks like

Placement is by skill, not birthday. A "grade 7 math workbook" is a publisher's scope, not a legal grade, and not a transcript line. The record, when you need one, is a dated notebook, the task named, plus an exit ticket, plus one diagnostic item. The title a stranger can read is Mathematics, or Pre-Algebra, or Algebra I — only if the year's work was that course. It is not Young Minds Math I. It is not a brand as the credit.

Checklist before moving on

- You can hear "of means multiply," and you can ask what kind of story this is.
- The student can sort three problems into ratio / percent / compare, draw one, and solve one without keyword circling.
- A mean (or other center) has been computed *and* paired with a sentence about what it hides.
- A chance or sports-rate number has been questioned for what it claims and does not claim.
- At least one in-home data try-it and one out-of-home stats/chance try-it happened.
- Mixed practice included a skill from an earlier chapter in the same week.
- 13–14, as it holds: multi-step percent with meaning, and/or a short sampling or bivariate peek labelled as an edge — not an AP Stats dump.
- Sits can be short. A sort plus a drawing plus one honest data sentence is enough.
- You can hear keyword theater, mean-as-destiny, and forecast-as-promise, and you can ask a good question, without taking the pencil.

Life-of-the-habit tips (moves, not a ninth chapter)

[70] See the math-hour AI box; Bastani et al. (2025) high-school math generative-AI trial — crutch sentence for adult tool use, not an 11–14 RCT.

These tips live here so the habit has a home without inventing a separate “life of the habit” teaching chapter. They are session moves you already met in the math hour; print them as a short list on the fridge if you want.

1. **Revisit next Tuesday.** Take last week’s schema — ratio, percent, or compare — and mix it with this week’s exit items. Mixing is how the choice stays alive. A single Friday success is not ownership.

2. **Mixed types on purpose.** When the student can sort cleanly in a blocked set, put ratio next to percent next to compare on the same scrap. Add one data sentence. Stop at three to five items. Quality beats a packet.

3. **Dated notebook habit.** Date the page. Name the task in ordinary words (“percent bar — sale price”; “mean of seven highs”). Keep the exit ticket and one diagnostic wrong that you repaired. That file is the record — not a percentile printout.

4. **Stop when the hour is honest.** Thirty honest minutes with a sort, a drawing, and a check beat ninety minutes of silent worksheet grinding. If the sit has become a fight, stop. Return tomorrow with a smaller number set. The math hour chapter already said this; this chapter repeats it as a done-enough move.

5. **Not a diploma. Not a percentile.** Finishing this chapter does not finish adolescence, high school, or a transcript. NAEP’s Grade 8 average of 272 and 27% at or above Proficient describe U.S. school populations on a national assessment — sobriety about difficulty, not a target score for your kitchen table.[71] Your promise is a path through middle-grades mathematics: numbers on a line, ratios with meaning, signed numbers, expressions, equations by same-to-both-sides, input–output, geometry with reasons, and stories sorted by structure. Keep walking that path. Place the next topic by skill, not by birthday.

If most of the checklist is true, go on to records and resources when you need them — how to title the year’s work, how to keep the file, which materials fit your load. If the birthday says “done with word problems” and keyword theater is still in the room, stay. The habit is the revisiting, not the cover of a workbook.

[71] NAEP Grade 8 mathematics 2024, nation (public): average scale score 272; 41% below Basic, 32% Basic, 19% Proficient, 8% Advanced — 27% at or above Proficient. NAEP Proficient does not represent grade-level proficiency as determined by other standards. Not a nationally representative homeschool score. nationsreportcard.gov materials opened for this project 4 September 2026.

A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

Chapter 9

Records

The math *work* can be as serious as any kitchen's. The *credit* is a family claim until a stranger can use it — a receiving middle-school counselor, a Pennsylvania evaluator, a New York quarterly reader, a dual-enrollment office two years later, or the high-school transcript the family will someday write. That stranger will not have sat at your table. They will have a page.

This chapter is how you make that page at ages 11–14. Lawful, honest, and readable to a stranger are three jobs. Keep them in view.

This week, date one equation page a stranger could almost read. Today the student tries it; you write the line.

Figure 12: An open notebook, a dated multi-step equation with a correction in the margin, a sticky note titled Algebra I, a pencil. No diploma seal. No brand logo as the title. No adolescent.

What you are making

You are making a **dated notebook of mathematics**, with the skill named, plus an **exit ticket**, plus **one diagnostic item**, plus optional publisher placement to start a purchased program or restart after a gap. You are not making a national diploma. You are not making Young Minds Math I.

There is often still **no** college-facing transcript at this age. When a stranger asks, the title is **Mathematics**, **Pre-Algebra**, or **Algebra I** — only if the year's work was that course a stranger can map. Never a brand. Never Singapore 7A, Beast Academy 5, Saxon 8/7, or **Young Minds Math I**. The dated notebook and the exit ticket live *inside* those ordinary titles.

There is no federal homeschool diploma and no national mathematics credit at this age.[72] Texas, North Carolina, and New York do not issue one. Pennsylvania's supervisor or approved-organization diploma after listed graduation courses is a high-school object, not a thirteen-year-old's file. Virginia's list of subjects is a list, not a transcript. A parent-issued diploma can be a real piece of paper later. It is not a registrar's national Algebra I credit now.

Two readers, if anyone outside the kitchen asks: the **state** (IHIP, affidavit, portfolio, math-inclusive test) and a **later stranger** (transfer, dual enrollment, future high-school transcript). State compliance is not a math diploma. Neither a thick portfolio nor a silent lawful year is a reason to fake a credit — or to skip the dated notebook.

A family that invents "Young Minds Math I, 1.0, A" for a twelve-year-old has created a slogan. A receiving counselor cannot map it. NCAA core worksheets, Advanced Placement, and CLEP College Algebra belong to an older year and to *Mathematics for Homeschooling*. A parent of a twelve-year-old who is already anxious about college sittings has been handed the wrong timeline. SAT Subject Tests were discontinued — U.S. sittings ended immediately in January 2021; last international sittings were May and June 2021.[73] One sentence. Stop. They are not this year's check.

The last published federal subject map for grades 6–8 still shows **arithmetic** taught that year to 66 percent and **Algebra I** to 41 percent.[74] That is why this book places

[72] No national homeschool diploma: Texas Education Agency Home Schooling and Alternative Schooling pages (state does not award a homeschool diploma); North Carolina DNPE (the home school issues any later diploma; the State does not); Pennsylvania 24 P.S. § 13-1327.1(d.1) diploma is a high-school object; New York 8 NYCRR 100.10 (home instruction does not yield a local or Regents diploma); Virginia list of subjects is a list, not a transcript. Access date for URLs in these notes: 4 September 2026.

[73] College Board, 19 January 2021 announcement: SAT Subject Tests discontinued; U.S. sittings ended immediately; last international administrations May and June 2021. No CLEP for middle grades as a diploma. No AP as this band's default year object. Those objects belong to *Mathematics for Homeschooling*.

[74] Jiashan Cui and Rachel Hanson, *Homeschooling in the United States: Results from the 2012 and 2016 Parent and Family Involvement Survey*, NCES 2020-001, Table 9: grades 6–8, subjects taught that year — arithmetic 66 percent; basic algebra (Algebra I) 41 percent; geometry 19 percent; probability 13 percent; advanced algebra and calculus reporting standards not met. Do not paste elementary arithmetic 83 (K–2) / 86 (3–5) into a middle-grades sentence. The 2023 First Look (NCES 2024-113) did not republish subject-taught tables. NCES 2024-113 table A-6: 3.4 percent homeschooled in 2022–23, about 1,765,000; 6th–8th grade equivalent 3.0 percent. Instruction-at-

by skill. If a thick state later wants a log: name the skill, date it, keep a sample.

Titles a stranger can read

Print these and only these as middle-grades names, if a name is needed at all:

Mathematics. Pre-Algebra (only if the year's work was that course). **Algebra I** (only if the year's work was that course — by skill, not birthday). **Reading / Language Arts. Science. Social Studies / History** (if taught). **Art, Music, Physical Education.**

Those names travel. They are the names public middle reports already use. They are the names homeschool middle files, when they exist, already follow.

Never a brand. Never Young Minds Math I.

The brand, if used at all, sits in a parenthetical or in a materials line: "Algebra I — text: Art of Problem Solving *Introduction to Algebra*; lessons 1–X; dated notebook." Or: "Pre-Algebra — Math-U-See Pre-Algebra; Integer Block Kit; tests on file." Or: "Mathematics — Dimensions Math 7A/7B; workbook samples in the grade 8 portfolio file." Combining objects is allowed. Combining does not mint a new title. Kitchen ratio table plus library text plus one video lesson is still Mathematics.

A publisher's "grade 7 math workbook" is a scope, not a legal grade, and not a transcript line. Placement is by skill, not birthday.

Here is an illustration, labelled as such, not a reported family:

> 4 September 2026. Algebra I. Task: solve $3x + 7 = 22$ by same-to-both-sides. Student subtracted 7 from both sides, divided by 3, wrote $x = 5$. Checked by substitution. Missed a later item with negatives on both sides; rebuilt on a balance sketch; wrote the check.

That is a record a parent, a Pennsylvania evaluator, or a New York quarterly narrative can actually use. It is not a credit factory. It is not a college sitting. It is not a diploma.

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home 5.2 percent is a different bucket (homeschool or full-time virtual).

The law is a pattern, not one form

U.S. homeschooling is state law. There is no federal math office. Dates and subject lists change. Read your current department page and the statute it cites. A color-coded chart on a membership site is not your statute. Five official-page examples show the range. **None of them requires a critical-thinking course.** Math is the usual required or assumed middle object.[75]

Texas. The Agency does not regulate, index, monitor, approve, register, or accredit home-school programs. Parents follow a bona fide written curriculum: reading, spelling, grammar, **math**, and good citizenship. *Leeper*, restated on TEA's Alternative Schooling page, names math. Critical thinking is not named. The state does not award a homeschool diploma. At 11–14 there may be no external reader at all. The kitchen still benefits from a dated notebook if the family later transfers. Transfer is treated like an unaccredited private school. "We did Saxon 8/7" is not, by itself, a public-school credit. A district may test for placement.

North Carolina. Notice of Intent. Nine calendar months. Each year, a nationally standardized test in English grammar, reading, spelling, and **mathematics** — not a franchise, and not critical thinking. The home school, not the State, issues any later diploma. A five-hour day is a recommendation, not the statute, and not a math-minutes table. DNPE tells the family not to name the homeschool after a curriculum brand. The annual battery is compliance. It is not this book's teaching check, not NAEP, and not a percentile promise.

Pennsylvania. Elementary **shall include arithmetic**. Secondary (grades 7–12) **shall include mathematics, to include general mathematics, algebra and geometry**. Ages 11–14 straddle that line: grade 6 is elementary; grades 7–8 are secondary. Portfolio tests in grades 3, 5, and **8** are in reading/language arts and **mathematics**, not a franchise sitting. Drop samples into the log of materials. Skip

[75] Texas *Leeper*, restated on TEA Alternative Schooling: reading, spelling, grammar, **math**, good citizenship. North Carolina DNPE: annual nationally standardized test includes **mathematics**; five-hour day is a recommendation. Pennsylvania 24 P.S. § 13-1327.1: elementary shall include **arithmetic**; secondary shall include **mathematics, to include general mathematics, algebra and geometry**; portfolio tests in grades 3, 5, and 8 in reading/language arts and mathematics; 900 elementary / 990 secondary hours are whole-program. New York 8 NYCRR 100.10: grades 1–6 require **arithmetic**; grades 7–8 require **mathematics (two units)** cumulative; a unit is 6,480 minutes; 990 hours grades 7–12; quarterly reports; annual assessment. Virginia § 22.1-254.1 pattern: parent writes a list of subjects; evidence of progress by 1 August. None of the five requires a named critical-thinking course. This chapter is not legal advice. The parent reads their own statute.

a parallel "Young Minds" portfolio. 180 days or 900 hours elementary, and 990 hours secondary, are whole-program hours, not a math-minutes table.[76] A supervisor diploma is Pennsylvania's, and it is high school.

Virginia. Notice by 15 August, a parent-written **list of subjects**, and evidence of progress by 1 August: a composite score at or above the fourth stanine, or an equivalent SAT, ACT, or PSAT score, or an evaluation or college transcript the superintendent accepts. Mathematics is not named as required because no subject is named as required; the parent writes the list. Mathematics is the ordinary word to write. Critical thinking is not named. Those sittings are not this book's short check. A publisher placement test does not, by itself, satisfy Virginia's fourth-stanine object unless the parent has chosen a nationally normed battery.

New York. Letter of intent, an Individualized Home Instruction Plan, quarterly reports, and an annual assessment. Grades 1–6 require **arithmetic**. Grades **7 and 8** require **mathematics (two units)** cumulative for both grades, plus English, history and geography, science, and the rest of the list. A unit is 6,480 minutes. Attendance hours for grades 7–12 are 990 per year — whole-program hours, not a math-minutes table. Put the work under Mathematics in the quarterlies: material covered in each IHIP subject. A kitchen ratio table and a written equation can be named under mathematics. Home instruction does not yield a local or Regents diploma. Annual assessment: named commercially published tests or, in some grades, a written narrative. Composite above the 33rd percentile or one year of growth is a probation threshold in that regulation. It is not a goal this parent manual advertises as success.

If you live in none of those states, you still have a department page. Open it.

What a short check is

Four objects, not a battery.

1. **A dated notebook.** Date, skill named, what the student built or wrote, one

[76] Hours of middle-grades math instruction are not in a nationally representative subject-minutes table opened for this book. North Carolina's five-hour day is a recommendation across the day. Pennsylvania's 900/990 and New York's 900/990 are whole-program hours. There is no national math-minutes table in these pages. NCES has never published a nationally representative homeschool NAEP mathematics score. NAEP Grade 8 2024 average 272, 27 percent at or above NAEP Proficient — NAEP, not a homeschool census.

sentence of what happened. Under **Mathematics**, **Pre-Algebra**, or **Algebra I** if a title is needed. Mistakes visible. A stranger can see growth across weeks. Best samples for this band: a corrected multi-step equation page; a proportion with a ratio table or double number line; a simple linear-graph sketch; one short written reason ("why this slope is negative").

2. **An exit ticket.** Three to five items at the end of a math hour or week, one of them not today's new skill. Right, or wrong-with-a-reason you can use tomorrow, is enough. Not a percentage identity. Include at least one item that requires written reasoning so the notebook stays richer than an app score.

3. **One diagnostic item.** A single-page skill probe — fraction operations; integer operations; solve one- and two-step equations; proportional reasoning — used to place or to re-place midyear. Not a forty-item diagnostic. Keep one wrong answer you can reteach. That is the file of the habit.

4. **Optional publisher placement** (the organization sells the test). Beast Academy unit tests; AoPS "Are you ready?" pretests; Math-U-See yes/no questionnaire; Singapore / Dimensions placement guidance; Saxon skill-level tests; Math Mammoth placement as the publisher describes it; CTCMath topic diagnostics; Teaching Textbooks trial lessons. Use to start a purchased program, or to restart after a gap. IES does not certify them as randomised trials. They place by skill. They do not license a national percentile. They do not satisfy North Carolina unless the chosen test is a nationally standardized achievement test covering grammar, reading, spelling, *and* mathematics — a publisher math placement PDF generally is not that battery. They do not satisfy Virginia's fourth-stanine object by themselves.

Date the work, keep one wrong answer, leave the pencil in the student's hand while they try.[77]

[77] Erin A. Maloney et al., "Intergenerational Effects of Parents' Math Anxiety on Children's Math Achievement and Anxiety," *Psychological Science* 26 (2015): 1480–1488. School homework-help study (grades 1–2), not a middle-grades homeschool trial. Cited for the kitchen move: leave the pencil with the student.

What a short check is not

NAEP. A percentile promise. A homemade high-stakes battery. Terra Nova as the kitchen model. MAP as an identity. CLEP as a middle-grades diploma. AP as this band's default year object. SAT Subject Tests — discontinued, last international sittings June 2021. A convenience-sample homeschool scoreboard. This book does not reprint that fight.

Dual enrollment, AP, and credit-by-exam are foreshadow only — the middle-grades title you choose now is the noun a later high-school transcript will build on.

Before you title a year Algebra I

Check — at minimum — fluency with multi-digit operations; fraction, decimal, and percent conversions and operations; integer operations; simple one-step equations; plotting on a number line and a coordinate plane. Publisher readiness lists (Math-U-See for Algebra 1; AoPS Are-you-ready; others) are one articulation of that gate. Neither is a state law. A thirteen-year-old who fails the gate does **Pre-Algebra** or **Mathematics** honestly. An eleven-year-old who clears the gate may do Algebra I honestly. Age is a weak instrument. Skill is the instrument.[78]

Re-placement midyear is allowed. Spiral programs and mastery programs both produce students who look "behind" or "ahead" of a birthday grade. The short check's job is to notice. Pride is not a placement instrument.

If they transfer

Transfer is a local placement problem. A brand in the title column will not be read as Algebra I. Another reason the title should already look like Mathematics, Pre-Algebra, or Algebra I.

A receiving counselor who sees "Young Minds Math I, A" has been given a slogan. A counselor who sees "Algebra I — dated notebook; same-to-both-sides; nega-

[78] Algebra I by skill, not birthday: NMAP Critical Foundations first; Finding 15 continuity. Math-U-See published readiness lists for Pre-Algebra and Algebra 1; AoPS "Are you ready?" pretests — vendor tools, not state law. IES / WWC does not certify publisher placement tests as randomised trials.

tives on both sides rebuilt on a balance; check by substitution" has been given a year they can place.

Place by skill, not birthday

Can the student place -3/4 on a line? If not, they are still in the fractions-as-numbers chapter, whatever their age. Can they build a ratio table before they cross-multiply? If not, they are still in the meaning stage of ratio. Can they say what an expression is saying before they chase letters? If not, they are still in expressions. Can they do the same thing to both sides and say why? If not, they are still in equations. High-sounding *titles* cannot skip those moves. A fourteen-year-old may still need fraction magnitude. An eleven-year-old may already clear an Algebra I gate. Skill first. Publishers opened for this book say the same in their own placement language: start about a level behind; Are-you-ready pretests; skill levels rather than ages; Grade 6 is grade 6, not traditional pre-algebra.[79]

What to keep in the file

1. A dated notebook: date, skill named, what the student did, one wrong answer you will reteach. Under **Mathematics**, **Pre-Algebra**, or **Algebra I**.
2. Exit tickets from the week, not a crate of every worksheet. App gradebooks are useful parent reports; they are not a substitute for a work sample a Pennsylvania evaluator can hold.
3. One diagnostic item, labelled — including the Algebra I gate when that title is on the table.
4. Optional publisher placement, if you used one to start or restart a program — as a placement, not as a yearly IQ.
5. State objects that actually apply: a North Carolina annual mathematics-inclusive battery if you are in North Carolina; a Pennsylvania grade 8 mathematics portfolio sample if you are in Pennsylvania; a New York quarterly

[79] Beast Academy FAQ: levels 4 and 5 cover many middle-grades standards; recommend starting about a level behind; after 5D, background for AoPS Prealgebra. AoPS choosing-a-course handbook: Are-you-ready pretests; Introduction to Algebra for grades 6–9. Math-U-See: skill levels, not ages. Singapore Math Dimensions 6–8 for Homeschool: Grade 6 is a grade 6 text, not traditional pre-algebra; Grades 7 and 8 together cover Pre-algebra and Algebra 1 with some Geometry. Placement language accessed 4 September 2026.

> under Mathematics if you are in New York; a Virginia list that includes Mathematics if you write the list. Compliance stays in its lane.

Life of the habit, as the file you keep, lives here: date the work; name the skill; keep one wrong answer you can reteach. Revisit next Tuesday is a move in Chapter 8. The notebook is how Tuesday leaves a trace.

A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not.

Chapter 10

Resources

This chapter names programs families actually meet at ages 11–14. It does not rank them. It does not sell them. It is a fit list so you can match an object to the table you already have: how much of the teaching you will carry, whether a kitchen can use it tonight, what it costs in money and in minutes, and whether the student will still build a ratio table, place a signed number, write an equation, and explain a reason.

A "grade 7 math workbook" is a publisher's scope, not a legal grade. Placement is by skill. The record, as Chapter 9 said, still says **Mathematics**, **Pre-Algebra**, or **Algebra I** — only if the year's work was that course. Combining two honest objects does not mint a new title. Kitchen ratio table plus library book plus one lesson from a purchased program is still Mathematics.

Independent trials of these programs *in homeschool* remain scarce. What Works Clearinghouse ratings, where they exist, are school-side — not a kitchen bake-off. Choose by fit: parent load, style, budget, age or grade label, and whether the student will generate the work.

This week, name *why* tonight's object fits — usually a scrap-paper ratio table and a number line both ways. Today the student tries that table.

Figure 13: A scrap-paper ratio table, a number line in both directions, a dated notebook, a blank fit card (load / style / age label). No logos. No portraits.

How to read a program

Four questions do the work.

Parent load. Are you asking one question on a ratio table you already drew? Sitting with a scripted lesson every day? Learning teaching notes well enough to hear a wrong move? Booking a video as a second mouth?

Style. Ratio table and double number line? Worktext the student can read? Blocks and a video? Comics and puzzles? Spiral mixed practice? Short videos for several siblings? Problem-solving depth?

Cost shape. Nothing tonight. A book you already own. A print set whose dollars sat on a page this access day and will stale. Fees this book did not verify. Confirm a current price on the page before you buy.

Age or grade label. An eleven-year-old is not automatically Algebra I. A fourteen-year-old may still need fraction magnitude. Birthday is a weak prior. Publisher age bands and grade labels are starting hints; skill gates decide.

Kitchen table, nothing spent

A number line on tape or paper, both directions. A ratio table on scrap. A double number line on register tape. Signed chips of two colors. A drawn balance. A generic unit-price tag from a store you were already visiting. A map scale. A ramp or stairs as rise-over-run. A dated notebook. That is tonight.

Fit. One student, low cash cost, parent load moderate to high if you invent the sequence — moderate if a free course map or a borrowed text carries scope. The store you were already visiting is the same shape: a unit price as a number, never a sermon about a food system. The walk you were already taking is the same shape: an angle, a similar shadow, a scale. The library you already use is the same shape: a used algebra text plus one question — what stays in the same ratio? In 2016, 61 percent of grades 6–8 homeschool households used the library as a curriculum source. In 2022–23, 53 percent of the Homeschool-total row had visited a library in the past month, against 29 percent of all K–12.[80] Those are different questions in different years. Both are a free hour.

Transcript. Mathematics, Pre-Algebra, or Algebra I by content. Never Young Minds Math I. Never "Consumer Math 7." Never Outdoor Math 1.

The teaching chapters of this book are written for this object first. A purchased program is a fit, not a requirement.

Beast Academy Levels 4 and 5 — age-labelled

Comic-guide plus practice from Art of Problem Solving. Puzzle-hard. Parent load is moderate: the comics are readable; the hard problems need patience; some

[80] NCES 2020-001, Table 6: library as a source of curriculum and books, grades 6–8, 61 percent. NCES 2024-113, Table A-5: Homeschool-total 53 percent visited a library in the past month versus 29 percent of all K–12. Do not swap 53 for 61. Access date for URLs in these notes: 4 September 2026.

students need a parent as coach. The publisher's age labels, copied not rounded: Level 4 ages **9–12**; Level 5 ages **10–13**.[81] The publisher also says levels 4 and 5 cover many middle-grades standards and can be a good fit for early middle school. Recommend starting about a level behind current grade. After Level 5 / 5D, the student has mathematical background for AoPS Prealgebra. Guide + Practice units were priced about **$30** per unit on an opened store page (4 September 2026).[82] Confirm before you buy.

Age-label. This track is not a crown for gifted, and it is not the default kitchen hour. A fourteen-year-old who finished 5D moves to AoPS Prealgebra or Introduction to Algebra by skill — not by birthday. Do not title Beast Academy 5 as Algebra I.

Fit. A family whose student thrives on hard puzzles, sits in the publisher's age bands for Levels 4–5, and can hear a wrong answer in that style.

Transcript. Mathematics during mixed Level 5 work. Pre-Algebra only when the year's content is that course. Never "Beast Academy 5D" as the credit.

Art of Problem Solving Introduction series

Deep secondary-edge curriculum: Prealgebra; Introduction to Algebra (publisher: thorough introduction for students in grades **6–9**); Introduction to Geometry; Introduction to Counting & Probability; Introduction to Number Theory.[83] Parent load is moderate to high: hard problems; the parent may not spot-check quickly; an online class shifts load to an instructor. Style: problem-solving, proof-leaning, contest-adjacent. "Are you ready?" and "Do you need this?" pretests place by content experience, not by age restriction. A student finishing Beast Academy 5D is pointed toward Math 6 Prealgebra on the publisher's path.

Book prices on an opened store page (4 September 2026): Prealgebra text + solutions **$59**; Introduction to Algebra **$67**.[84] Confirm online tuition on the current

[81] Art of Problem Solving / Beast Academy FAQ and store list pages, accessed 4 September 2026. Ages: Level 4, 9–12; Level 5, 10–13. Levels 4 and 5 cover many middle-grades standards; recommend starting about a level behind; after 5D, background for AoPS Prealgebra.

[82] Beast Academy Guide + Practice unit prices ~$30 per unit on opened store page, 4 September 2026. Vendor prices at access; they will stale.

[83] AoPS Online choosing-a-course handbook and store, accessed 4 September 2026. Introduction series for grades 6–10; Introduction to Algebra for grades 6–9; Are-you-ready pretests.

[84] AoPS store: Prealgebra text+solutions $59; Introduction to Algebra $67, displayed 4 September 2026. Online tuition not extracted as a clean list that day.

page.

Fit. Problem-solving appetite; pretest placement; coach or online seat. If the year was Intro Algebra end-to-end, title **Algebra I** — not “AoPS.” If Algebra I was already granted and AoPS is for depth, say enrichment in the description; do not mint a silent duplicate credit.

Singapore Dimensions Math 6–8

Concrete–pictorial–abstract lineage, mastery within a level, problem variety. Parent load is moderate: textbooks are more student-facing at 6–8 than in the primary years; Teaching Notes still matter. The publisher’s own map, copied honestly: **Dimensions Math 6** includes some pre-algebra topics but is a Grade 6 text, not a traditional pre-algebra text. **Grades 7 and 8 together** cover Pre-algebra and Algebra 1 topics, with some Geometry. Grade 7 goes further into algebra than a traditional pre-algebra text; Grade 8 has greater expectations (especially linear equations) than a traditional Algebra 1 text.[85] After Grade 8, Geometry and Algebra 2 programs. Opened set prices (4 September 2026): Grade 6 Set **$166.00**; Grade 7 Set **$190.40**; Grade 8 Set **$190.40**.[86] Those are prices at access. They will stale. Confirm before you buy. Discovering Mathematics is a legacy secondary edge Dimensions 6–8 replaced; families who already own it may continue it.

The What Works Clearinghouse Singapore Math intervention report, December 2015, Primary Mathematics protocol, K–8: no studies of Singapore Math that fall within the scope of that protocol meet WWC group design standards. Because no studies meet WWC group design standards, the WWC is unable to draw any conclusions based on research about the effectiveness or ineffectiveness of Singapore Math.[87] Ratio tables, double number lines, and strip diagrams are representations. That sentence is not “Singapore works.”

[85] singaporemath.com Dimensions Math 6–8 for Homeschool, accessed 4 September 2026. Grade 6 is grade 6, not traditional pre-algebra; Grades 7 and 8 together cover Pre-algebra and Algebra 1 with some Geometry.

[86] Same page: Grade 6 Set $166.00; Grade 7 Set $190.40; Grade 8 Set $190.40, displayed 4 September 2026. Vendor prices at access; they will stale.

[87] What Works Clearinghouse, *Singapore Math*® intervention report, Primary Mathematics protocol, K–8, December 2015. Quoted finding: no studies within scope meet WWC group design standards; WWC unable to draw conclusions about effectiveness or ineffectiveness. Copy this finding whenever Singapore Math is named. Do not launder bar models into a curriculum endorsement.

Fit. Singapore-family secondary path; title years honestly — Grade 6 is not Algebra I; Algebra I only when Grade 8 (or equivalent) completes the algebra arc.

Transcript. Mathematics / Pre-Algebra / Algebra I by content. Never "Dimensions 7A."

Saxon Math 6/5–8/7 and Algebra 1/2

Incremental, spiral, mixed practice as the point. Parent load is moderate to high early in a course (lesson plus mixed practice), lower when the student is independent. Homeschool conversation commonly uses 6/5, 7/6, 8/7, then Algebra 1/2 or Algebra 1. Algebra 1/2 in older Saxon naming is a pre-algebra bridge — place by skill; do not auto-title it Algebra I without reading the contents against a gate. 2026 kit list prices were not cleanly extracted for this book — confirm on the current vendor page.[88]

School-side, labelled school, not a kitchen trial: the What Works Clearinghouse Saxon Math report, May 2017, found **mixed effects**, five studies all *with reservations*, grades **1–3 and 6–8**, average improvement index **+8** percentile points (range −**1 to +16**).[89] That is mixed, school, not home. Publisher ESSA language is school-implementation evidence from the organization that sells the program, not a homeschool randomised trial.

Fit. A family that wants spiral mixed practice and can live with review that returns last week's skill.

Transcript. Pre-Algebra or Algebra I by content. Never "Saxon 8/7" as the credit.

Math-U-See Pre-Algebra and Algebra 1

Build, Write, Say — still live at this edge. Integer blocks and Algebra/Decimal inserts named for Pre-Algebra; video plus worktext. Parent load is moderate if the

[88] Saxon Math homeschool naming (6/5–8/7, Algebra 1/2) from catalogue consensus; 2026 HMH homeschool kit dollars not cleanly extracted — confirm current vendor price. HMH school-side ESSA language is not a homeschool RCT.

[89] What Works Clearinghouse, *Saxon Math* intervention report, May 2017, Primary Mathematics protocol. Mixed effects; 5 studies all with reservations; grades 1–3 and 6–8; improvement index average +8 (range −1 to +16). School-side.

video carries the first explanation; you still need to hear a wrong integer move or a letter-chase with no structure. Sequence uses skill levels, not ages: Zeta (decimals and percents) → **Pre-Algebra** (negative numbers, order of operations, solving for the unknown) → **Algebra 1** / Algebra 1: Principles of Secondary Mathematics. Interactive yes/no placement tool. Publisher readiness lists emphasize fraction/decimal/percent fluency and one-step equations — skill gates, not birthdays.[90] 2026 kit dollars were not extracted as a clean table — confirm on the current page.

Fit. A family that wants mastery, blocks still on the table at Pre-Algebra, and a video as a first mouth.

Transcript. Pre-Algebra or Algebra I. Never "Math-U-See." Never "Zeta."

Teaching Textbooks, CTCMath, and Khan Academy

Teaching Textbooks. Homeschool math app; vendor claims include graded problems, step-by-step audiovisual solutions, tutor helpline, offline work for a short stretch of lessons, daily parent email. Free trial: first lessons of a level. Parent load is **low** — the vendor's main pitch. Homepage did not display a clean 2026 price table on the day opened for this book; confirm current price.[91] Style: screen, often described in catalogues as a gentler pace — relevant for later transfer; keep a parallel skill diagnostic in the notebook so placement is not a surprise. Transcript: Mathematics / Pre-Algebra / Algebra I by content — never "TT" as the credit.

CTCMath. K–12 video tutorials, interactive questions, worksheets and solutions, reports, topic diagnostics. Parent load is **low** if the student can watch and work; parent reads reports. Homepage showed no 2026 list price on the day opened; confirm current price.[92] Modular topics help catch-up; they make year titles harder — the parent must still write a coherent title based on dominant work. Transcript: not "CTCMath Grade 7" as the credit.

[90] Math-U-See curriculum, placement, Pre-Algebra, and Algebra 1 / Principles of Secondary Mathematics pages, accessed 4 September 2026. Skill levels; readiness lists; Integer Block Kit and Algebra/Decimal Insert Kit named for Pre-Algebra. List prices not extracted as a clean table — confirm current page.

[91] teachingtextbooks.com homepage, accessed 4 September 2026: app features and free trial described; no clean public price table on the fetched view that day.

[92] ctcmath.com homepage, accessed 4 September 2026: K–12 video, interactive questions, reports; list price not displayed that day.

Khan Academy. Free K–12 courses and practice maps. Parent load is low to moderate; you must build the record. Cost: free. Efficacy language on About pages is organization copy, not a homeschool RCT opened as a PDF for this chapter.[93] Do not treat Khan alone as a complete program of record unless you date the notebook and title the year honestly. COPPA still applies for ages 11–12; parent holds the account. At 13–14 the parent still co-holds and stays in the room — turning 13 is not a licence. Transcript: Mathematics / Pre-Algebra / Algebra I by content. Never “Khan” as the credit.

Math Mammoth

Complete worktext curriculum through middle grades (Light Blue series); the publisher discusses Algebra 1 options after grade 7 or 8. Parent load is low to moderate if the student can read the page. A live fetch of the complete-curriculum page timed out on the research day for this book; details beyond the search index are not claimed as opened HTML.[94] Placement tests are described in publisher materials. Confirm the current page and price before you buy.

Fit. A reader who likes a worktext that teaches to the student. Algebra I is a next-step recommendation, not automatic with birthday.

Transcript. Mathematics / Pre-Algebra / Algebra I by content. Never “Math Mammoth 7.”

Other school programs you will see named

These are school-side What Works Clearinghouse ratings, labelled school, copied not rounded, not a reason to crown a franchise at the table.[95]

[93] Khan Academy About page pattern (sibling streams / prior lock): free K–12 courses. Organization efficacy copy is not a homeschool RCT opened as a PDF for this chapter. COPPA: parent holds account ages 11–12; 13–14 co-holds, still parent in the room.

[94] Math Mammoth Light Blue: search-indexed publisher description of complete worktext through middle grades; live fetch of complete-curriculum HTML timed out on the research access day. Confirm current page and price.

[95] WWC intervention reports: Saxon Math (May 2017) mixed, +8 (−1 to +16), grades 1–3 and 6–8; Connected Mathematics Project (February 2017) no discernible effects, grades 6–8, +2 (0 to +4); Odyssey Math (January 2017) potentially positive, grades 4–8, +12 (+1 to +18); Singapore Math (December 2015) no studies meet group design standards. All school-side. None is a parent-

Saxon Math — mixed effects, grades 1–3 and 6–8, improvement index average **+8** (range −**1 to +16**). Named again so the school label stays attached.

Connected Mathematics Project (CMP) — no discernible effects for grades 6–8 studies meeting standards with reservations; improvement index average **+2** (range 0 to +4). Problem-based lessons are not automatically "what works."

Odyssey Math — potentially positive, grades 4–8, improvement index average **+12** (range +1 to +18). Vendor software path does not replace the parent's named moves.

Singapore Math, named again so the sentence stays attached: WWC December 2015, **no studies meeting group design standards**; the WWC is unable to draw conclusions about effectiveness or ineffectiveness. Bar models remain a usable representation. That is not a curriculum endorsement.

None of these is a parent-at-home randomised trial. None is a kitchen winner.

Extra help, age-labelled — school maths, not a kitchen promise

Named English *maths* trial only, copied honestly, labelled by year group / age, as an analogy for coherent focus — not as a ranking of kitchen curricula, and not as "+1 month at your table."

Ark Mathematics Mastery **Secondary**, Year 7 (~age **11–12**): **+1 month**, **4 of 5 padlocks**, 7,712 pupils, 40 schools, on the Education Endowment Foundation project page.[96] Whole-school English secondary programme in a first adoption year. Portable morals only: coherent focus on fewer topics with depth can move attainment modestly in school trials; systematic language and representations appear as design features — consistent with themes you already use (ratio table, number line, same-to-both-sides), not proof of a home franchise.

Philosophy for Children is **not** a mathematics program; its attainment results are not copied here as math findings.[97]

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at-home RCT.

[96] Education Endowment Foundation, Ark Mathematics Mastery Secondary project page: Year 7 (~age 11–12), +1 month, 4/5 padlock, 7,712 pupils, 40 schools. School programme; not a kitchen promise. Named *maths* trial only.

[97] EEF Philosophy for Children attainment trials (including effectiveness findings reported in

When you are the ceiling, a tutor, co-op algebra hour, district à la carte seat, or video as a second mouth is ordinary (NCES 2016 grades 6–8: tutor 29%; co-op 29%).[98] Unless the outside provider issues a transcript, you date the notebook. Title remains Mathematics, Pre-Algebra, or Algebra I.

Placement tests are tools, not percentiles

Beast Academy unit tests; AoPS Are-you-ready; Math-U-See yes/no; Dimensions / Singapore placement guidance; Saxon skill-level; Math Mammoth placement as described; CTCMath topic diagnostics; Teaching Textbooks trial lessons. IES does not certify them as randomised trials. They place by skill. They do not license NAEP. They do not write Young Minds Math I. Use one to start a purchased program or to restart after a gap, as Chapter 9 said.

A student of thirteen may place into three different "grades" on three tests the same afternoon. That is three grains of "grade," not a contradiction. Start where the student can succeed, then add. Algebra I by skill, not birthday.

A short chooser, not a ranking

- If you want nothing spent tonight: ratio table, number line both ways, dated notebook, library text. Title by content.
- If you want puzzle comics in the publisher's middle-adjacent bands: Beast Academy Level 4 (ages 9–12) or Level 5 (ages 10–13); after 5D $\rightarrow$ AoPS Prealgebra by pretest. Title: Mathematics (or Pre-Algebra when content matches). Never BA as the credit.
- If you want problem-solving depth: AoPS Prealgebra / Introduction to Algebra, Are-you-ready first. Title: Pre-Algebra or Algebra I by content.
- If you want a Singapore-family secondary path: Dimensions Math 6–8. Grade 6 is grade 6; 7+8 together $\approx$ Pre-algebra + Algebra 1. WWC Singapore Math, December 2015: no studies meet group design standards. Title honestly across years.

months on reading, with no evidence treated here as a maths programme result) are not mathematics programs and are not copied as math findings.

[98] NCES 2020-001, Table 3: grades 6–8 — mother as main provider 82 percent; any tutor 29; any co-op 29; formal curriculum 75. 2023 First Look did not republish these tables.

- If you want spiral mixed practice: Saxon 6/5–8/7 $\rightarrow$ Algebra by skill. School-side WWC mixed, +8 (-1 to +16), grades 1–3 and 6–8.
- If you want mastery, blocks, and video: Math-U-See Pre-Algebra / Algebra 1 after the readiness list.
- If you want low parent load on a screen: Teaching Textbooks or CTCMath — confirm current price; keep paper samples for portfolio states.
- If you want free practice: Khan — you still write the record; parent holds or co-holds the account.
- If you want a worktext a reader can use: Math Mammoth, confirm current page.

None of those bullets is "best." Fit is whether you can hear a wrong answer in that program's pictures, and whether the student will generate the work. When the student has unfinished fraction magnitude, go back, whatever the calendar says. When you are the ceiling, buy a tutor, a co-op seat, or a video as a second mouth. When the destination is a stranger with a page in hand, title the year Mathematics, Pre-Algebra, or Algebra I — the mathematics, not the franchise.

A path through middle-grades mathematics is the promise. A diploma is not. A percentile is not. A catalogue star is not a finding.[99]

[99] No homeschool-curriculum randomised trial comparing these named programs was opened for this book. Catalogue reviews are not trials.

A Note on Sources

Studies named in the chapters are listed in Notes, in one series at the back. That is where the full citations live, so the teaching pages can stay a teaching voice.

Some items the research behind this book did not open stay out of the teaching voice. I did not invent a coefficient, a statute, or a product feature to fill a hole. If a study is in the chapter, it is in the notes. If we could not open it, it is not used as a finding here.

Program hours, credit claims, and product descriptions in the resources chapter are taken from the companies' own pages. Access date 4 September 2026.